

Yarn strength behaviour at different gauge lengths

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A theoretical model has been developed for predicting yarn strength at different gauge lengths as a summation of two mutually independent stationary, ergodic, Markovian and Gaussian stochastic processes and then experimentally verified with different cotton yarns produced from different spinning technologies. A new methodology to measure yarn strength at a gauge length longer than that of the longest fibre in yarn has been devised and special data evaluation techniques developed. With this, it is possible to obtain a new characterization of yarn strength as well as to predict actual yarn strength behaviour at different gauge lengths. It is experimentally observed that the strengths of neighbouring short sections along a yarn are correlated and this correlation is different in different yarns. Depending on the degree of this correlation, the empirical equations relating yarn strength and gauge length are found to be different in different yarns.

Keywords: Autocorrelation function, Cotton, Gauge length, Stochastic process, Theoretical model

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1 Introduction

It is a common practice among the textile technologists to perform only standard measurement of yarn strength at 500 mm gauge length and based on this result, a few basic statistical parameters are calculated. These characteristic parameters are generally used to compare strength between yarns. However, while comparing strength between the yarns, we often neglect a fact that two yarns can show same strength in the standard test, and may possess very dissimilar strength properties at other lengths¹. Moreover, the importance of understanding the yarn strength behaviour at different gauge lengths can be further appreciated by considering the post spinning and subsequent technological operations, where yarns are stressed at different lengths. Hence, the strength measurements carried out only at 500 mm gauge length are not sufficient. Therefore, it is necessary to determine yarn strength behaviour at different lengths. However, due to technical limitations, it is not possible to perform strength measurements at very long lengths by using today's tensile testers. This difficulty can be overcome by using the following well-known empirical relations of strength and strength variability between long and short specimens, as obtained by Peirce²:

$$\bar{S}^* = \bar{S} + 4.2\sigma_s \left[\left(l/l_0 \right)^{-1/5} - 1 \right], \quad \dots (1)$$

$$\sigma_{S^*} = \sigma_s \left(l/l_0 \right)^{-1/5}; \quad \dots (2)$$

where $\bar{S}^*$ and $\bar{S}$ are the mean values of strength measured at gauge lengths l and l_0 ($l > l_0$) respectively; and σ_{S^*} and σ_s , the standard deviations of strength measured at gauge lengths l and l_0 respectively. But these relations are not precise^{3,4}. From time to time, the textile researchers tried to investigate the causes of these deviations. Knox and Whitwell⁵ observed that Peirce's hypothesis of the weakest-link theory was not very appropriate on cotton yarns; however, they did not completely deny this hypothesis. Kapadia⁶, Kaushik *et al.*⁷ and Hussain *et al.*⁸ accepted the existence of the weakest-link effect on staple fibre yarns. Peirce's another assumption of Gaussian (normal) distribution of yarn strength at short gauge length was also critically studied by several researchers. Kapadia's observation⁹, based on some statistical tests on a huge number of experimental data, showed that the heterogeneous variation in yarn caused inadequacy to fit the experimental strength distributions by the normal curve. But when corrections were made for count by using count-strength product, the distributions were adequately represented by the

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Pearson type of curves. On the other hand, based on other statistical tests, Truevtsev *et al.*¹⁰ found no reason to deny Gauss's law for describing the yarn strength distribution. A similar conclusion was earlier documented by Pozdniakov¹¹ and Perepelkin¹². According to Peirce's third and last assumption, the strength of one section in yarn does not influence the strength of the neighbouring sections. In opposite to this, Spencer-Smith¹³ hypothesized that the adjacent fracture zones, where the actual breaking takes place in yarn, were related; though no attempt was reported to verify this experimentally. After Peirce, a few studies^{6,8,14} were conducted to investigate the relation between strength and gauge length in yarns but those were empirical only. Hence, there is a need to find out a new scientific way to predict actual yarn strength behaviour at different gauge lengths.

In the present work, an original theoretical model for predicting yarn strength at different gauge lengths has been developed and verified with a lot of different technological yarns. Here, first the model is presented very briefly and then its experimental verification is discussed with a few yarns.

