

Pressure and density of air in mines

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Mines are the deepest cavities in the Earth's crust, where man has set foot and worked. The observed temperature gradients within the mines range from 10 to 50 K/km. In this paper, the pressure and density of air inside mines are calculated by integrating the governing equation for the number density obtained from the hydrostatic equation and the perfect gas laws. The variation of the acceleration of gravity with depth is seen to have only minimal effects on the pressure and air density. The higher temperature gradients inside the mines are found to depress both the pressure and density of air there. Whereas the pressure must necessarily increase with depth due to the hydrostatic equation, the density of air can increase, remain constant, or actually decrease with depth, for temperature gradients of less than, equal to, or greater than 34 K/km, respectively. Air conditions in the deepest mine in the world are discussed.

Keywords: Air pressure, Air density, Mines, Mine air

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1 Introduction

The deepest levels below the Earth's surface where man has set foot are the depths of the deepest mines. Currently, the deepest mine in the world is the Western Deep gold mine near Johannesburg in South Africa¹. It is over 3,500 m (or 3.5 km) deep from the ground level¹ and efforts are being made to deepen this mine to 4,000 m, and eventually² to 5,000 m. Of course, the blistering temperatures encountered at these levels, among many other factors, have to be contended with². The temperature below the Earth's surface rapidly increases with depth. Observed temperature gradients³ in the Earth's crust ranges from 10 K/km to as high as 50 K/km. Typical temperature gradients⁴ are between 20 and 30 K/km, with an average value^{5,6} of about 25 K/km. Further, the temperature gradients in the continental crusts are generally lower than those in the oceanic crusts⁷. This paper estimates the pressure and density of air in mines for given temperature profiles.

2 Theoretical considerations

The pressure and density of air in the atmosphere are governed by the hydrostatic equation and the perfect gas laws. Let z denote the vertical distance from the ground level, reckoned positive when downwards. The hydrostatic equation in this coordinate system is written as

$$\frac{dP}{dz} = \rho g \quad \dots (1)$$

where P is the pressure, ρ the mass density of air and g the acceleration due to gravity. Assume perfect gas equations to hold one can write

$$P = NkT \quad \dots (2)$$

where N is the number density (number of molecules per unit volume), k the Boltzmann constant and T the absolute temperature. The terms ρ and N are related by the equation

$$\rho = mN \quad \dots (3)$$

with m denoting the mean molecular weight of air.

In the Earth's atmosphere, m is constant (equal to 28.8 amu) in the homosphere (up to about 85 km altitude) due to turbulent mixing of the constituent gases. The same condition is assumed to hold within the mines, which are only 3.5 km deep at the deepest. Equations (1)-(3) then give

$$\frac{1}{N} \frac{dN}{dz} + \frac{1}{T} \frac{dT}{dz} = \frac{mg}{kT} \quad \dots (4)$$

Equation (4) is the governing equation for the number density of air. If the dependence of T and g

are known as functions of the depth, then this equation can possibly be integrated to give N . Once that is achieved, the pressure P and density ρ are obtained from Eqs (2) and (3), respectively.

In this paper, it is assumed that the temperature gradient with depth α is constant, so that one can write

$$T = T_0 + \alpha z \quad \dots (5)$$

where T_0 is the temperature at ground level (i.e., mouth of the mine). Substitution of Eq. (5) in Eq. (4) gives

$$\frac{dN}{N} = \left(\frac{mg}{k} - \alpha \right) \frac{dz}{T} \quad \dots (6)$$

To find the dependence of g on z , one has to know the density distribution inside the Earth up to its center, which will necessarily remain speculative. In this paper, the authors consider two models for g . In the first model (Model A), the density of the Earth is considered uniform. In that case, g will decrease linearly with z up to the centre of the Earth, where it is zero⁸. In Model A, one can write

$$g = g_0 \frac{r_0 - z}{r_0} \quad \dots (7)$$

where g_0 is the acceleration of gravity at the surface (mouth of the mine) at a radial distance r_0 from the centre of the Earth.

