

A new rapid approach in assessing slope stability beneath a random field

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Slope stability of soil (Zhu *et al.*²⁰ method used) has been studied under a random field. Spatial variability of soil's properties (random variables (RV)) is decomposed in term of deterministic trend and the random fluctuations of short duration of soil round this deterministic trend. Spread of variation of these fluctuations from one point to another in the soil is a very important parameter for describing the natural variability of soil's properties. In this paper, to quantify tendency value and dispersion of safety factors (FS) in function of trend and dispersion of RV, one uses Point Estimated Method (PEM) validated by Monte Carlo Method (MCM). It is known that the MCM has a prohibitive machine time compared to PEM, where the advantage to use this last one. Unfortunately, the PEM don't take explicitly the fluctuations of RV into account. Therefore, one proposes to incorporate it in the PEM.

[Keywords : Slope, Pseudo-static method, Reliability Index, Point Estimated Method, Monte Carlo, Fluctuation]

Introduction

In slope stability computations, various sources of uncertainties are encountered, such as geological details missed in the exploration program, estimation of soil properties that are difficult to quantify, i.e. the spatial variability in the field cannot be reproduced accurately, fluctuation in pore water pressure, testing errors and many other relevant factors¹². The limit of spatial continuity is defined as the separation distance between field data at which there is no, or insignificant, spatial correlation. This limit can be expressed in terms of the spatial range²¹, the scale of fluctuation²², or the autocorrelation distance²³.

Therefore, probabilistic methods give us a solution, by reasoning not in terms of a single point value of soil properties, but in terms of the tendency and of dispersion of these soil properties. These are considered in the probabilistic model as random variables (RV). Thus, for engineering purposes, a simplification is introduced in which spatial variability of soil properties is decomposed into deterministic trend, and to random component describing the variability about that trend^{9,6,14,15,16}. In such decomposition, the trend represents phenomenon effect, which influences the formation of soil during

long periods, whereas the random component describes the fluctuations of short duration of soil formation conditions. Spread of variation of these fluctuations from one point to another is also a very important parameter for describing the natural variability of the mechanical soil properties. Spatial correlation function appears to be an efficient tool for describing the variation of these fluctuations¹⁴.

Two RV are used in this study; which are the cohesion and the tangent of the angle of effective friction ($Tan(\phi')$)^{8,10}. They are independents (it is safer to assume the two RV as independents, that to suppose a correlation error^{15,9,3}) and have a lognormal distribution. This choice is motivated by the fact that the two RV are positives and are used by the Mohr-Coulomb criterion, which regulates the comportment of the sliding surface for a given sloping soil.

For the formulation of FS used in this study to estimate the stability of the slope, one has used the method of Zhu *et al.* (2005)²⁰. This last one is based on the method of the GLEM (general limit equilibrium method). The choice of this method is justified by the fact that, when performing analyses of slope stability with established procedures for the Morgenstern-Price¹³ method, tedious computations are often involved, resulting in unduly long computation times. This situation necessitates the improvement of the algorithm for computing²⁰.

Moreover, the introduction of the fluctuations of the RV around the mean is often used in connection with the finite element method^{19,14,15,16}, which has the characteristic to be very heavy to develop. As part of the sloping soil stability, the GLEM is generally more favoured than the FEM (Finite Element Method). In this study, one proposes to use the concept of fluctuations in the framework of the GLEM. This is, because the FEM can be very cumbersome to develop. The GLEM is more flexible, so the execution time is optimized.

At the end, to achieve the objectives set, a visual code of calculating, based on the concept of object-oriented programming (P.O.O) and interactive with Access database, was developed. This allowed us to increase the speed of managing of the mass information processed; hence, the speed is more important. The exchange information between the program and the Access database is faster compared to the conventional programming.

Materials and Methods

Recall of classical Point Estimated Method (PEM)

The Point Estimated Method (PEM) is an alternative way of taking into account inputs random variables (RV). PEM does not require knowledge of a particular form of the probability density function (Pdf) of the inputs (RV). PEM is essentially a weighted average method reminiscence of numerical integration formulas involving “sampling points” and “weighting parameters”. PEM reviewed here will be the PEM developed by Rosenblueth (1975, 1981)^{17,18}. PEM seeks to replace a continuous probability density function, with a discrete function having the same first three central moments (mean μ , variance σ^2 , skewness ν)¹⁰.

If $f_X(X)$ is a Pdf with only a single RV (mean, variance and skewness known) (Fig. 1), Rosenblueth gave four conditions that must be satisfied by the RV X in question to be correctly modelled⁴.

$$\begin{cases} P_+ + P_- = 1 \\ X_+ P_+ + X_- P_- = \mu_X \\ P_+(X_+ - \mu_X)^2 + P_-(X_- - \mu_X)^2 = \sigma_X^2 \\ P_+(X_+ - \mu_X)^3 + P_-(X_- - \mu_X)^3 = \nu_X \sigma_X^3 \end{cases} \quad \dots (1)$$

where, σ_X is standard deviation of RV X , ν_X is skewness of RV X .

A solution of Eq. (1) is:

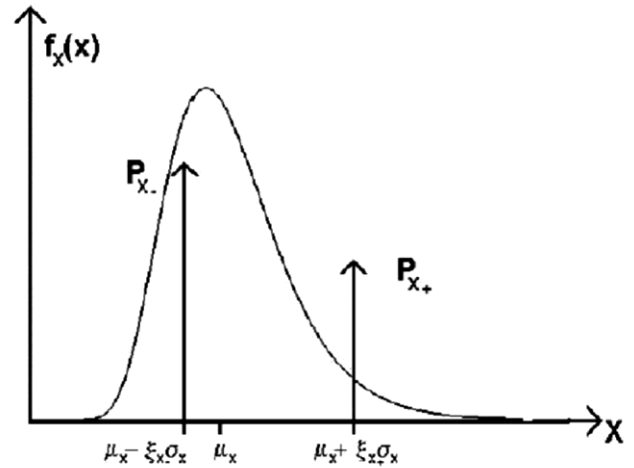


Fig. 1 — PEM for the case of one RV

$$X_+ = \mu_X + \xi_+ \sigma_X \quad \dots (2)$$

$$X_- = \mu_X - \xi_- \sigma_X \quad \dots (3)$$

$$P_- = 1 - P_+ \quad \dots (4)$$

$$P_+ = \frac{1}{2} \left[1 - \frac{\left(\frac{\nu_X}{2}\right)}{\sqrt{1 + \left(\frac{\nu_X}{2}\right)^2}} \right] \quad \dots (5)$$

$$\xi_{X+} = \frac{\nu_X}{2} + \left(1 + \left(\frac{\nu_X}{2}\right)^2 \right)^{1/2} \quad \dots (6)$$

$$\xi_{X-} = \xi_{X+} - \nu_X \quad \dots (7)$$

In the case where the Pdf of RV X is symmetric, $\nu_X = 0$, $\xi_+ = \xi_- = 1$.