2 Theory

2.1 Stochastic Process of Yarn Strength

Let us divide a randomly selected long length of yarn into several successive short sections of equal length l_0 , as shown schematically in Fig. 1. These sections are designated by the serial numbers $i=1,2,3,\dots,k,\dots$. Each i^{th} section possesses some value of strength S_i . These values are found to depend on the serial number of section i . This procedure can be independently repeated many times from the other places of the same yarn. In all repetitions, however, a section having the same serial number can be found. Therefore, S_i values, where $i=1,2,\dots$, are stochastic variables that constitute a stochastic process with a discrete argument i . Each of such repetition is called as a realization of the stochastic process. It is also possible to describe a general section by its distance x from section No. 1 (Fig. 1) as $x=l_0(i-1)$, thereby the discrete parameter x is another argument of the stochastic process.

Consider this stochastic process as a summation of two independent stationary, ergodic, Markovian and Gaussian (SEMG) stochastic processes, as defined below:

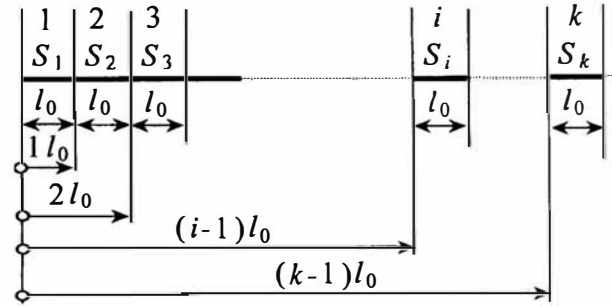


Fig. 1—Schematic diagram of a long yarn divided into equal shorter sections [S_i —strength of the i^{th} section of length l_0 , situated at a distance of $(i-1)l_0$ from the first section]

$$S_i = {}^{(1)}S_i + {}^{(2)}S_i, \quad \dots (3)$$

where the top left superscript represents individual stochastic process. The above process is usually described by the following statistical and probability characteristics.

Statistical Characteristics of the SEMG-Stochastic Process

The statistical characteristics of this process are usually as follows:

Mean:

$$\bar{S} = \bar{{}^{(1)}S} + \bar{{}^{(2)}S}, \quad \dots (4)$$

Variance:

$$\sigma_s^2 = {}^{(1)}\sigma_s^2 + {}^{(2)}\sigma_s^2, \quad \dots (5)$$

Standard deviation:

$$\sigma_s = \sqrt{\sigma_s^2}, \quad \dots (6)$$

Autocovariance:

$$\text{cov}(S_i, S_{i+k}) = \text{cov}_s(k) = {}^{(1)}\sigma_s^2 {}^{(1)}r^k + {}^{(2)}\sigma_s^2 {}^{(2)}r^k, \text{ and} \quad \dots (7)$$

Autocorrelation coefficient:

$$\rho(S_i, S_{i+k}) = \rho_s(k) = \frac{{}^{(1)}\sigma_s^2}{{}^{(1)}\sigma_s^2} {}^{(1)}r^k + \frac{{}^{(2)}\sigma_s^2}{{}^{(2)}\sigma_s^2} {}^{(2)}r^k. \quad \dots (8)$$

The step-by-step derivations of these expressions can be found in Neckář's book¹⁵.

Probability Characteristics of the SEMG-Stochastic Process

The distribution of strengths S_i is usually described by the probability density function $f(S_i)$

of the Gaussian (normal) distribution, as shown below:

$$f(S_i) = (1/\sqrt{2\pi}\sigma_s) \exp\left\{-\left(S_i - \bar{S}\right)^2 / 2\sigma_s^2\right\}. \quad \dots (9)$$

