

Sensitivity and predictive capability studies on a finite element surge simulation model

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Received 15 December 1998, revised 9 February 2000

This paper reports the sensitivity and predictive capability studies carried out on the finite element model for storm surge simulation along the east coast of India. In this model, an explicit finite element scheme with 9-noded Lagrangian quadratic isoparametric elements was used to discretize the study area and the governing equations. The integration of dependent variables over an element was performed using two-dimensional Simpson's rule, which leads to uncoupling of the system of equations and yields an explicit scheme for determining the flow parameters evolving with time. In the present work, the model has been benchmarked using an International software MIKE21. Sensitivity studies are carried out to evaluate the influence of the length of the ocean shelf and its depth at the coast in predicting the storm surges along the coast. The results show that the model is not sensitive to these parameters. Then the model is calibrated by simulating the storm surges induced by the 1977- Andhra cyclone, which is one of the severe cyclones in recent years along the east coast of India. The predictive capability of the calibrated model is assessed by simulating the surges for a few past cyclonic storms, which have crossed the east coast of India.

The east coast of India is cyclone prone and frequently affected by storm surges. Since storms and surges are natural phenomena, they cannot be prevented or modified. But, if a well-validated model is available, the time history of the sea level near the coast (i.e. storm surges) can be forecast before storm landfall so that necessary precautions can be taken before the arrival of the storm and the losses to life and property in the coastal areas can be minimized. Several numerical models have been developed in India for the simulation of surges along the east coast of India¹⁻⁵. All these surge simulation models use the finite difference method to solve the equations governing the ocean flow field subjected to the wind forcing associated with a storm.

In recent years finite element method (FEM) has become more attractive in shallow water flow simulation problems because of the flexibility it provides for representing irregular boundaries and complex topographies of the study area in the model. Further, this method is also capable of accommodating elements of different sizes over the flow domain of interest. Because of these advantages, Ajoykumar⁶ developed a finite element model, which uses three noded triangular elements for discretising the problem for surge simulation along the east coast of India. But it was observed that this model requires too, much of computational efforts to be useful for real time prediction of surges. Hence, Mahadevan &

Aswathanarayana⁷ developed a surge simulation model using an explicit finite element scheme⁸ which uses higher order elements and Simpson's rule for the integration of dependent variables over an element, which leads to an explicit scheme requiring less computational time.

The results of the sensitivity studies carried out on this explicit finite element model to evaluate the influence of the length of the coastal shelf and the shelf depth at the coast in predicting storm surges along the coast, are reported in this paper. The model is then calibrated by simulating the storm surges induced by the 1977-Andhra cyclone (Fig. 1). The predictive capability of the calibrated model is assessed by simulating the surges for a few past cyclonic storms, which crossed the east coast of India. Prior to sensitivity studies the FEM model is first used to simulate the wind induced flow fields in ocean basins with regular geometries and the resulting flow fields are compared with those simulated by MIKE21, which is an advanced mathematical modelling software, developed by Danish Hydraulics Institute, Netherlands, for the simulation of two-dimensional free surface flows in shallow regions.

Methodology

The vertically integrated forms of the shallow water equations governing the ocean flow field in an ocean shelf, in a Cartesian co-ordinate frame fixed to the rotating earth, are the continuity equation:

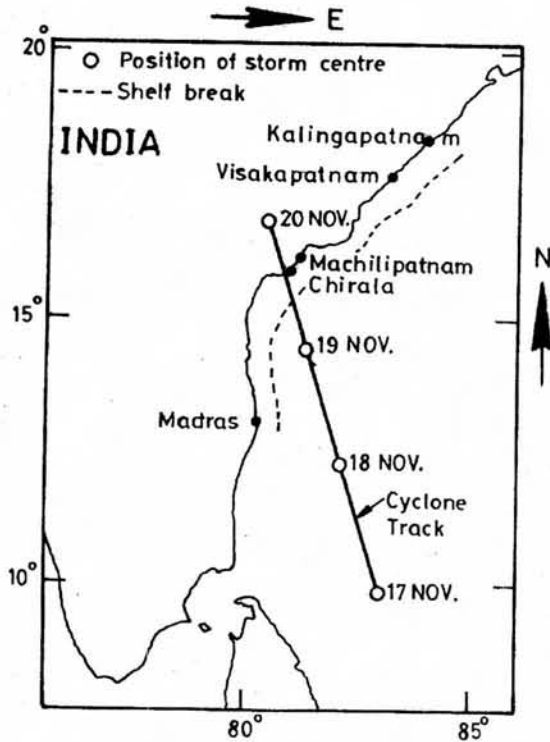


Fig. 1 — Shelf between Madras and Kalingapatnam with an idealised track of 1977 Andhra cyclone.