If n RV are treated, there will be 2^n sampling points corresponding to all combinations of the two sampling points for each variable.

For $n=2$ (2VA) used, there are four sampling points given by:

$$\begin{cases} F_{S_{++}}(\mu_X + \xi_{X+} \sigma_X, \mu_Y + \xi_{Y+} \sigma_Y) \\ F_{S_{+-}}(\mu_X + \xi_{X+} \sigma_X, \mu_Y - \xi_{Y-} \sigma_Y) \\ F_{S_{-+}}(\mu_X - \xi_{X-} \sigma_X, \mu_Y + \xi_{Y+} \sigma_Y) \\ F_{S_{--}}(\mu_X - \xi_{X-} \sigma_X, \mu_Y - \xi_{Y-} \sigma_Y) \end{cases} \quad \dots (8)$$

The weight P_{XS1} and P_{YS1} (for two independent RV x and y) are bound to have the weight P_{S1S2} corresponding for the same two dependents RV (Fig. 2) as:

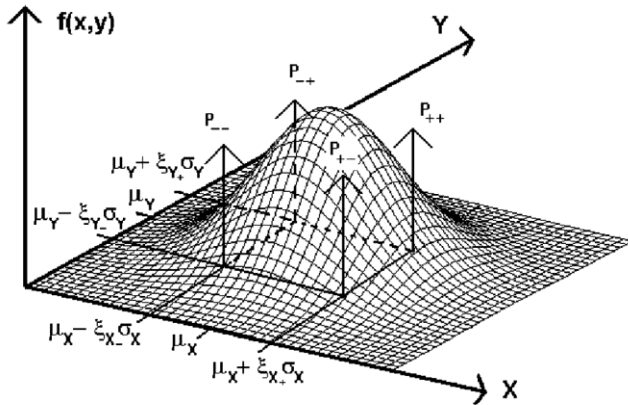


Fig. 2—PEM for the case of two RV

$$P_{S1S2} = P_{XS1}P_{YS1} + S1S2(\rho_{XY}/((1+(\nu_X/2)^3)) \\ (1+(\nu_Y/2)^3))^{1/2} \quad \dots (9)$$

where ρ_{XY} is the cross correlation coefficient between the two RV X and Y.

The (Si) terms (Eq. 9) are the sign (+) for points greater than the mean and (-) for points smaller than the mean. The product (S1S2) (Eq. 9), determines the sign of cross correlation coefficient and the subscripts of the weight P indicate the location of the points that is being weighted¹⁰.

Equation (9) is valid for a non symmetric Pdf; i.e. when the two RV are dependents ($\rho_{XY} \neq 0$). In the case where ($\rho_{XY} = 0$, P_{XS1} and P_{XS2} (Eq. 9) will be the weights of the two independents RV.

Mean and variance of the Fs (Safety factor of a slope) are established as:

$$\mu_{Fs} = \sum_{i=1}^{2^n} P_i F_{Si} \quad \dots (10)$$

$$\sigma_{Fs}^2 = \sum_{i=1}^{2^n} P_i (F_{Si} - \mu_{Fs})^2 \quad \dots (11)$$

In the frame of the Lognormal law, Eq. (10, 11) become^{14,15 and 16}:

$$\mu_{Ln(Fs)} = Ln(\mu_{Fs}) - \frac{\sigma_{Ln(Fs)}^2}{2} \quad \dots (12)$$

$$\sigma_{Ln(Fs)}^2 = Ln(1 + Cv_{Fs}^2) \quad \dots (13)$$

$$Cv_{Fs} = \sigma_{Fs} / \mu_{Fs} \quad \dots (13.1)$$

Cv_{Fs} , μ_{Fs} , σ_{Fs} , being respectively the coefficient of variation, the mean and the standard deviation of Fs.

Monte Carlo Method (MCM)

Monte Carlo Method consists in sampling at random the vector of input parameters, running the system model computer code for each sample of that vector and getting a sample of the vector of output variables. Later on, the characteristics of the output variables may be estimated using the output samples obtained. One of the advantages of using the Monte Carlo Method is that all statistical standard methods we need to estimate the output variable distributions and to test any hypothesis may be used. This makes it the most straightforward and powerful method available in the scientific literature to deal with uncertainty propagation in complex models, as it is the case of performance assessment models. This method is valid for models that have static and also dynamic outputs. It is adequate for working with discrete and continuous inputs and outputs, and the implementation of computational algorithms required has no fundamental complexity.

In this technique a large number of soil variables such as shear strength, angle of internal friction of the soil can be sampled from their known (or assumed) probability distribution. For this purpose, the probability density function for each of these soil variables must be specified. Then, the corresponding safety factor of each set is calculated¹².

Randomness in the generation of samples of RV is insured in one term of the equation describing the fluctuation (Dfp) (it will be defined a little further (paragraph 5)).

Mean and variance of a RV, governed by the lognormal distribution, are established as follows^{14,15,16}:

$$\mu_{Ln(X)} = Ln(\mu_X) - \frac{\sigma_{Ln(X)}^2}{2} \quad \dots (14)$$

$$\sigma_{Ln(X)}^2 = Ln(1 + Cv_X^2) \quad \dots (15)$$

$$X = \exp(\mu_{Ln(X)} + Dfp\sigma_{Ln(X)}) \quad \dots (16)$$

X: is a RV (Cohesion or $Tan(\phi')$ considered, Dfp is the fluctuation with mean zero and standard deviation equal to unity ($= 1$), $Cv_X = \sigma_X/\mu_X$ being the coefficient of variation of RV X and ϕ' : effective friction angle.

Improvement of PEM in this study

Under this method, the behaviour Lognormal of the RV is insured in Eq. (6) and (7) by the coefficient of

asymmetry. This last one is given for the lognormal distribution by¹¹:

$$\nu_x = \left(2 + e^{\sigma_{\ln(x)}^2}\right) \sqrt{e^{\sigma_{\ln(x)}^2} - 1} \quad \dots (17)$$

If one substitutes Eq (15) in Eq (17), Eq (17) will have the following new form¹⁰:

$$\nu_x = 3CV_x + CV_x^3 \quad \dots (18)$$

The Eq (18), more simple, is used in the frame of this study. CV_x : Coefficient of variation of the RV X .