This distribution has two parameters, namely mean value $\bar{S}$ and variance σ_s^2 . The probability density function of strengths S_{i+1} of the next section, when strengths S_i of its previous section are given, is found from the conditional probability density function $\varphi(S_{i+1}|S_i)$, as given below:

$$\varphi(S_{i+1}|S_i) = \left(1/\sqrt{2\pi}\sigma_s\sqrt{1-r^2}\right) \exp\left\{-\left(S_{i+1} - [\bar{S} + r(S_i - \bar{S})]\right)^2 / 2\sigma_s^2(1-r^2)\right\}, \quad \dots (10)$$

where $r = \rho(S_i, S_{i+1})$. The conditional probability density function also follows Gaussian distribution with mean value $[\bar{S} + r(S_i - \bar{S})]$ and variance $\sigma_s^2(1-r^2)$. Using these two functions and by applying some basic relations from the theory of probability, it is possible first to know the probability density function $f(S_i, S_{i+1})$ of strengths S_i and S_{i+1} of any two successive sections, and then by generalizing it, one can obtain the probability density function $f(S_i, S_{i+1}, S_{i+2}, \dots, S_{i+k})$ of strengths $S_i, S_{i+1}, S_{i+2}, \dots, S_{i+k}$ of any k successive sections. Finally, the probability density function $f(S_i, S_{i+k})$ of strengths S_i and S_{i+k} of any two sections can be obtained. These derivations are shown in Neckář's book¹⁵.

Using the above characteristics, it is possible to perform a huge number of realizations of the SEMG-stochastic process by a suitable simulation technique within a very short time.

Realizations of the SEMG-Stochastic Process

The statistical characteristics $\bar{S}$, σ_s and r can be obtained from the actual (experimental) strength results corresponding to the short gauge length l_0 . Using these parameters, a huge number of strength values of adjacent sections of short gauge length can

be generated by using a suitable computer simulation program under the assumption that the yarn strength distribution corresponding to the short gauge length l_0 is Gaussian. In this way, we obtain a dataset $\{S_i, S_{i+1}, \dots, S_{i+k}\} \equiv \{S_{i+j}\}_{j=0}^{j=k}$ containing strengths of $k+1$ successive sections of length l_0 , which altogether form a longer section of length l , such that $l = l_0(k+1)$. Then, by applying the weakest-link principle² on the above dataset, one can obtain strengths S^* corresponding to length l , as shown below:

$$S_i^* = \min\{S_{i+j}\}_{j=0}^{j=k}. \quad \dots (11)$$

Thus, a huge number of strength values S_i^* can be generated, and the probability density function $g(S^*, k)$ and the statistical characteristics of yarn strength S^* at a particular value of k can be obtained. The same technique can be repeated for different values of $k \in \{0, 1, 2, \dots\}$, and the probability density functions $g(S^*, k)$ and the statistical characteristics can also be evaluated for different values of k , i.e. for different gauge lengths $l = l_0, 2l_0, \dots$. One can use interpolation technique if it is necessary to find strengths corresponding to a length that is not a whole number multiple of l_0 .

3 Materials and Methods

3.1 Materials

A variety of yarns with varying linear density produced from different varieties of cotton fibres using different spinning technologies was used for the experimental verification of the presented theoretical model. The study dealt with the three different varieties of cotton yarns, namely three combed ring yarns (10 tex, 14.5 tex, and 20 tex), three carded ring yarns (20 tex, 25 tex, and 29.5 tex), and three rotor yarns (20 tex, 35.5 tex, and 42 tex).

3.2 Methods

3.2.1 Special Measurement of Yarn Strength at Short Gauge Length

A special methodology was developed to measure yarn strength at gauge length $l_0 = 50$ mm. No fibre could be clamped by both jaws of a tensile tester. This methodology can be understood from Fig. 1. Let us mark the successive short sections, each of 50 mm

(l_0) length by using the serial numbers 1,2,3,...,60, and then measure the strength of the sections marked by the numbers 1,3,5,...,59, the remaining sections will be used for clamping. Thus, we will have the strength of 30 alternate sections along the yarn. This procedure can be repeated 30 times at different places of the yarn randomly chosen from 6 packages. As a result, a time series containing 900 strength values will be obtained. This methodology was realized by attaching a special device to the Instron tensile tester (Model 4412) for semi-automatic feeding of the yarn in between the jaws of the tensile tester.