Substitution of Eqs (5) and (7) in Eq. (6) gives

$$\frac{dN}{N} = \left(\frac{mg_0}{k} - \alpha \right) \frac{dz}{\alpha z + T_0} - \frac{mg_0}{kr_0} \frac{z dz}{\alpha z + T_0} \quad \dots (8)$$

Equation (8) can readily be integrated from the surface ($z = 0$) up to a depth z , giving

$$N = N_0 \left(\frac{T}{T_0} \right)^{\left[\frac{mg_0 T_0 + mg_0}{kr_0 \alpha^2 + k\alpha} - 1 \right]} \exp \left(- \frac{mg_0 z}{kr_0 \alpha} \right) \quad \dots (9)$$

whence, from Eqs (2) and (3)

$$P = P_0 \left(\frac{T}{T_0} \right)^{\left[\frac{mg_0 T_0 + mg_0}{kr_0 \alpha^2 + k\alpha} \right]} \exp \left(- \frac{mg_0 z}{kr_0 \alpha} \right) \quad \dots (10)$$

and

$$\rho = \rho_0 \left(\frac{T}{T_0} \right)^{\left[\frac{mg_0 T_0 + mg_0}{kr_0 \alpha^2 + k\alpha} - 1 \right]} \exp \left(- \frac{mg_0 z}{kr_0 \alpha} \right) \quad \dots (11)$$

The actual density distribution within the Earth, however, is far from uniform. From a plausible density distribution consistent with earthquake data, Bullen⁹ calculated the variation of g with depth and found surprisingly that g should remain nearly constant down to a depth of 2,500 km. In the second model (Model B), g is assumed to be constant. Equation (6) then readily integrates to

$$N = N_0 \left(\frac{T}{T_0} \right)^{\frac{mg}{k\alpha} - 1} \quad \dots (12)$$

Thus

$$P = P_0 \left(\frac{T}{T_0} \right)^{\frac{mg}{k\alpha}} \quad \dots (13)$$

and

$$\rho = \rho_0 \left(\frac{T}{T_0} \right)^{\frac{mg}{k\alpha} - 1} \quad \dots (14)$$

3 Model Results

In this study, model results for a representative mine are given. The surface value of gravity g_0 is taken as the world-wide average¹⁰ of 9.8 m/s². The surface value of temperature T_0 is taken as the world average¹¹ of 15°C or 288 K. Three model temperature gradients are assumed: $\alpha = 10, 30$ and 50 K/km. First, the results given by Eqs (10) and (11) of Model A and those given by Eqs (13) and (14) of Model B agreed to within 1% of each other up to a depth of 3.5 km, even though their functional forms are entirely different. This signifies that the variation of g with depth did not have much impact on the pressure and density of air in the mines at the depths under consideration. Thus, temperature was the main factor responsible for the state of air in the mines. The following results are representations of both models.

Figure 1 shows the variation of pressure with depth for the three temperature gradient models given by $\alpha = 10, 30$ and 50 K/km. In all three models, the pressure increased rapidly with depth reaching values of 1.48, 1.42 and 1.38 times the surface pressure at 3.5 km. The highest magnitudes of pressure were attained for the lowest temperature gradient, showing

thereby that the temperature gradient in the mines actually acted to suppress the pressures within. However, the hydrostatic equilibrium condition ensures that, no matter how high the temperature gradient is, the pressure of air must necessarily increase with depth. This will not be true for the density of air, as discussed in the following.