The fluctuation Dfp not explicitly used in the PEM. One proposes to include it explicitly in this study. Indeed, Eq. (2), (3), (4) and (5) are obtained as solutions of the system of Eq. (1). If one repeats the same operation by substituting σ_x by $(Dfp \cdot \sigma_x)$ in Eq. (1), relations (2) and (3) become:

$$X_+ = \mu_x + \xi_+ Dfp \sigma_x \quad \dots (19)$$

$$X_- = \mu_x + \xi_- Dfp \sigma_x \quad \dots (20)$$

One believes that this approach is reasonable, because in the process of solving Eq. (1), σ_x (constant) can be easily replaced during the stages of resolution, by $Dfp \cdot \sigma_x$ without changing the final form of the solution. Thus, Eq. (8) will be revised for the new following form:

$$\begin{cases} Fs_{++}(\mu_x + \xi_{x+} Dfp \sigma_x, \mu_y + \xi_{y+} Dfp \sigma_y) \\ Fs_{+-}(\mu_x + \xi_{x+} Dfp \sigma_x, \mu_y - \xi_{y-} Dfp \sigma_y) \\ Fs_{-+}(\mu_x - \xi_{x-} Dfp \sigma_x, \mu_y + \xi_{y+} Dfp \sigma_y) \\ Fs_{--}(\mu_x - \xi_{x-} Dfp \sigma_x, \mu_y - \xi_{y-} Dfp \sigma_y) \end{cases} \quad \dots (21)$$

The Eq. (21) is equivalent to Eq. (8) if the fluctuation $Dfp=1$; i.e. the conventional PEM.

This change is interesting, because machine time computation of MCM is very expensive compared to the PEM. Here, one notes the advantage of using this last method, because the classical formulation suffers to not incorporate of the fluctuation (Dfp) in its formulation, and the MCM has a prohibitive machine time calculation.

Random field for soil properties

Because of the uncertainty shown when one measures the in situ soil's proprieties (RV), the spatial variability of these RV are decomposed into two parts. The first one is the deterministic trend, and the

second one is a random component describing the variability about that trend. In such decomposition, the trend represents phenomenon effect, which influences the formation of soil during long periods, whereas, the random component describes the fluctuations of short duration of soil formation conditions^{14,15}. Let the variable $f_{op}(x,y)$ describing a deterministic trend soil propriety in space, taken in practice as the mean of measured values, and also function of zero mean, unit variance Gaussian random field $Dfp(x,y)$. One writes^{6,14,15 and 16}.

$$f_p(x,y) = \Re[f_{op}(x,y) + \sigma_p Dfp(x,y)] \quad \dots (22)$$

$\Re$: is a transformation taking the Gaussian process $Dfp(x,y)$, into the distribution appropriate for $f_p(x,y)$, σ_p is the standard deviation, hence:

$$Dfp(x,y) = \sum_{n=1}^2 Dg_{pn}(x,y) \quad \dots (23)$$

Here $p=1$ corresponds to cohesion and $p=2$ stands for $Tan(\phi')$. The zero mean, unit variance, two-dimensional bivariate Gaussian random field $Dg_{pn}(x,y)$, can be simulated as follow^{14,15,16}:

$$Dg_{pn}(x,y) = \sqrt{2} \sum_{k=0}^{Nx-1} \sum_{l=0}^{Ny-1} A_{kl,pn} [Cos(\kappa_{xk}x + \kappa_{yk}y + \Omega_{kl,n}) + Cos(\kappa_{xk}x - \kappa_{yk}y + \Psi_{kl,n})] \quad \dots (24)$$

$$\text{With, } A_{kl,pn} = H_{pn}(\kappa_{xk}, \kappa_{yl}) \sqrt{2\Delta\kappa_x \Delta\kappa_y} \quad \dots (25)$$

Eq. 24, ensures also the inter-property correlation between two RV cohesion and $Tan(\phi')$. Ω_{kl} , Ψ_{kl} are random phase angles distributed uniformly on the interval $[0 \ 2\pi]$. The randomness in the generation of samples of RV is insured by these two angles. Hpn (Eq. 25) coefficients are obtained from the wave number cross spectral density matrix $S(k_x, k_y)$ below:

$$S(\kappa_x, \kappa_y) = \begin{bmatrix} S_{CC} & S_{C\mathcal{E}} \\ S_{\mathcal{E}C} & S_{\mathcal{E}\mathcal{E}} \end{bmatrix} = HH^T; \quad \mathcal{E} = Tan(\phi')$$

$$S(\kappa_x, \kappa_y) = \begin{bmatrix} H_{11} & 0 \\ H_{21} & H_{22} \end{bmatrix} \cdot \begin{bmatrix} H_{11} & H_{12} \\ 0 & H_{22} \end{bmatrix} \quad \dots (26)$$

Here, H is lower triangular matrix deduced from the Cholesky decomposition of cross spectral density matrix $S(k_x, k_y)$. k_x and k_y are the wave number in x and y directions respectively given below as:

$$\begin{cases} \kappa_{xk} = k\Delta\kappa_x; \kappa_{yl} = l\Delta\kappa_y \\ \kappa_{xu} = N_x\Delta\kappa_x; \kappa_{yu} = N_y\Delta\kappa_y \end{cases} \quad \dots (27)$$

The wave numbers steps $\Delta\kappa_x$ and $\Delta\kappa_y$ are evaluated from the representation of $S(k_x, k_y)$ and by evaluating

the cut-off wave numbers values K_x and K_y for N_x and N_y increments respectively.

The cross correlation elements of the power spectral density matrix S are given by:

$$S_{c\varepsilon} = \sqrt{S_{cc}S_{\varepsilon\varepsilon}} \cdot \rho_{c\varepsilon} \quad \dots (28)$$

$$\varepsilon = \tan(\phi')$$

$$\rho_{c\varepsilon} = \frac{\text{mean}(C\varepsilon) - \text{mean}(C)\text{mean}(\varepsilon)}{\sigma_\varepsilon \sigma_C} \quad \dots (29)$$

$$\begin{cases} H_{11} = \sqrt{S_{cc}} \\ H_{22} = \sqrt{S_{\varepsilon\varepsilon}} \sqrt{1 - \rho_{c\varepsilon}^2} \\ H_{21} = \sqrt{S_{\varepsilon\varepsilon}} \cdot \rho_{c\varepsilon} \end{cases} \quad \dots (30)$$

$\rho_{c\varepsilon}$: stands for the inter-property correlation of the two RV. However, one considers in this study that $S_{cc} = S_{\varepsilon\varepsilon} = S$. The S expression is given for a unit variance as:

$$S(\kappa_x, \kappa_y) = \frac{\sigma^2}{4\pi} \exp \left[-\left(\frac{\kappa_x a}{2} \right)^2 - \left(\frac{\kappa_y b}{2} \right)^2 \right] \quad \dots (31)$$

Also, constants (a) and (b) in Eq (31), represent a correlation length respectively horizontal and vertical. In this study, one considers $a_{Ln(C)} = a_{Ln(\varepsilon)}$ and $b_{Ln(C)} = b_{Ln(\varepsilon)}$ and the mean and the standard deviation of two RV will be considered as constants.