3.2.2 Conventional Measurement of Yarn Strength at Other Gauge Lengths

In accordance with the conventional strength measurement technique, three hundred strength measurements, each at different gauge lengths (100, 200, 350 and 500 mm), were performed with different yarns.

4 Results and Discussion

4.1 Descriptive Statistical Parameters of Actual Yarn Strength

The basic statistical strength parameters were obtained from the actual measurements carried out at different gauge lengths with different yarns. The results are presented, however for brevity, only with 25 tex cotton carded ring yarn (Table 1). Evidently, the mean value and the coefficient of variation of yarn strength decrease with the increase in gauge length. Similar trend was also observed with the other yarns¹⁶. This phenomenon, as explained earlier by several researchers^{2,6-8}, is a result of the weakest-link effect².

4.2 Frequency Distributions of Actual Yarn Strength

The strength values corresponding to 50 mm gauge length were standardized according to $U_i = (S_i - \bar{S})/\sigma_s$ and the strength values corresponding to other gauge lengths were transformed according to $U_i^* = (S_i^* - \bar{S})/\sigma_s$. Here, $\bar{S}$ and σ_s are related to gauge length $l_0 = 50$ mm, but S_i^* is related to gauge length l . The frequency distributions of these quantities related to three different gauge lengths with 25 tex cotton carded ring yarn are shown in Fig. 2 together with the probability density function of the standardized Gaussian distribution as expressed by Eq (9). It can be observed

that as the gauge length increases, the shape of the histogram becomes higher and narrower. This is because of the decrease in strength variability with the increase in gauge length. The relative shifting of the histogram to the left hand side direction at higher

Table 1—Descriptive statistical parameters of actual strength of 25 tex cotton carded ring yarn

Gauge length mm	50	100	200	350	500
Mean value cN/tex	14.37	13.72	13.82	13.14	13.25
Coefficient of variation, %	9.26	8.83	9.12	8.01	7.66

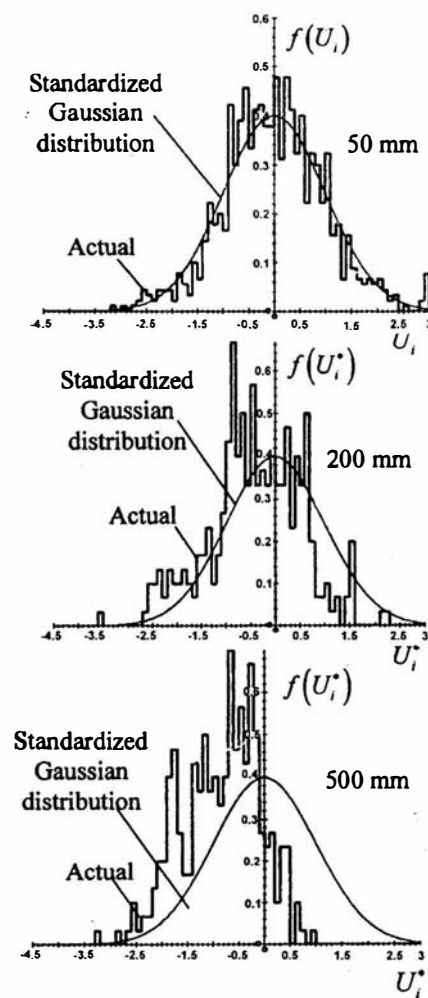


Fig. 2—Frequency distributions of the strength of 25 tex cotton carded ring yarn measured at different gauge lengths [U_i and U_i^* — standardized and transformed strength quantities respectively, and $f(U_i)$ and $f(U_i^*)$ — corresponding probability density functions]

gauge length is ascribed to the decrease in mean strength value with the increase in gauge length. Similar observations were found with the other yarns¹⁶.