In Fig. 2, the variations of density of air with depth are shown for the three temperature gradient models given by $\alpha = 10, 30$ and 50 K/km. The effect of the temperature gradient was again to suppress the density of air, but the effect was quite far reaching. For $\alpha = 10$ K/km, the density increased rapidly with depth, reaching a value 1.3 times that of the surface density at 3.5 km. For $\alpha = 30$ K/km, the density of air

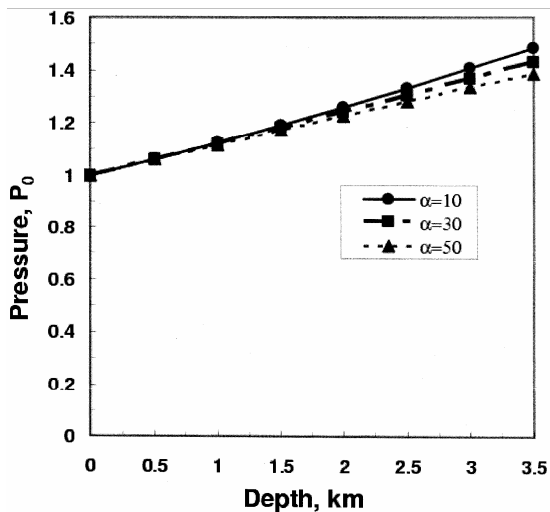


Fig. 1 — Variation of air pressure with depth inside mines for temperature gradients of $\alpha = 10, 30$ and 50 K/km

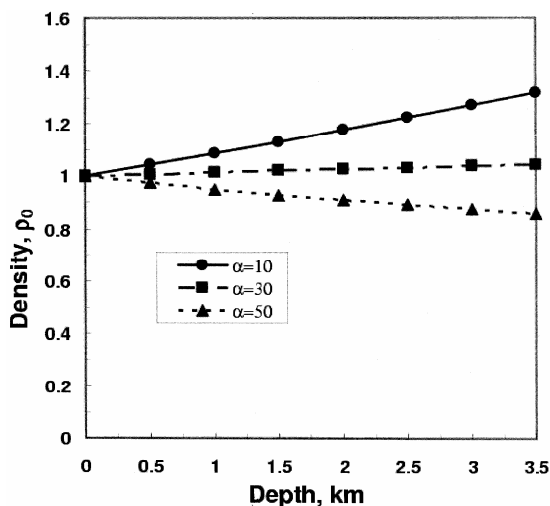


Fig. 2 — Variation of air density with depth inside mines for temperature gradients of $\alpha = 10, 30$ and 50 K/km

remained nearly constant in depth. However, the most interesting result was observed for $\alpha = 50$ K/km: the density of air actually decreased with depth, attaining a value of about 86% that of the surface density at 3.5 km.

One can estimate the temperature gradient for which the density of air would remain constant in depth. This would happen when the exponent in Eq. (14) is zero, i.e.

$$\alpha = \frac{mg}{k} \quad \dots (15)$$

Taking the value of $m = 28.8$ amu and $g = 9.8$ m/s², one obtains $\alpha = 34$ K/km. Thus, the density of air in mines would increase, remain constant, or actually decrease with depth depending upon whether the temperature gradient α was less than, equal to, or greater than 34 K/km.

4 Comparison with actual conditions

The authors emphasize that detailed pressure and density measurements within deep mines could not be found in the literature despite an intensive search, and probably do not exist. Thus, the validation of the present results with experimental data could not be made. One can surmise that this is because increased pressure and density of air are not of major concern to the health and safety of the miners, as high temperatures are. In the Western Deep mine, refrigerated air is pumped into the depths, where the miners work through an insulated shaft in order to bring the temperature² down to 28°C. As a result, the pressure of air increases to twice¹² that at the surface. This was the sole reference to the pressure inside the Western Deep that the authors could find.

The Western Deep is located at a latitude of 26°S at an elevation¹³ of $h = 1740$ m. For T_0 , the authors take the average value of the temperature¹⁴ of the nearest meteorological station of Johannesburg, which is 15 °C or 288 K. Note that this happens to be the same as the average surface temperature¹¹ on the globe. From the reported virgin rock temperatures of about 57°C or 330 K at 3.5 km in the Western Deep mine², the temperature gradient is estimated to be about 12 K/km, as it is in most stable old continental crusts^{7,15}. Thus the uppermost lines in Figs 1 and 2 are appropriate for comparison.