However, the fluctuation of RV is generally used in the context of finite element method (FEM)^{14,15,19}; it will be used as part of this study in the frame of general limit equilibrium method (GLEM). To calculate the FS of the slope, one will use the Zhu *et al.* (2005)²⁰ method. In our approach, one will replace the finite element (FEM) by the slice of soil (GLEM). The two RV (cohesion and $\varepsilon = \tan(\phi')$) are considered independent. So, in Eq. (9), (28), (29) and (30), the cross correlation coefficient between the two RV is equal to zero throughout the study. At the end, one will consider that in the iterative process to calculate the FS of Zhu *et al.*, the fluctuation (Dfp) is constant for the same slice of soil and the same Fs during the steps process to calculate the same Fs.

In the sense to reducing a computationally time, the digital generation of sample functions of equation (24) is computed with the aid of the fast Fourier transform (FFT).

Reliability Index

The idea of having confidence in one single Fs only for saying that a given slope is stable or not is not enough. Indeed, the dispersion of the values of Fs (precisely due to uncertainties inherent in the soil for the RV) plays a significant role on the stability of soil slope. Results of a probabilistic interpretation may contradict the deterministic findings, because of the dispersion of measured values of the RV. Thus, a reliability study will be more effective.

The reliability index in this case is given by^{12,1}:

$$\beta = (\mu_{Fs} - 1) / \sigma_{Fs} \quad \dots (32)$$

A Lognormal form of Eq. 32 is given by^{2,4}, and⁷:

$$\beta = \mu_{Ln(Fs)} / \sigma_{Ln(Fs)} \quad \dots (33)$$

$\mu_{Ln(Fs)}$ and $\sigma_{Ln(Fs)}$, are respectively defined by Eq. (12) and (13).

A common part between MCM and PEM

The first thing to do is to choose the circle of failure most probable (Fig. 4). This last one, in this study, is assumed known. Because, finding the circle in question is not the aim of this study. The sliding corner of the slope bounded by the chosen circle of failure is divided into slices. The two RV chosen are evaluated at the base of each slice.

For the Monte Carlo Method (MCM)

By using the MCM, one must verify that the number of samples chosen gives a stable value of reliability index (Fig. 6). For each selected sample of RV (the equations used for the two RV are Eq. (14), (15) and (16), a single safety factor is calculated using the method of Zhu (2005) *et al.*²⁰. The randomness of the values of the two RV is insured by the fluctuation (Dfp). Once all FS (corresponding to the couples of RV) have been calculated, the mean and the standard deviation (μ_{Fs}, σ_{Fs}) of all safety factors (Fs) will be calculated. Here of course, for the two RV, the Cv_X (coefficient of variation of RV X) must be known in advance.

In the case of Fig. 6, 600 Fs (where reliability index is stable) are chosen. One will first calculate their mean and standard deviation (STD). The mean and standard deviation thus computed are reused in the relations Eq. (12), (13) and (13-1) to have a Lognormal form. At the last step, one calculates the reliability index using Eq. (33) (or Eq. 32, according the context).

For the Point Estimated Method (PEM)

For the PEM, one computes the two RV by using Eq. (19) and (20). Thus, by using Eq. (21), one computes four FS. The Lognormal shape of the two RV is ensured by Eq. (18). At the last, the randomness of the two RV is provided by the fluctuation (Dfp).

The mean and standard deviation (STD) of F_s are calculated by Eq. (10) (11). In these last equations, the F_{s_i} used are stated in equation 21. Afterwards, one calculates the shape of the lognormal mean and standard deviation by using Eq. (12), (13) and (13-1). Of course, at the last step, one will calculate the reliability index using Eq. (33) (or Eq. 32, according the context).

Example

By using, for the calculation of F_s , the formulation of Zhu et al (2005)²⁰, the pseudo-static analysis is studied in term of reliability based on the following assumptions:

Data for the slope

To calculate the F_s of Zhu et al.²⁰, the following data are used:

- One will use the example of the Fig. 3, which data have been stated in Table 1.

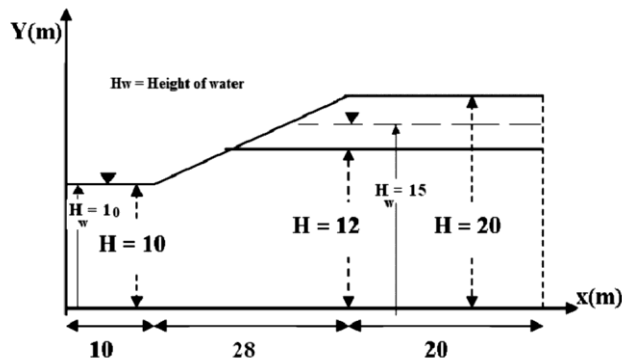


Fig. 3—Example of slope with two layers (For details see section 8)

Table 1— Soil characteristics

N° Layer	1	2
H (m)	8	12
C' (KN/m ²)	32.74	27
γ_{sat} (KN/m ³)	11.852	14.23
ϕ' (Deg)	12	10

- 280 slices were used in this example
- γ_{sat} : Saturate density of soil (KN/m)
- ϕ' : Effective friction angle (Degree)
- C' : Effective Cohesion (KN/m²)
- H : Height of layer (m)

- The coordinates of the circle of failure most probable: Radius $R = 20.6$ m, circle centre C ($x_C = 16$ m and $y_C = 22$ m) and the point A ($x_A = 34.8$ m and $y_A = 20$ m) and B ($x_B = 2$ m and $y_B = 10$ m). (see Fig. 4).
- The tolerance used for the process of iterations for the safety factor (F_s) and inclination of the inter-slice forces (λ) (Fig. 5) is 0.0001. ($\Delta F_s = \Delta \lambda = 0.0001$).
- The function inter slice is taken constant $f_i = f_{i-1} = 1.85$ (Fig. 5).
- The horizontal seismic coefficient $K_c = 0.02$ (Fig. 5).
- The concentrated load $Q_i = 0$; i : represents the number of the slice of soil of the corner sliding. W_i is the weight of the slice i ; α_i angle of slice i ; E_i is the horizontal force inter slice; b_i is the width and the height h_i of the slice i ; u_i is the pressure of the water, (C') effective cohesion (Fig. 5).