To investigate whether the actual strength data of short specimens (50 mm) follows the theoretical normal distribution, the quantile-quantile plot was used as a diagnostic tool¹⁷. This plot compares the empirical quantiles Q_E with the theoretical quantiles Q_T . Here, the empirical quantiles Q_E correspond to the quantiles of orderly (ascending order) arranged actual strength data (without standardization) and the theoretical quantiles Q_T correspond to the quantiles of the standardized normal distribution. This plot for 25 tex cotton carded ring yarn is shown in Fig. 3, where the straight line is indicative of what the strength data would look like if it were perfectly normally distributed. It is evident that the strength data with 25 tex carded ring yarn can be reasonably regarded as a sample from a population following normal distribution. Similar conclusion can be drawn from the similar results with the other yarns¹⁶. Therefore, the assumption of Gaussian distribution in the present model is justifiable.

4.3 Autocorrelation Characteristics of Actual Strength of Short Specimens

The autocorrelation coefficients $\rho_s(x)$ were calculated for different distances (x) from the actual strength data corresponding to 50 mm gauge length. These coefficients with 25 tex cotton carded ring yarn are presented in Fig. 4. It is observed that the strengths of neighbouring short sections in this yarn are correlated. This was evident with the other yarns¹⁶ also. It is thus proved that the Peirce's assumption of strength independency is not true. By using standard regression technique, it was observed with all yarns that these coefficients were represented very satisfactorily by the following double exponential function:

$$\rho_s(x) = ae^{-bx} + (1-a)e^{-cx}, \quad \dots (12)$$

where a is the coefficient; b and c , the two exponents of the function. Thus, our assumption of yarn strength as a summation of two mutually independent stochastic processes is verified. The behaviour of this function in the case of 25 tex carded ring yarn is shown in Fig. 4. It is also possible to describe the above function in terms of lag k , as shown below:

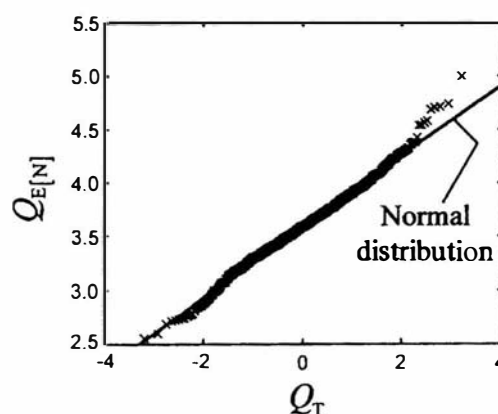


Fig. 3—Quantile-quantile plot corresponding to the strength of 25 tex cotton carded ring yarn measured at 50 mm gauge length [Q_E — empirical quantiles, expressed in Newton, and Q_T — theoretical quantiles]

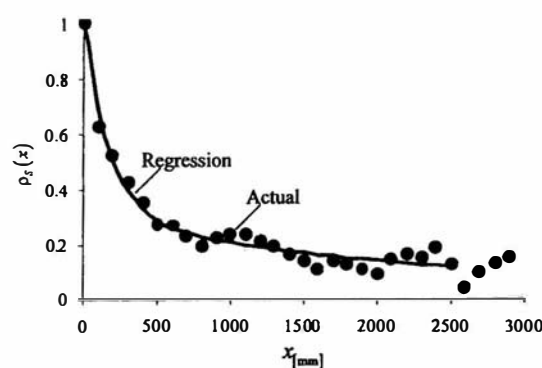


Fig. 4—Strength correlogram of 25 tex cotton carded ring yarn [$\rho_s(x)$ — autocorrelation coefficient at distance x]

$$\rho_s(k) = ae^{-bk} + (1-a)e^{-ck}, \quad \dots (13)$$

where $k = x_{[mm]}/50$. The values of these parameters (a, b , and c) for different yarns are shown in Table 2. By comparing Eq (13) with Eq (8), the statistical characteristics $\left({}^{(1)}\sigma_s, {}^{(1)}r^k \text{ and } {}^{(2)}\sigma_s, {}^{(2)}r^k \right)$ of these two processes were calculated (Table 2). It is remarkable to observe that these two processes are highly different from each other. It infers that two highly different and mutually independent phenomena are acting together on the yarn to impart variability to its strength. Also, it is noticeable from Table 2 that the standard deviations of both of these processes are following fairly similar increasing trend with the increase in yarn linear density. However, no evidence is found to figure out any relation between the characteristics of those processes and yarn spinning

