Systematics of alpha decay hindrance factors in doubly-even nuclei

Sushil Kumar\(^a\), Sukhjeet Singh\(^a\), Balraj Singh\(^b\) & A K Jain\(^c\)

\(^a\)Department of Physics, Akal University, Talwandi Sabo 151302, India
\(^b\)Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada
\(^c\)Department of Physics, Amity Institute of Nuclear Science and Technology, Noida 201 313, India

Received 17 February 2020

In present work, we have calculated the hindrance factors of 182 even-even alpha emitters using Preston’s formulation of alpha decay probabilities and presented HFs systematics of 1\(^+\) and 3\(^+\) states in reflection asymmetric even-even quadruple-octupole deformed nuclei (A=216-230). The calculated HFs of both 1\(^+\) and 3\(^+\) states decrease with reduction in neutron number and this decrease is attributed to onset of intrinsic reflection asymmetry. There is a trend reversal for 1\(^+\) states at N=132 (\(^{218}\)Ra) and N=134 (\(^{220}\)Rn), which might be a possible indication of departure from static octupole deformation. Similarly, HFs systematics is discussed for \(^{224,226}\)Th and \(^{232,234}\)U isotopic chains along with 2\(^+\) states observed in daughter nuclei in N=132-146 isotonic chains.

**Keywords:** Even-even alpha emitters, Hindrance factors, Reflection asymmetry

1 Introduction

Alpha hindrance factor (HF), the ratio of experimental to theoretical partial half–lives of alpha transitions, is found to be an important tool in extracting nuclear structure information\(^1\)\(^-\)\(^4\). Various theoretical techniques have been developed in order to understand the alpha–decay process and hence to calculate the penetrability of alpha particles through a barrier\(^1\)\(^,\)\(^5\). The alpha transitions for which HF lies between 1 and 4, called favored transitions and take place between nuclear states having similar configurations and hence it is promising to ascertain both J\(^\pi\) and nucleonic configurations assignments for a given daughter (parent) state if those of parent (daughter) are known\(^6\). Similarly, the alpha HFs quantifies the correlation between nuclear wave functions of the initial state of parent and final state of daughter nuclei; larger wave function’s overlap gives a lower HF\(^3\). The systematics of alpha-decay HFs can be used to deduce a variety of quantities like total alpha branching ratio, intensities of unobserved alpha groups and excitation energy of level in daughter nucleus\(^5\). In the present study, the spin-independent part of Preston’s equations\(^1\) have been used for the calculations of alpha decay probabilities. This formalism contains radius of the daughter nuclide, r\(_0\), as a free parameter. By setting HF=1 for the ground-state to ground-state alpha branch for an even-even nuclide\(^2\), the radius parameter for the daughter nuclide can be deduced\(^7\) that can be used to deduce alpha hindrance factors for alpha-fed excited states in even-even nuclides. We have calculated HFs of 182 even-even alpha emitters in the framework of Preston’s formulation\(^1\) by using ALPHAD_RadD program\(^8\) and present the systematics of alpha HFs with daughter neutron number in reflection asymmetric (RA) mass region (A=216-230) i.e. for quadruple-octupole deformed nuclei. Additionally, the systematics of HFs for 2\(^+\) states observed in N=132-146 isotonic chains is also presented

2 Methodology

In the present study, a well established Preston’s spin-independent formulation\(^1\) with only essential steps described here, has been used. In this formulation, a preformed alpha particle is considered to be moving inside a nucleus having rectangular potential field of depth of V\(_0\); V\(_0\) = constant for \(r < r_0\) where \(r\) is the distance from the center of the product nucleus and \(r_0\) is the radius of the product nucleus. The field beyond effective nuclear radius was assumed to be generated by a Coulomb potential (2Ze\(^2\)/r, where Z is the atomic number of product nucleus and e is the elementary charge) between alpha particle and daughter nucleus. The solution of the time dependent Schrödinger equation is assumed to have a form\(^1\):

\(^a\)Corresponding author (E-mail: sukhjeet.dhindsa@gmail.com)
\[ u = \psi(x, y, z) \exp(-iEt/\hbar) \]

The wave-function \( \psi \) should obey following boundary conditions (a) at \( r = 0 \) at \( \psi \) is finite, (b) at \( r = r_0 \), \( \psi \) and \( d\psi/dr \) are continuous, (c) \( \psi \) represents as outgoing wave for \( r > r_0 \). The Schrödinger equation for alpha particle in afore said potential field can be written as:

\[
\frac{d^2X_i}{dr^2} + \frac{2m}{\hbar^2} \left[ E - V - \frac{\hbar^2l(l+1)}{2mr^2} \right] X_i = 0
\]

where, \( E = E_\alpha - \frac{1}{2}ih\lambda \); \( E_\alpha \) is the energy of \( \alpha \)-particle and \( \lambda \) the time constant; are complex eigen values. In the interior of the nucleus with \( V=U \), we have

\[
X_i^f = \left\{ \frac{2m(E-U)}{\hbar^2} \right\}^{1/2} r^{l+1/2} J_l^{1/2} \left\{ \frac{2m(E-U)}{\hbar^2} \right\}^{1/2} r
\]

where, \( J \) denotes Bessel functions and superscript \( f \) refer to interior of the nucleus. The solution \( X_i^f \) represents a standing wave and imaginary part of \( E \) related to the leak of alpha particle through the potential barrier.