Data for the random field

- The coefficient of variation of cohesion Cv_c varies between (0.1 to 1). For the coefficient of variation of $\varepsilon = \tan(\phi')$; Cv_ε varies between (0.05 to 0.2)¹².
- The two RV (cohesion, $\tan(\phi')$) are incorporated in the Mohr-Coulomb criterion to evaluate the stability of the slope using the formulation of Zhu and al (2005)²⁰.
- The two probabilistic methods used are: The PEM and the MCM.
- $a_{Ln(C)} = a_{Ln(\varepsilon)} = 10$ m (horizontal correlation length) et $b_{Ln(C)} = b_{Ln(\varepsilon)} = 1.2$ m (vertical correlation length).
- The mean and the standard deviation are considered constants for the two RV.

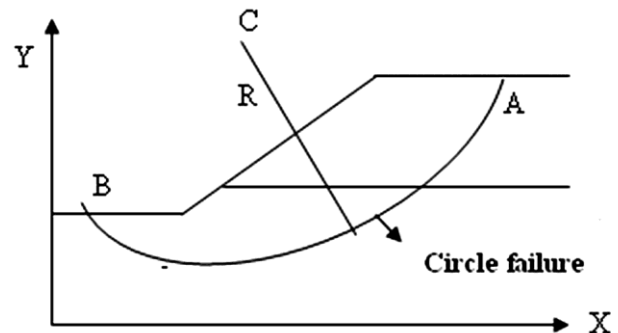


Fig. 4— Probable circle failure (For details see section 8)

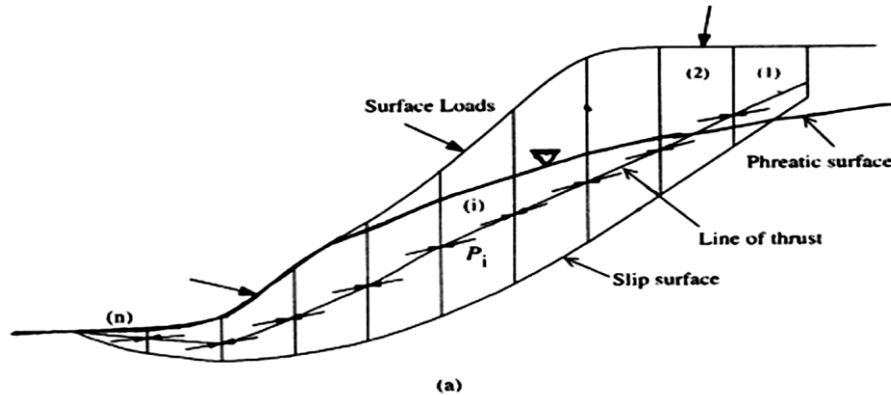


Fig. 5 — (a) Sliding body, (b) Typical slice. P_i : Inter slice force

Table 2 — Time of implementation

Monte Carlo Method (MCM)	Point Estimated Method (PEM)	
Dfp (Eq. 23)	Dfp (Eq. 23)	$Dfp=1$
4 H 07 mn	4 mn 43 s	1 mn 13 s

for one couple of ($C_{vc} = 0.6$, $C_{v\varepsilon} = 0.14$)
Using Eq 33 (With Pentium 4 CPU: 2.8 GHz)
H: Hours; mn: Minutes; s: Seconds

- The two RV (cohesion and $\varepsilon = \tan(\phi')$) are independents and calculated at the foot of every slice of a sliding corner.

Results

The results obtained by programming the classical PEM and the PEM improved (by introducing the fluctuations of RV) and MCM are concentrated in Table 2 and Fig. 6, 7, 8, 9, 10, 11, 12. So, the Table 2 shows the execution time found by calculating of the PEM (Dfp #1, $Dfp=1$) and the MCM under a fixed coefficients of variation $C_{vc}=0.6$ and $C_{v\varepsilon}=0.14$ of the two RV. The computer used for these calculations is a Pentium 4 with a CPU of 2.8 GHz. And the Fig. 6 is plotted to verify the necessary number of samples (to use in MCM) to have a stable value of the reliability index; for fixed values of $C_{vc}=0.6$ and $C_{v\varepsilon} = 0.14$. In this case, one has chosen 600 samples.

The Fig. 7, shows the comportment of reliability index (by using Eq. 33) and the standard deviation of F_s (version Lognormal) versus of the variation of the C_{vc} (with a fixed $C_{v\varepsilon}=0.14$). For the same objective, the Fig. 8, is given to show the reliability index (by using Eq. 33) and the standard deviation of F_s (version Lognormal) versus different values of $C_{v\varepsilon}$ (with a fixed $C_{vc}=0.6$). For the Fig. 9, the numerator of Eq 33 is compared to the reliability Index (Eq. 33)

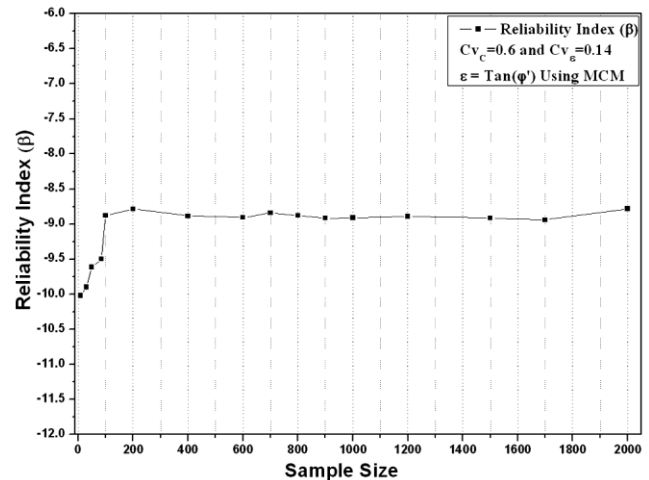


Fig. 6 — Number of samples used in MCM (One chooses 600 samples) using Eq. 33 ($C_{vc}=0.14$); $C_{vc}=0.6$)

versus the variation of the C_{vc} (with a fixed $C_{v\varepsilon}=0.14$). Also, as on had made it for the Fig. 8, the Fig. 10 has been plotted to compare the mean (Lognormal) of F_s with the reliability Index (Eq. 33) versus the variation of the $C_{v\varepsilon}$ (with a fixed $C_{vc}=0.6$).