Table 2—Autocorrelation characteristics of strength of different yarns

Yarn	Linear density, tex	a	b	c	$^{(1)}\sigma_s, N$	$^{(2)}\sigma_s, N$	$^{(1)}r^k$	$^{(2)}r^k$
Combed ring	10	0.6214	1.1042	0.0411	0.1698	0.1325	0.3314	0.9597
	14.5	0.6601	0.4378	0.0310	0.1735	0.1245	0.6455	0.9695
	20	0.7032	0.1934	0.0167	0.2311	0.1502	0.8242	0.9834
Carded ring	20	0.6659	0.4532	0.0164	0.2363	0.1674	0.6356	0.9837
	25	0.7048	0.2796	0.0182	0.2792	0.1807	0.7561	0.9820
	29.5	0.7450	0.8122	0.0122	0.3956	0.2314	0.4439	0.9879
Carded rotor	20	0.8563	0.9302	0.0165	0.1962	0.0804	0.3945	0.9836
	35.5	0.5968	1.2041	0.0409	0.3754	0.3086	0.3000	0.9599
	42	0.5507	0.6360	0.0022	0.3148	0.2843	0.5294	0.9978

a , b and c are the parameters of the autocorrelation function.

$^{(1)}\sigma_s$ and $^{(2)}\sigma_s$ are the standard deviations of the individual stochastic processes, expressed in Newton.

$^{(1)}r^k$ and $^{(2)}r^k$ are the autocorrelation coefficients of the individual stochastic processes corresponding to lag k .

technology. Nevertheless, the characteristics of these two processes will be useful in future to investigate the physical bases of these processes, which, in turn, will lead to deeper understanding of yarn strength variability.

4.4 Simulations of Yarn Strength

The simulation of strengths was performed with 30000 yarn sections, each of 5000 mm length. Thus, 3×10^6 strength values each corresponding to strength of 50 mm length of yarn were generated. By applying the weakest-link principle on the generated strength values, as shown by Eq (11), it was possible to obtain strength values S^* corresponding to yarn specimens of different lengths ranging from 50 mm to 5000 mm.

The frequency distributions of the simulated strengths for 25 tex cotton carded ring yarn corresponding to three selected lengths are presented in Fig. 5. It is remarkable to observe the changes in shape and behaviour of histograms with the increase in length of yarn section. Similar trends were observed with the actual strength values (Fig. 2). Also, it is noticeable that the smootiness of the histogram is reducing as the length of yarn section is increasing. This is due to the relative reduction in the number of simulated strength values for longer yarn sections. Similar results were obtained with other yarns.

Also, it was possible to obtain the basic statistical parameters of simulated strength values for yarn sections of different lengths. In the case of 25 tex cotton carded ring yarn, these parameters are very well approximated by the following empirical relations:

$$\bar{S}^* = \bar{S} + 5.45\sigma_s \left[(l/l_0)^{-1/10.55} - 1 \right], \quad \dots (14)$$