3 Input Parameters

In the present work, the daughter radius parameter \( r_0 \) is the main input used to calculate the HFs of alpha-fed excited states in even-even nuclides. In order to deduce \( r_0 \), a set of experimental quantities such as \( Q_\alpha \) value, parent nuclide's half-life (\( T_{1/2} \)), total alpha–decay branching ratios (\( \%\alpha \)), and alpha intensities (\( I_\alpha \)) are used. The \( Q_\alpha \) energies are taken from recent atomic mass evaluation of M. Wang et al., total alpha-decay branching ratios and half-lives of parent nuclides are taken from the ENSDF database supplemented by recent data from literature.

In order to calculate HFs of various excited states, the recently developed ENSDF analysis code namely, ALPHAD_RadD, which is based on Preston's equations for alpha decay transition probabilities, has been used. This program can also be used to deduce HFs in odd-\( A \) and odd-odd nuclei by employing recently evaluated list of even-even daughter radius parameters.

4 Results and Discussion

In Reflection Asymmetric even-even quadruple-octupole deformed nuclides (\( A \sim 216-230 \)), two separate bands with opposite parity i.e. \( \Gamma = 0^+, 2^+, 4^+ \ldots \) and \( \Gamma = 1^-, 3^-, 5^- \ldots \) are generally observed. In present paper, we studied the systematics of alpha HFs for \( 1^- \) and \( 3^- \) states with daughter neutron number in above said RA mass region (\( A \sim 216-230 \)). The results of HF systematics of \( 1^- \) and \( 3^- \) states observed in \( ^{218-226} \text{Ra} \) isotopic chains with daughter neutron number are presented in Fig.1(a). From this figure, it is clear that, the HFs of both \( 1^- \) and \( 3^- \) states smoothly decreasing with reduction in neutron number. This decrease of HFs is attributed to onset of
static Quadruple-Octupole deformation, but there is a trend reversal at N=132 \(^{218}\text{Ra}\), which might be a possible indication of departure from static Quadruple-Octupole deformation\(^3\).

Similar trend reversal in HFs systematics of \(^1\) state at N = 134 is also observed for \(^{218-222}\text{Rn}\) nuclides (Fig.1 (b)), but such reversal could not be observed in \(^{224-230}\text{Th}\) (Fig.1(c)) as experimental data for \(^1\) state in lower mass region is not accessible. The HF systematics of \(^1\) state in \(^{232-236}\text{U}\) nuclides (Fig.1(d)) shows a minimum at N=142 and may be due to similar shape transition as discussed for Ra and Rn isotopic chains.

Additionally, we have also presented the systematics of HFs of \(^2\) states observed in N=132-146 isotonic chains as shown in Fig. 2(a-i). From Fig. 2(a-c), a smooth decrease in HFs of \(^2\) states with increase in proton number is observed till Z=88 in N=130, 132 and till Z=90 in N=134 isotopic chain. This smooth decrease in HFs indicates more probable alpha penetration through coulomb barrier and hence could be attributed to decrease in stability beyond Z=82 shell closure. The abrupt increase of HF at Z=90 in N=130, 132 and at Z=92 in N=134 isotopic chain is still an open problem. Although there is increase in HFs in other isotonic chains (N=136-146) as shown in Fig. 2(d-i), but minima corresponding to certain Z number could not be identified due to inaccessibility of experimental data for these isotonic chains. On the basis of above systematics (Fig. 2(a-i)), we suggest that all the experimentally observed alpha decays branches (\(^0\) to \(^2\)) are favored.

5 Conclusions

The HFs systematics of \(^1\) and \(^3\) states observed in RA mass region is presented. The HFs of both \(^1\) and \(^3\) states decrease with reduction in neutron number and this decrease of HFs could be attributed to onset of intrinsic reflection asymmetry, but there is a trend reversal for \(^1\) state at N=132 \(^{218}\text{Ra}\) and N=134 \(^{220}\text{Rn}\), which might be a possible indication of departure from static octupole deformation. A smooth decrease in HFs of \(^2\) states with proton number is observed N=130, 132 and N=134 isotonic chains, which indicates the enhanced alpha penetration probability beyond Z=82 shell closure.

Fig. 2(a-i) — HF’s systematics of \(^2\) states observed in daughter nuclei in N=132-146 isotonic chains.
Acknowledgement

The financial support from DAE-BRNS, Govt. of India (Sanction No. 36(6)/14/60/2016-BRNS/ 36145) is gratefully acknowledged.

References
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