$$q_{x,x} + q_{y,y} + \zeta_{,t} = 0, \quad \dots (1)$$

and the momentum equations in the x and y directions:

$$q_{x,t} + H^{-1} q_x q_{x,t} + H^{-1} q_y q_{x,y} - f q_y = -\rho^{-1} H p_{a,x} - g H \zeta_{,x} + \rho^{-1} (\tau_{ax} - \tau_{bx}), \quad \dots (2)$$

$$q_{y,t} + H^{-1} q_y q_{y,t} + H^{-1} q_x q_{y,x} + f q_x = -\rho^{-1} H p_{a,y} - g H \zeta_{,y} + \rho^{-1} (\tau_{ay} - \tau_{by}) \quad \dots (3)$$

Here the origin of the coordinate system is chosen at the undisturbed sea surface and z is measured positive upwards. The pressure is assumed hydrostatic and the astronomical tide generating forces are neglected. The volume transport components are defined as $q_x = \int u dz$ and $q_y = \int v dz$, where the integration limits are from $-h$ to ζ . (u , v) are the components of velocity in the (x , y) directions respectively and t is the time. The suffixes preceded by comma indicate partial derivatives. $H = h + \zeta$ is the total depth of water, h is the undisturbed depth of water and ζ is the elevation of the sea surface, measured from the undisturbed sea surface. p_a is the atmospheric pressure, f is the Coriolis parameter, ρ is the density of water and g is the acceleration due to gravity. (τ_{ax}, τ_{ay}) and (τ_{bx}, τ_{by}) are the stresses at the air-sea interface (wind stresses)

and at the bottom surface (bottom stresses) respectively.

The wind and the bottom stresses are evaluated using the conventional quadratic law as follows:

$$\tau_a = K_a \rho_a |W| W, \quad \tau_b = K_b \rho |q| q, \quad \dots (4a,b)$$

where, K_a and K_b are the wind and bottom stress coefficients respectively, ρ_a is the air density, $W \equiv (W_x, W_y)$ is the wind velocity measured 10 m above the sea level and the volume transport $q \equiv (q_x, q_y)$.

Initial and boundary conditions

The usual practice in storm surge simulation studies is to assume the ocean to be initially at rest, before the introduction of the wind stresses at the ocean free surface, i.e. $\zeta, q_x, q_y = 0$ for $t \leq 0$. In the present study, the zero gradient condition is assumed for all the flow variables along the lateral boundaries. Along the open ocean boundary the clamped condition ($\zeta = 0$) is used in view of the observations of Jossy *et al.*^{4,5} and the conventional impermeable vertical side wall assumption is made along the coastal boundary.

Explicit finite element model

In this model, the flow domain is first divided into 9-noded quadrilateral elements. The variables (ζ, q_x, q_y) over the elements are represented by Lagrangian quadratic basis functions (i.e. piece-wise continuous polynomials) in the form,

$$\zeta(x, y, t) \approx \sum_{j=1}^N \zeta_j(t) \phi_j(x, y), \quad \dots (5a)$$

$$q_x \approx \sum_{j=1}^N q_{xj}(t) \phi_j(x, y), \quad \dots (5b)$$

$$q_y \approx \sum_{j=1}^N q_{yj}(t) \phi_j(x, y), \quad \dots (5c)$$

where, ζ_j, q_{xj}, q_{yj} are the unknown nodal values of the dependent variables in an element, $\phi_j(x, y)$ are the shape functions and N is the number of nodes in each element. The approximate solutions (Eqs 5a-c) are substituted into the governing equations (Eqs 1-3) and the Galerkin's method of weighted residuals is employed to get the element equations. In this method, the shape functions are themselves used as the weighting functions and the integrated weighted residuals, $\epsilon_\zeta, \epsilon_x$ and ϵ_y , in each element or the element equations become,