The comparison between the reliability index of Eq. 33 and Eq. 32, have been plotted in the Fig. 11 (with different values of $C_{v\varepsilon}$ and a fixed $C_{vc}=0.6$) and in the Fig. 12 (with different values of C_{vc} and a fixed $C_{v\varepsilon}=0.14$).

Discussions

The study of sloping soils is safer using the reliability approach, because our reasoning will be optimized by calculating the mean and standard deviation of F_s . Slope stability is sensitive to these two parameters. Judicious combination of these two parameters requires a great reality about the reliability of the results compared with the ground realities.

Unlike the reliability approach, a classical approach (deterministic), in the calculation of slope stability, occults the real behaviour of these slopes to the extent that it ignores the uncertainties (human and/or equipment) related to inputs (RV). This neglect means that the outputs (FS) calculated will be biased, because the inputs (RV) are far from the reality *in situ* of ground. Indeed, if, for a given building built on a broad surface or a highly heterogeneous environment, one does not take into account the dispersion of the values of RV throughout the vast area, then one makes a deterministic calculation with punctual values of soil proprieties, perhaps one will find $FS > 1$, but the reliability study may interpret differently the deterministic results obtained. In this case, in the wide area where construction is built, the soil is rarely homogeneous. Hence, the dispersion and the random (fluctuation) will emerge as an inescapable reality.

However, the cost of calculating the stability of the slope is generally prohibitive if one uses reliability methods (especially by using MCM) compared to the conventional deterministic approach. Also, according to an idea of a reasonable rate optimized between the computation time and accuracy of results, one has decided to introduce the fluctuation of RV in the conventional PEM by reformulating it. This will save computation time consuming observed in the MCM and give us a margin of acceptable accuracy compared to conventional deterministic study.

Thus, the Fig. 7, 8 are drawn to highlight and to validate the insertion of the fluctuation into the PEM. So, the Fig. 7 shows three curves for the reliability index, and three others curves for the standard deviation (STD) of FS. For each parameters, two curves are plotted to validate the MCM (fluctuation $Dfp \# 1$) with PEM (fluctuation $Dfp \# 1$) and the third curve is plotted to highlight the effect of the absence of fluctuations ($Dfp=1$) on the PEM; i.e. the third curve represents the classical PEM.

Note that the effect of the absence or the presence of the fluctuation is very clear and the margin of validation between the MCM ($Dfp \# 1$) and the PEM ($Dfp \# 1$) is acceptable. The same conclusion can be advanced to the Fig. 8, but with the remark that the effect of the coefficient of variation is more important for the cohesion (curves are not straight lines and horizontals (Fig. 7) compared to the effective friction angle (trend of curves are almost straight lines and horizontals). Therefore, the stability of the slope is

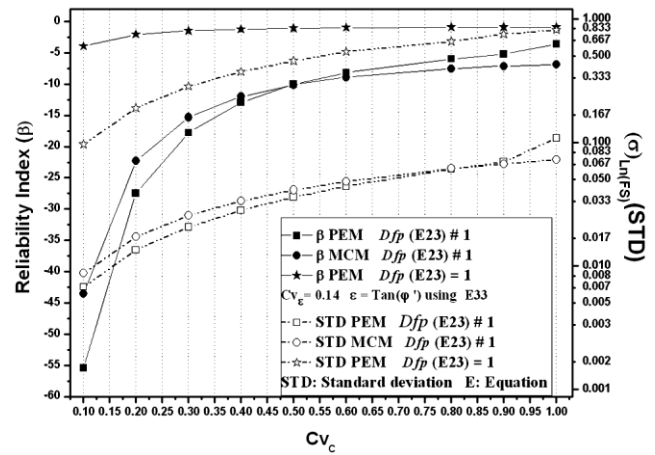


Fig. 7—Reliability Index and STD versus the variation of CV_c by using Eq. 33 ($C_{v\varepsilon}=0.14$)

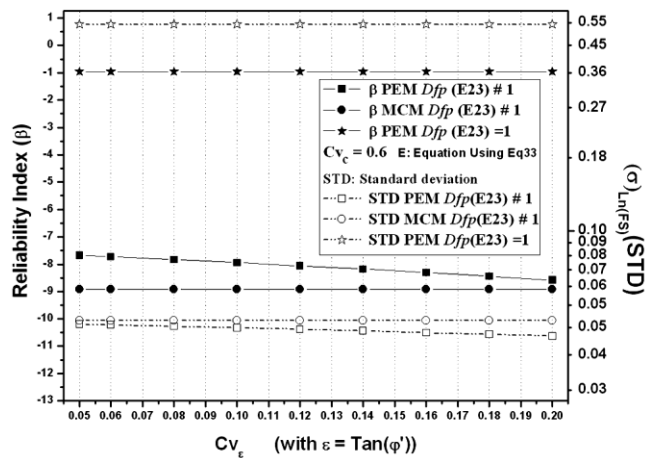


Fig. 8—Reliability Index and STD versus the variation CV_ε by using Eq. 33 ($CV_c=0.6$)

much more influenced by the dispersion of the values of the cohesion than it is influenced by the dispersion of the effective friction angle.

It is noticed however, that the Fig. 7 shows the reliability index which evolves by increasing with increasing of cohesion's coefficient of variation. Same remark is confirmed for the standard deviation (STD). For the Fig. 8, it seem that the reliability index evolves by decreasing (for PEM $Dfp \# 1$), with increasing of the CV_ε ($\varepsilon = \tan \phi'$).

The Table 2 (The computer used to make these calculations is Pentium 4 CPU: 2.8 GHz) below shows a good time performance of the PEM ($Dfp \# 1$; four minutes and 43 seconds) compared to the MCM ($Dfp \# 1$; four hours and 07 minutes). Although, the computation time of the classical PEM ($Dfp=1$; one minutes and 13 seconds) is better compared to the

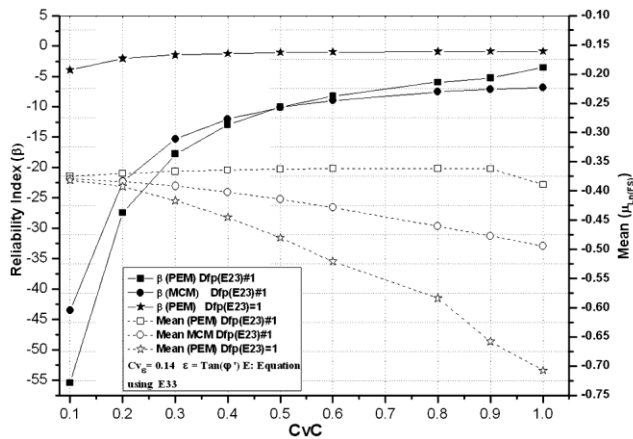


Fig. 9— Reliability Index and Mean versus the variation of Cv_c by using Eq. 33 ($Cv_c=0.14$)