The distance r_0 in Eqs (10) and (11) is equal to

$$r_0 = r_\lambda + h \quad \dots (16)$$

where r_λ is the radial distance from the centre of the Earth to the geoid (which would be the sea level over the oceans) at the latitude under consideration. Considering the Earth to be an ellipsoid of revolution having equatorial radius r_e and polar radius r_p , one can have¹⁶

$$r_\lambda^2 = \frac{r_p^2 r_e^2}{r_p^2 \cos^2 \lambda + r_e^2 \sin^2 \lambda} \quad \dots (17)$$

Substituting for $r_e = 6,378$ km, $r_p = 6,357$ km and $\lambda = 26^\circ$, one obtains $r_\lambda = 6,373.95$ km.

Taking both the centrifugal acceleration and ellipsoidal shape of the Earth into consideration, the value of acceleration due to gravity at any latitude λ at sea level, in cgs units, is given by¹⁰

$$g_\lambda = 978.03185 (1 + 0.0053024 \sin^2 \lambda - 0.0000059 \sin^2 2\lambda) \quad \dots (18)$$

For $\lambda = 26^\circ$, $g_\lambda = 979.02484$ cm/s². Assuming an inverse square variation of g above sea level, one can have

$$g_0 = g_\lambda \frac{r_\lambda^2}{(r_\lambda + h)^2} \quad \dots (19)$$

Upon substituting the values of g_λ , r_λ and h , one gets $g_0 = 978.4905$ cm/s². This is not far from the model values of $g = 9.8$ m/s² used earlier.

The resulting variations of pressure and density inside the Western Deep mine should be close to those given by the upper lines in Figs 1 and 2. According to theory and observation, the temperature, pressure and density of undisturbed air at 3.5 km depth in the Western Deep should be 330 K, 1.48 P_0 and 1.3 ρ_0 , respectively. It is understood that after the introduction of refrigerated air, the temperature is brought down to 28°C or 301 K and the pressure rises to 2 P_0 as a result. One can verify that 0.52 moles of

refrigerated air at 220 K and 1.48 moles of ambient air at 330 K can account for the working condition temperature and pressure of 301 K and 2 P_0 , respectively.

One should point out that atmospheric pressure at the surface of the mine (elevation 1740 m) is only about 82% that at sea level¹⁷. However that does not affect the results, which are expressed in terms of surface pressure and density.

Acknowledgement

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References

- 1 <http://www.hypertextbook.com/facts/2003/YefimCavalier.shtml>.
- 2 <http://www.deepmine.csir.co.za/Press%20Clips/clip02.htm>.
- 3 <http://www.du.edu/~jcalvert/geol/termag.htm>.
- 4 http://www.hww.geol./sec.edu/Faculty/Nunn/gl17065/Chp4_notes.html.
- 5 http://www.uky.edu/AS/Geology/howell/220/notes06_metamorphic-s.
- 6 <http://www.madsci.org/posts/archives/mar2002/1017275290.Es.r.html>.
- 7 <http://Ideo.columbia.edu/users/jcm/Topic3/Topic3.html>.
- 8 Kittel C, Knight W D & Ruderman M A, *Berkeley Physics Course*, Vol. 1 (McGraw-Hill, New York) 1965, p. 266.
- 9 Bullen K E, *Month Not R Astr Soc, Geophys Suppl (UK)*, 3 (1936) 395.
- 10 Tsuboi C, *Gravity* (George Allen and Unwin, London), 1983, p. 78.
- 11 McBride N & Gilmour I G (Eds), *An Introduction to the Solar System* (Cambridge University Press, Cambridge), 2004, p. 376.
- 12 <http://www.scienceblog.com/older/2001/C/200113499.html>.
- 13 <http://www.britannica.com/eb/article9046293>.
- 14 <http://www.world66.com/africa/southafrica/johannesburg/lib/climate>.
- 15 http://www.geology.wise.edu/~chuck/Mtn_and_plates.html.
- 16 Weisstein E W, *CRC Concise Encyclopedia of Mathematics* (Chapman & Hall/CRC, Boca Raton), 2003, p. 868.
- 17 *US Standard Atmosphere* (NASA, NOAA, USAF, Washington), 1976.