$$\int_A [\phi]^T \{ \zeta_{,t} + q_{x,x} + q_{y,y} \} dA = \epsilon_\zeta, \quad \dots (6)$$

$$\int_A [\phi^e]^T \{q_{x,t} + H^{-1}q_x q_{x,x} + H^{-1}q_x q_{x,y} - f q_y + \rho^{-1} H p_{a,x} + g H \zeta_{x,y} - \rho^{-1}(\tau_{ax} - \tau_{bx})\} dA = \varepsilon_x \quad \dots (7)$$

$$\int_A [\phi^e]^T \{q_{y,t} + H^{-1}q_y q_{y,x} + H^{-1}q_y q_{y,y} + f q_x + \rho^{-1} H p_{a,y} + g H \zeta_{y,x} - \rho^{-1}(\tau_{ay} - \tau_{by})\} dA = \varepsilon_y \quad \dots (8)$$

To perform these integration in the above equations, a transformation is made element-wise from the (x-y) system to a (ξ-η) system, such that each element in the (ξ-η) system is a square with corners at (ξ, η) = (±1, ±1) (see Fig. 2). The transformation between these co-ordinate systems is defined in the form,

$$x = x(\xi, \eta) = \sum_N x_j \phi_j(\xi, \eta) \quad \text{and} \quad y = y(\xi, \eta) = \sum_N y_j \phi_j(\xi, \eta) \quad \dots (9)$$

where, (x_j, y_j) are the co-ordinates of the nodal points of the elements in the (x-y) system. The basis functions, φ_j, in this transformation have the form⁸,

$$\phi_i = \frac{1}{4}(\xi \xi_i + \xi^2)(\eta \eta_i + \eta^2), \quad (\xi_i, \eta_i) = (\pm 1, \pm 1), \quad \dots (10)$$

$$\phi_i = \frac{1}{2}(\xi \xi_i + \xi^2)(1 + \eta^2), \quad (\xi_i, \eta_i) = (\pm 1, 0), \quad \dots (11)$$

$$\phi_i = \frac{1}{2}(\eta \eta_i + \eta^2)(1 + \xi^2), \quad (\xi_i, \eta_i) = (0, \pm 1), \quad \dots (12)$$

$$\phi_i = (1 + \xi^2)(1 - \eta^2), \quad (\xi_i, \eta_i) = (0, 0). \quad \dots (13)$$

Introducing the above transformation element-wise in Eqs 6-8, systematically assembling them and equating the integrated residue over the whole flow domain to zero, we get the system equations.

The integrals in the system equations are to be evaluated exactly by a numerical scheme, for which two-dimensional Simpson's rule, involving 9 integration points, is used. For a given function, G(ξ, η), the two-dimensional Simpson's rule is given as:

$$\int G(\xi, \eta) d\xi d\eta = \frac{\Delta \xi \Delta \eta}{9} \left(G_{-1,1} + 4G_{0,1} + G_{1,1} + 4G_{-1,0} + 16G_{0,0} + 4G_{1,0} + G_{-1,-1} + 4G_{0,-1} + G_{1,-1} \right) \dots (14)$$

In the Simpson's rule of integration, as the values of the integrands are required at the nodal locations and as the basis function, φ_j appears as a factor in the integrand of all the integrals in the system equations, the integrands will be non-zero only at one nodal point. Thus, when the Lagrangian basis functions are used, the nine-point Simpson's integration formula

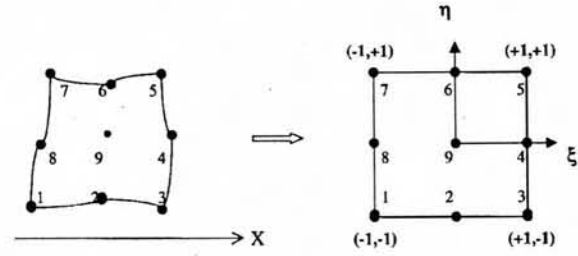


Fig. 2 — A 9-noded Lagrangian quadratic isoparametric element and the relationship between (x,y) and (ξ-η) co-ordinate systems.

becomes, effectively, a one-point formula. Consequently the system equations become diagonal and we get a set of first order differential equations with respect to time for the unknown nodal values of the variables, which leads to an explicit finite element scheme⁸ for the determination of the field variables. We use the Leapfrog scheme for the integration of the system equations over time.