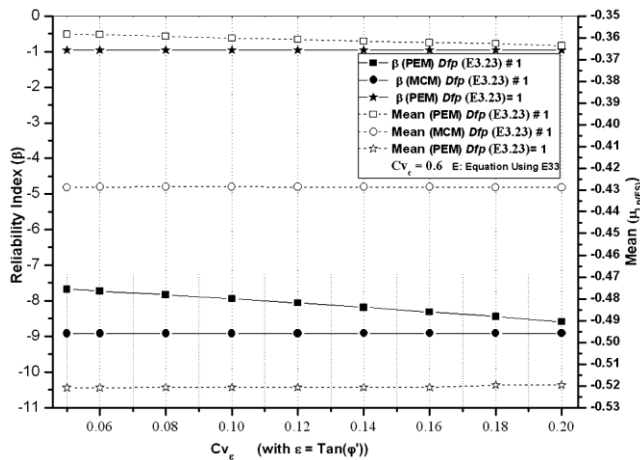


Fig. 10— Reliability Index and Mean versus the variation of Cv_ϵ by using Eq. 33 ($Cv_c=0.6$)

PEM ($Dfp \# 1$) reformulated, this last one, however, is better in accuracy of results (see Fig. 7 and 8), because, the MCM ($Dfp \# 1$) is the reference in terms of accuracy.

In the probabilistic approach, an improvement is added in this study, by introducing the fluctuation of soil properties explicitly in the classical PEM ($Dfp=1$). This reformulation of the classical PEM ($Dfp=1$) is beneficial because there is a gain of accuracy of the results and a gain in the computing of the machine time. Furthermore, the use of RV fluctuations in the general method of limit equilibrium (GLEM) (method of slices) is, in this sense, welcome, since these fluctuations are generally used in the context of finite element method^{14,15,19}. Unfortunately in the formulation of the GLEM, the fluctuation is not used in general, which reduces the effectiveness of this method.

However, to highlight the effect of the fluctuations on the different terms of Eq. 33, one has plotted the curves of Fig. 7, 8, 9, 10. In this sense, the numerator of Eq. 33 represents the mean of F_s in its version lognormal. The Fig. 9 and 10 are plotted to show the evolutions of these trends and the reliability index with respect to the variation of coefficients of variation for the two RV. Similarly, the denominator of Eq. 33 represents the standard deviation of F_s (Lognormal version). In this case, the Fig. 7, 8, are plotted.

One observes that globally, the shape of the curves plotted for the standard deviations of F_s marry better to the shapes of curves of the reliability index that the curves of the trend of F_s (especially for the coefficient of variation of cohesion). This suggests the idea that the standard deviations of F_s have more influence on the reliability index. So, the denominator of Eq. 33 evolves faster than the numerator. This suggests that the randomness in the dispersion of values imposes the pace of change on the variation of reliability index compared to the change in trend of F_s . Consequently, the reliability index reflects better the behaviour of the slope compared to a punctual safety factor in a purely deterministic study. Indeed, the formulation deterministic ignores the influence of dispersions of RV.

The effect of the fluctuation is, hence, very clear on all the curves plotted of the reliability index. Note, however, that the influence of standard deviation on the reliability index is prevailing compared to tendency. Graphs of Fig. 11 and Fig. 12 are plotted to show the effect of Eq. 32 and Eq. 33. It is noteworthy that the behaviour of the reliability index is essentially the same, by using Eq. 32 or Eq. 33. However, one notes that the intensities are different. This difference in the intensity of the reliability index is little more felt when the change of Cv_ϵ (Fig. 11) compared to the change in Cv_c (Fig. 12). Indeed, the curves of the reliability index (PEM ($Dfp \# 1$) and MCM ($Dfp \# 1$)), by using Eq. 33 (Fig. 11), are closer than their analogues Eq. 32 (Fig. 11). For the Fig. 12, the proportions are almost the same for the two equations.

As it is pointed out at the end of the introduction, one code of calculation is created and interactive with the Access database software. Programming language used is the visual basic.6 (vb6) based on the concept of object-oriented programming (OOP). Design of this language is made to facilitate flexible and rapid interaction with the Access database (DB); which is

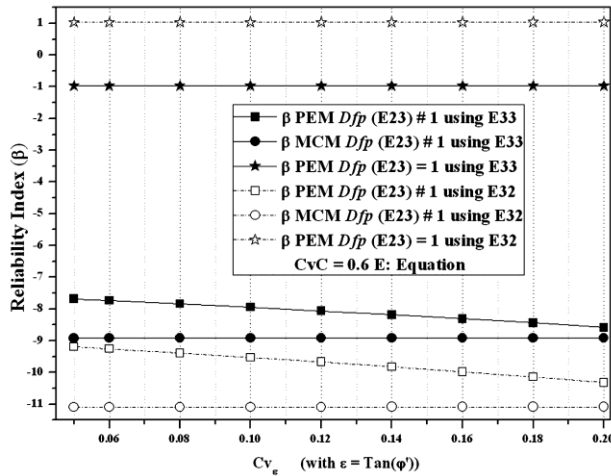


Fig. 11— Calculation of the Reliability Index, versus the variation of $C_{v\varepsilon}$ by using Eq. 33 and Eq. 32 ($C_{vC}=0.6$)

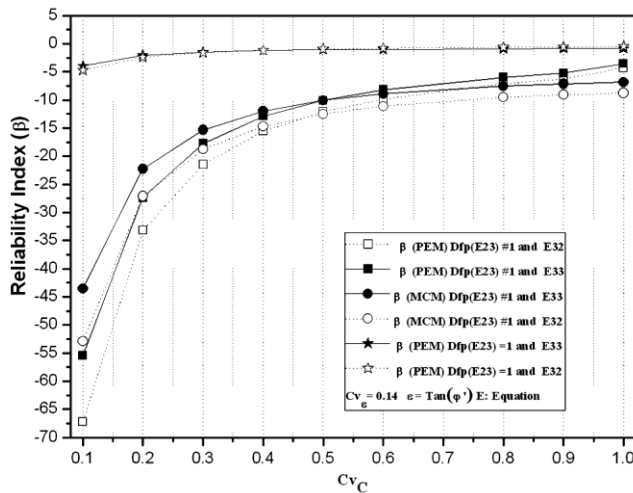


Fig. 12— Calculation of the Reliability Index, versus the variation of C_{vC} by using Eq. 33 and Eq. 32 ($C_{v\varepsilon}=0.14$)

considered as an object. With this valuable flexibility between the VB6 and the Access database, it is possible to facilitate the operations of using of the different parameters in the tables of the Access software. The treatment of the mass of information of the various parameters of the slices of the slippery corner will be flexible and easy. We will avoid, therefore, the heavy interactions and tiring of the conventional programs with text files containing formatted data used as input. The speed of the program will be then more optimized and information can be extracted by Access queries that have the precious ability to be easily adaptable to the information sought. The graphical interface of vb6, well known for its elegance and simplicity, adds one

more significant convenience and one time savings in the study treated.