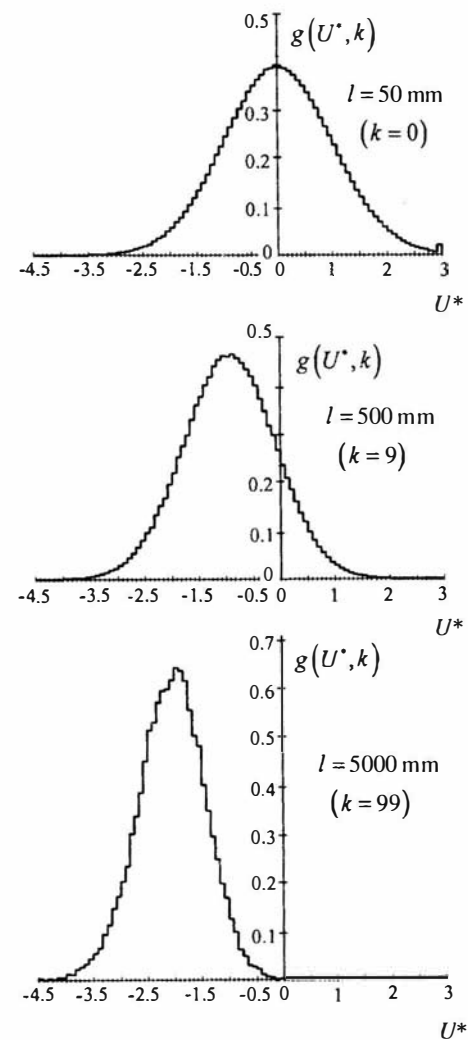


Fig. 5—Histograms of simulated strengths of 25 tex cotton carded ring yarn corresponding to different lengths [$g(U^*, k)$ —probability density function of the transformed strength quantities U^* corresponding to the gauge length parameter k]

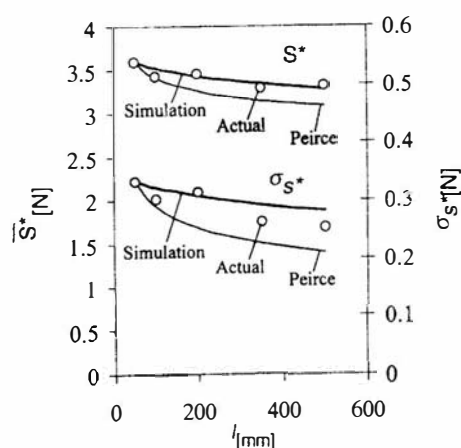


Fig. 6—Simulation results vs Peirce's equations vis-à-vis actual results corresponding to the strength of 25 tex cotton carded ring yarn [$\bar{S}^*$ — mean strength value, and σ_{S^*} — standard deviation of strengths]

$$\sigma_{S^*} = \sigma_S \left(l/l_0 \right)^{-1/10.55} \quad \dots (15)$$

Similar relations with different values of coefficient and exponent of the empirical equation were observed with the other yarns, depending on the degree of correlation among the strengths of neighbouring short sections¹⁶. Peirce also observed similar relations but with two unique values, as shown in Eqs (1) and (2), for all yarns, irrespective of material and technology used for their production.

The basic statistical parameters obtained from the simulated strength values for yarn sections of different lengths were compared with those calculated from Peirce's equations [Eqs (1) and (2)] with a reference to the actual results. This comparison in the case of 25 tex cotton carded ring yarn is shown in Fig. 6. Evidently, the simulation results are better than Peirce's equations in terms of predicting the actual results. This finding is in line with the observations on the other yarns¹⁶. This difference is ascribed to the fact that the simulation results were obtained on the basis of verified assumption of strength dependency; on the contrary Peirce's equations respected the imagination of strength independency, which is not real.

In some yarns, it was found that the actual strength at longer gauge length is nearly the same or a little higher as compared to the strength at short gauge length (50 mm). Imaginatively, this might happen because the "micro-slippage" of fibres and/or fibre segments was not fully realized in a region near to the jaw gripping line. Fibres could not slip inside the jaws. Evidently, this phenomenon was relatively more

prominent during tensile testing at short gauge length (50 mm).

5 Conclusions

The new approach for non-standard measurement and evaluation of yarn strength introduces not only a new characterization of yarn strength in terms of its autocorrelation characteristics, but also a new way of predicting actual yarn strength behaviour corresponding to different gauge lengths. It is experimentally observed that the strengths of neighbouring short sections along the yarns are related and the degree of this correlation is different in different yarns, accordingly the relation between strength and gauge length is different in different yarns. This observation contradicts Peirce's assumption of strength independency. This is the most probable reason why Peirce's independent strength model is far from the reality as compared to the presented stochastic strength model.

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