MIKE21 hydrodynamic module

The hydrodynamic module of MIKE21 has been used to simulate a wide range of shallow water flow fields. It employs the alternate direction implicit scheme (i.e. a finite difference method) to integrate the full non-linear equations of continuity and conservation of momentum.

In this software, the grids are specified with uniform spacing along the coordinate axes and stair step approximation is used to represent the irregular boundaries of the study area. The boundary separating the water and land is specified using positive values on the land. Conditions along the open boundaries are prescribed by specifying the water levels or fluxes as a function of time.

Results and Discussion

The finite element software developed for storm surge simulation is first used for simulating the flow field in idealized ocean shelf regions. Three numerical experiments are carried out and the flow parameters simulated by the present model are compared with those simulated by MIKE21 hydrodynamic module.

In the first experiment, the wind induced flow fields over two rectangular ocean shelves of width 30 km and 60 km are simulated. The length and the depth of the shelves are fixed as 200 km and 20 m respectively. Grid spacing of 5 km across the shelf and 10 km along the coastline and a time step of 60 seconds are used, in both finite element and MIKE21 models.

The time histories of water surface elevations, ζ(t), at the coast (Fig. 3), predicted by the two models for

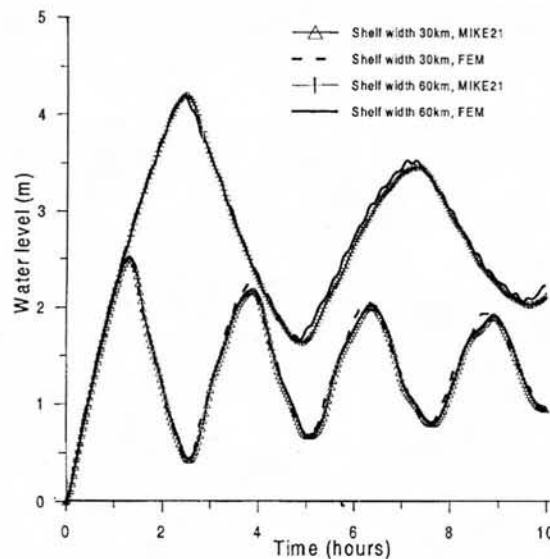


Fig. 3—Time series of water levels over a constant depth shelf when a uniform wind of 100 knots blows normal to the coast.

both the shelves show good agreement. For the 30 km wide shelf, the water surface elevation, $\zeta(t)$, exhibits slowly damped oscillations of period 150 minutes over the steady state surface elevation of 1.47 m. This steady state estimate is verified using bathystropic approximation. For the 60 km wide shelf, the period of oscillations in $\zeta(t)$ and the steady state surface elevation are found to be ≈ 300 minutes and 2.94 m respectively, which are twice of those for the 30 km wide shelf. The periods of oscillations observed in the predicted $\zeta(t)$ for the two shelves compare well with those of the first free mode of oscillations of the shelf waters (see the Appendix). This suggests the possible excitation of the first mode of free oscillations when the wind stresses are introduced suddenly in the model.

In the next experiment, again rectangular ocean shelf but with water depths varying across the shelf as given in Table 1, are considered. The flow field induced by a uniform wind of 100 knots blowing normal to the coast is simulated first in a 30 km wide shelf. Then the simulation experiment is repeated in a 60 km wide shelf, keeping the water depth in the extended region as 200 m. In the case of variable depth shelf, unlike in the case of a constant depth shelf, increasing the shelf width to 60 km does not lead to a proportional increase in the peak surge estimates or in the period of oscillations in $\zeta(t)$ (see Fig. 4). The periods of oscillations in $\zeta(t)$ predicted by the finite element model (115 and 131 minutes for the 30 km and 60 km wide shelves respectively) are

Table 1—Depth variations across the shelf

Distance from the coast (km)	Depth (m)
0	10.0
5	10.0
10	13.5
15	20.3
20	44.3
25	200.0
30	200.0

observed to agree well with those (120 and 130 minutes) predicted by MIKE21 software (see Fig. 4). The periods of the free oscillations of the shelf waters (see the Appendix) are also found to confirm this result. The deviations observed between the time histories (Fig. 4) predicted by the two models may be due to the method of approximations used in the models.