Conclusion

In practice, we are often faced with situations of use of highly efficient methods of calculation, but their execution time is tedious. By contrast, there are methods where the time execution is efficient, but suffers from poor solutions in terms of accuracy. This compromise between timeliness and accuracy of solutions to find will prove decisive in practice. This study responds to this problem. Namely, find an acceptable compromise between the use of MCM and the PEM (by introducing the fluctuation of RV in the classical PEM). State function (reliability index in our case) computed, shows that it is very sensitive to changes in RV around their trends. Moreover, it is out of question to avoid introducing these fluctuations in the calculations, given its critical impact on the state functions; especially for highly heterogeneous soils and/or extended soils in terms of area. In fact, the history of the stresses is very sensitive to this problem. Indeed, it is possible to find performing software based on powerful formulations, but, if the inputs which have been injected are far from the reality of the ground, the collected results (state function) will be simply erroneous. Similarly, we had incorporated in the formulation of the GLEM, the fluctuation of RV. This is used in the frame of finite element method. This last method is very heavy (in term of formulation) compared to the GLEM to solve the problems of sloping soil. This has added a speed in the execution of our program.

References

- 1 Baecher Gregory B & Christian John T, (2003), "*Reliability and statistics in geotechnical engineering*" John Wiley & sons; 2003.
- 2 Bafghi Ali-Reza Yarahmadi, (2003), « La méthode des groupes-clef probabiliste. » En vue d'obtenir le grade de Docteur de l'INPL En Génie Civil-Hydrosystèmes—Géotechnique Institut National Polytechnique de Lorraine. (2003).
- 3 Bauer J, Puła W, (2000), "Reliability with respects to settlement limit states of shallow foundation on linearly deformable subsoil." *Computers and Geotechnics*, 2000; 26 (3-4) 281-308.
- 4 Duncan J Michael & Wright Stephen G, (2005), "*Soil strength and slope stability*," John Wiley & Sons; 2005.
- 5 Elkateb Tamer, Chalaturnyk Rick & Robertson Peter K, (2003), "An overview of soil heterogeneity: quantification and implication on geotechnical field problems." *Can Geotech J*, 40, 1-15 (2003).

- 6 Fenton G A, (1990), "Simulation and analysis of random fields." *PhD thesis, Department of Civil Engineering and Operations Research*, Princeton University, 1990.
- 7 Fahd A, Jimenez R., (2008), "A Genetic Algorithm for Identification of Slip Surfaces with Minimum Reliability." *The 12th International Conference of International Association for Computer Methods and Advances in Geomechanics (IACMAG)*, 1-6 October, 2008 Goa, India.
- 8 Fenton G A, Griffiths D V & Williams M B, (2005) "Reliability of traditional retaining wall design", *Géotechnique*, 55(1) 55--62, 2005.
- 9 Fenton G A & Vanmarcke E H, (1998), « Spatial variation in liquefaction risk." *Géotechnique* 1998; 48(6) 819-31.
- 10 Griffiths D V, Fenton G A & Tveten D E, (2002), "Probabilistic geotechnical analysis: How difficult does it need to be?", *Proceedings of the International Conference on Probabilistics in Geotechnics: Technical and Economic Risk Estimation*, R Pottler, H Klapperich & H Schweiger (eds.), Graz, Austria, United Engineering Foundation, New York, Sep 2002.
- 11 Gilson de F N & Gitirana Jr. (2005), "Weather-related geo-hazard assessment model for rail way embankment stability." *PHD Thesis University of Saskatchewan Saskatoon, Saskatchewan Canada*, (2005).
- 12 Malkawi A I H, Waleed.F. Hassan & Fayez. A. Abdulla (2000), "Uncertainty and reliability analysis applied to slope stability." *Structural Safety*, 22 (2000) 161-187 Elsevier Science Ltd (2000).
- 13 Morgenstern N R & Price V E, (1967), "A numerical method for solving the equations of stability of general slip surfaces." *Computer Journal*, 9, 388--393.
- 14 Nour Ali, Slimani Abdennasser & Laouami Nasser, (2002), "Foundation settlement statistics via finite element analysis." *Computers and Geotechnics*, 29 (2002) 641-672 Elsevier Science Ltd (2002).
- 15 Nour Ali, Slimani Abdennasser, Laouami Nasser & Afra Hamid, (2003), "Finite element model for the probabilistic seismic response of heterogeneous soil profile." *Soil dynamics and Earthquake Engineering*, 23 (2003) 331-348 Elsevier Science Ltd. (2003).
- 16 Nour Ali (2004), « Investigation stochastique sur les milieux hétérogènes par la méthode des élément finis. » Thèse présentée pour l'obtention du Doctorat d'état. Université de Science et technologie Houari Boumediene Mars 2004.
- 17 Rosenblueth E, (1975) "Point estimates for probability moments." In *Proc. Nat Acad. Sci. USA*, number 10, pages 3812-3814. 1975.
- 18 Rosenblueth E, (1981), "Two point estimates in probabilities." *App. Math. Modelling*, 5:329-335, 1981.
- 19 Yun Hang Chok B E (Hons), MIEAust (2008). "Modeling the effects of soil variability and vegetation on stability of natural slopes." Thesis submitted for the Doctor of philosophy. University of Adelaide School of civil, *Environmental and Mining Engineering*; October 2008.
- 20 Zhu D Y, Lee C F, Qian Q H & Chen G R, (2005), "A concise algorithm for computing the factor of safety using the Morgenstern-Price method." *Can Geotech. J.*, 42, 272-278 (2005).
- 21 Deutsch C V, 2002, "Geostatistical reservoir modelling". Oxford University Press, Oxford, N.Y.
- 22 Vanmarcke E, 1977, "Probabilistic modeling of soil profiles". *Journal of the Geotechnical Engineering Division*, ASCE, 103 (GT11): 1227--1245.
- 23 DeGroot D J & Baecher G B, 1993, "Estimating autocovariance of in-situ soil properties". *Journal of the Geotechnical Engineering Division*, ASCE, 119 (GT1): 147--166.