In the third experiment, cyclone wind induced flow fields are simulated, in a rectangular shelf (500 km along and 30 km across the shore) with the depth profile as shown in Table 1. Wind distribution associated with the cyclone is computed using the following wind model⁹:

$$\begin{aligned}
 |W| &= W_m (r/R)^{3/2} && \text{for } r \leq R, \\
 &= W_m \exp[(R-r)/\alpha] && \text{for } R < r < R_1, \\
 &= W_m \exp[(r-R_1)/\alpha] \exp[(R_1-r)/\beta] && \text{for } r > R_1, \quad \dots (15)
 \end{aligned}$$

where, W_m is the maximum wind speed inside the cyclone, R is the radius at which the wind speed is maximum, $|W|$ is the wind speed at a radial distance ' r ' from the center of cyclone and α and β are numerical constants having dimensions of length. The cyclone parameters used in this experiment are (those used in by Johns *et al.*⁹ for the 1977-Andhra cyclone): $W_m = 70$ m/sec; $R = 80$ km; $R_1 = 175$ km and the constants α and β are 200 km and 10 km respectively. The cyclone is initially centered at a distance of 175 km away from coast along the center line of the shelf when the simulation commences and is moved normal to the coast at a speed of 4 m/s. The wind stress coefficient $K_a = 0.0028$. The same wind field is used when the flow simulation experiment is repeated with MIKE21 module. Figure 5 shows the predicted surge profile along the coast, half an hour before landfall, simulated by the finite element model agreeing very well with those predicted by the MIKE21 software.

The surge profile along the coast shows negative surges on the left side of the cyclone track and positive surges on the right side. Far away from the

cyclone center where the wind velocities are insignificant, the surge heights along the coast are observed to be small. The results above and the simulated flow field presented in the form of surge height contours and vector plots of flow fields (not presented here) show that the FEM model simulates all the expected features of wind induced flow fields in ocean shelves with idealized geometry.

Then the sensitivity studies are carried out on the finite element model to evaluate the influence of the

length of the ocean shelf and the shelf depth at the coast in predicting the surges along the coast. In these experiments, the flow field in the ocean shelf along the east coast of India between Madras and Kalingapatnam is simulated and the wind field associated with the 1977-Andhra cyclone is used as the driving force in the model. The cyclone is initially positioned at longitude $83^{\circ} 7' E$ and latitude $10^{\circ} N$ and is moved at a speed of 4 m/s along an idealized straight line track as shown in Fig. 1, with the landfall occurring at Chirala.

Then the length of the ocean shelf on both sides of the landfall point is gradually reduced in the model and the surge envelopes induced by the 1977-Andhra cyclone are predicted. The predicted surge envelopes for the 1977-Andhra cyclone are presented in Fig. 6 for three segments of the shelf considered in the model. It is observed that by varying the shelf length in the model, the estimates of surges along the coast are not altered (surge envelopes are indistinguishable), except at the two ends of the segment of the shelf.

In these studies a representative depth profile, obtained as an average of four typical depth profiles near Chirala, normalized with local shelf width, has been used. To accommodate the anticipated negative surges on the left side of the cyclone track, the depth of the shelf at the coast is taken as 10 m. It can also be seen from the hydrographic charts that the 10 m depth contour is very close to the coastline, which justifies the assumption of a vertical wall of 10 m height at the coast. Though the assumption of 10 m depth at the coast is justified, experiments are conducted to see

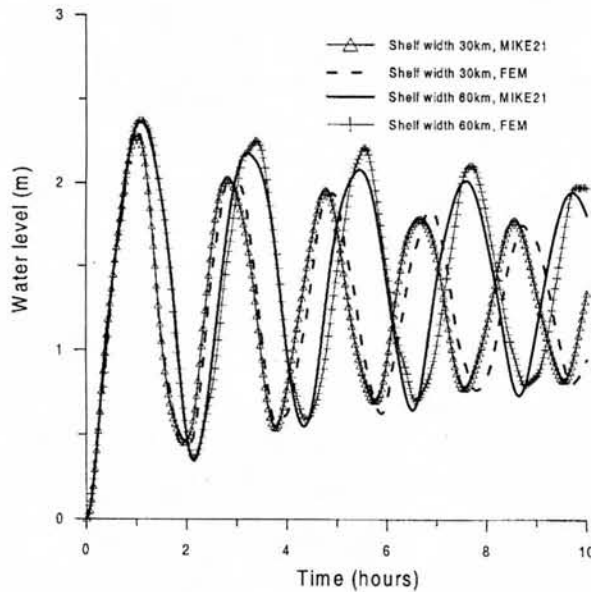


Fig. 4—Time series of water levels over a varying depth shelf when a uniform wind of 100 knots blows normal to the coast.

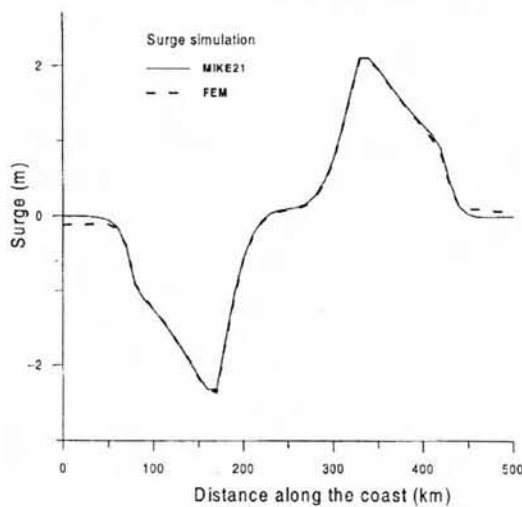


Fig. 5—Instantaneous surge along a straight coast, caused by a cyclone initially centered at 175 km away from the coast and started moving normal to the coast at 4 m/s. $W_m = 70$ m/s, $R = 80$ km.

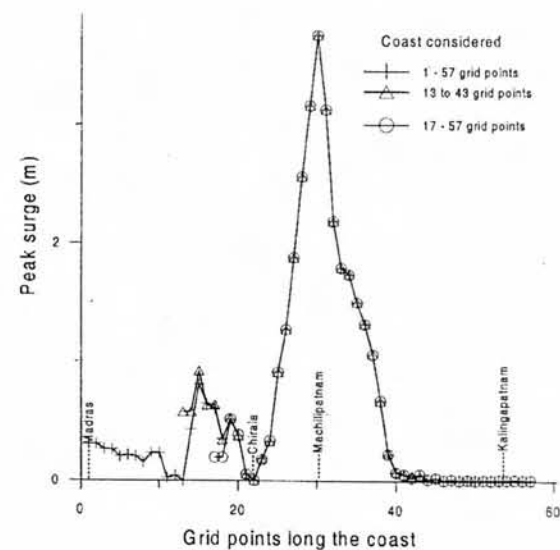


Fig. 6—Surge envelopes for the 1977-Andhra cyclone predicted with different lengths of ocean shelf considered in the model.

Table 2—Peak surge estimates for the 1977-Andhra Cyclone with different wind stress coefficients K_a

K_a	Peak surge (m)	
	Depth at the coast	
	5 m	10 m
0.0028	3.91	3.83
0.0003	4.15	4.07
0.0034	4.60	4.53
0.0036	4.82	4.75
0.0038	5.04	4.98
0.0040	5.26	5.20

Table 3—Predicted and reported peak surges for different cyclonic storms along the east coast of India

Year/Landfall of the cyclone	Predicted peak surge (m)		Observed peak surge (m)
	IIT Madras model	IIT Delhi model ¹²	
	K_a	K_a	
	0.0028	0.0038	
1927, Nellore	1.30	1.70	3.07
1949, Machilipatnam	1.80	2.30	2.09
1969, Kakinada	0.93	1.21	2.82
1972, Sriharikota	0.76	0.99	1.23
1977, Chirala	3.80	4.98	4.93
1984, Sriharikota	1.80	2.30	2.40
1989, Kavali	2.72	3.50	3.80
1990, Divi	3.80	4.90	4.41
1996, Kakinada	0.90	1.13	1.60

whether a reduction in depth at the coast influences the peak surge estimate. The depth at the coast on the right side of the landfall point is changed to 5 m in the model and it is observed (Table 2) that the predicted peak surges show no significant difference from those estimates based on the model with the shelf depth of 10 m at all grid points along the coast.

Some of the earlier studies on surge simulation (ref. 7,4,11), have identified that among the various model and shelf parameters, the wind stress coefficient, K_a , is the dominant one. Hence K_a has been used as the tuning parameter for calibrating the model. In the above experiments K_a is assumed as a constant ($K_a=0.0028$). With this value of K_a , the model estimate of peak surge for the 1977-Andhra cyclone was 3.8 m, which is significantly lower than the recorded value of 5 m. So the peak surges are predicted for different values of K_a and they are shown in Table 2. It is observed that for the value of $K_a=0.0038$, the model predicts a peak surge height of 5.04 m and 4.98 m in the two experiments with the depth at the coast on the right side of the landfall point as 5 m and 10 m respectively, which is comparable with the observed peak surge of 5 m.

The predictive capability of the calibrated model is assessed by simulating the surges for a few past cyclonic storms, which have crossed the east coast of

India. Past cyclone data are collected from IMD and the present model is used to predict the surges. The model predicted peak surges for five out of eight storms considered in the analyses are observed to compare well with the post-storm survey estimates (Table 3).

Note that, while calibrating the model and while assessing its predictive capability, we need to keep the following in our view. The existing tide gauges along the east coast of India are not designed for measuring the sea levels during severe storms and the available estimates of peak surges for the past-storms are based on post-storm surveys. These post-storm survey estimates based on the level of high water marks are subjective in nature and might include the contributions from the surface gravity waves present during the storm, to the peak surge levels. Further the model surge estimates are based on idealized axisymmetric cyclone wind fields and a constant wind stress coefficient, though it is known to depend on the prevailing wind and the wave fields.

Acknowledgement

The authors are thankful to K Chitra for the help rendered in FEM simulations.

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Appendix

Free oscillations of shelf waters

The one-dimensional linearized equations governing the flow field in a rectangular ocean shelf in the absence of wind forcing are,

$$\zeta_{,t} + q_{x,x} = 0, \quad \dots (A1)$$

$$q_{x,t} - f q_y = -gh \zeta_{,x} - (r/h) q_x, \quad \dots (A2)$$

$$q_{y,t} + f q_x = -(r/h) q_y, \quad \dots (A3)$$

where, q_x and q_y are the volume fluxes in the x - and y -directions respectively, h is the water depth and r is the linear bottom friction parameter. The x - and y -axes are taken across and along the shelf and the origin of the co-ordinate system at the coast. Eliminating q_y and ζ from the equations A1 – A3 and seeking a periodic solution for q_x (i.e. $q_x(x, t) = \hat{q}_x(x) \exp(i\omega t)$ and omitting 'cap' for convenience), we get,

$$q_{x,xx} = -\gamma^2 q_x. \quad \dots (A4)$$

where, $\gamma^2 = \omega[(\omega - ir/h)^2 - f^2]/[gh(\omega - ir/h)]$.

In the case of an *ocean shelf with constant depth*, γ is a constant and the solution to equation A4 satisfying the no flux condition at the coastal boundary (i.e. at $x = 0$) is,

$$q_x(x) = A \sin(\gamma x) \quad \dots (A5)$$

Applying the clamped condition, $\zeta = 0$, at the offshore boundary ($x = b$), we get,

$$(A\gamma/i\omega)\cos(\gamma b) = 0,$$

$$\text{i.e. } (\gamma b) = (2n+1)\pi/2, \quad \dots (A6)$$

where, $n=0, 1, 2, 3, \dots$. For different values of n , equation A5 defines different free modes of oscillation of the shelf waters. Since Coriolis force and bottom friction are generally very small (i.e. f and $r \ll 1$), we get $\gamma^2 = \omega^2/(gh)$. For $n=0$, using Eq. (A6),

$$\gamma^2 = \omega^2/(gh) = \pi^2/4b^2,$$

$$\text{and } T = 4b/(gh)^{1/2}, \quad \dots (A7)$$

where, $T = (2\pi/\omega)$ is the period of free oscillation of the shelf waters. Hence, for a shelf of depth 20 m and of width, 30 km, $T = 143$ minutes, and for a shelf of same depth, but of width, 60 km, $T = 285$ minutes.

In the case of an *ocean shelf with variable depth*, γ is not a constant and we need to use a numerical method to solve equation A4. For an ocean shelf of width 30 km and with variable depth as given in Table 1, the period of free oscillation was found to be 105 minutes. When the width of this shelf was increased to 60 km, the period of free oscillations was found to be 125 minutes.