

Control charts for autocorrelated colemanite data

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In this study, two types of control charts have been used to show the effect of autocorrelation (AC) to determine process stability of Espey and Hisarcik Colemanite concentrators. Individuals Control (IC) chart has shown that both concentrators are statistically out of control. On the other hand, according to Special Cause Control (SCC) chart, which considers impact of AC, only Hisarcik concentrator is out of control. SCC chart has been found more appropriate for autocorrelated data since it provides a higher probability of coverage than IC chart.

Keywords: Autocorrelation, Colemanite, Special cause control chart

Introduction

Control charts are used extensively in minerals industry for continuous monitoring of grade variation to ensure conformance with product specifications. Possible causes of grade variations¹⁻⁵ are inherent variation of grade in the deposit, mineralogy of ore and waste, chemical and metallurgical properties of ore, blending and mining method etc. Constructions of charts used in these studies¹⁻⁵ were based on independent and identically distributed observations. However, assumption of independence is often violated in mining. There is a great deal of studies⁶⁻⁹ considering assumption of independence for different industries. However, only few studies^{10,11} have considered impact of autocorrelation (AC) in minerals industry.

This study compares results of Individuals Control (IC) Chart and Special Cause Control (SCC) Chart using data from concentrators of Emet Boron Works (EBW) in order to help mine managers in control chart selection when dealing with autocorrelated data.

Special Cause Control (SCC) Chart

In case value of a particular variable is dependent on its previous values, AC is present in data. If quality characteristic exhibits various levels of AC, control

charts that are based on the assumption of independence give a higher false alarm rate⁷. AC function (ACF) and partial AC function (PACF) are used to detect presence of AC in the data. In order to visually assess AC in a set of time-ordered data, ACF and PACF plots that show non-zero or significant ACs are usually used. Among two main approaches [standard control charts¹⁰ and special cause control (SCC) chart¹²] for constructing control charts for autocorrelated data, SCC chart plots residuals obtained after fitting the process to an autoregressive integrated moving average (ARIMA) model rather than plotting actual observations. ARIMA involves following steps: i) Specification of number of differences (d) of original series needed to produce stationarity, whereas nonstationary series have an ACF that remains significant for half a dozen or more lags, rather than quickly declining to zero; ii) Identification of orders p and q for autoregressive and moving average operators using ACF and PACF; iii) Estimation of parameters of possible models; iv) Model selection using information criteria such as Akaike's Information Criterion (AIC); and v) Model diagnostic checking.

Analysis

Data

Most important boron minerals (borax, ulexite and colemanite) contain different amounts of B_2O_3 in their structures and can be used in industries as concentrates.

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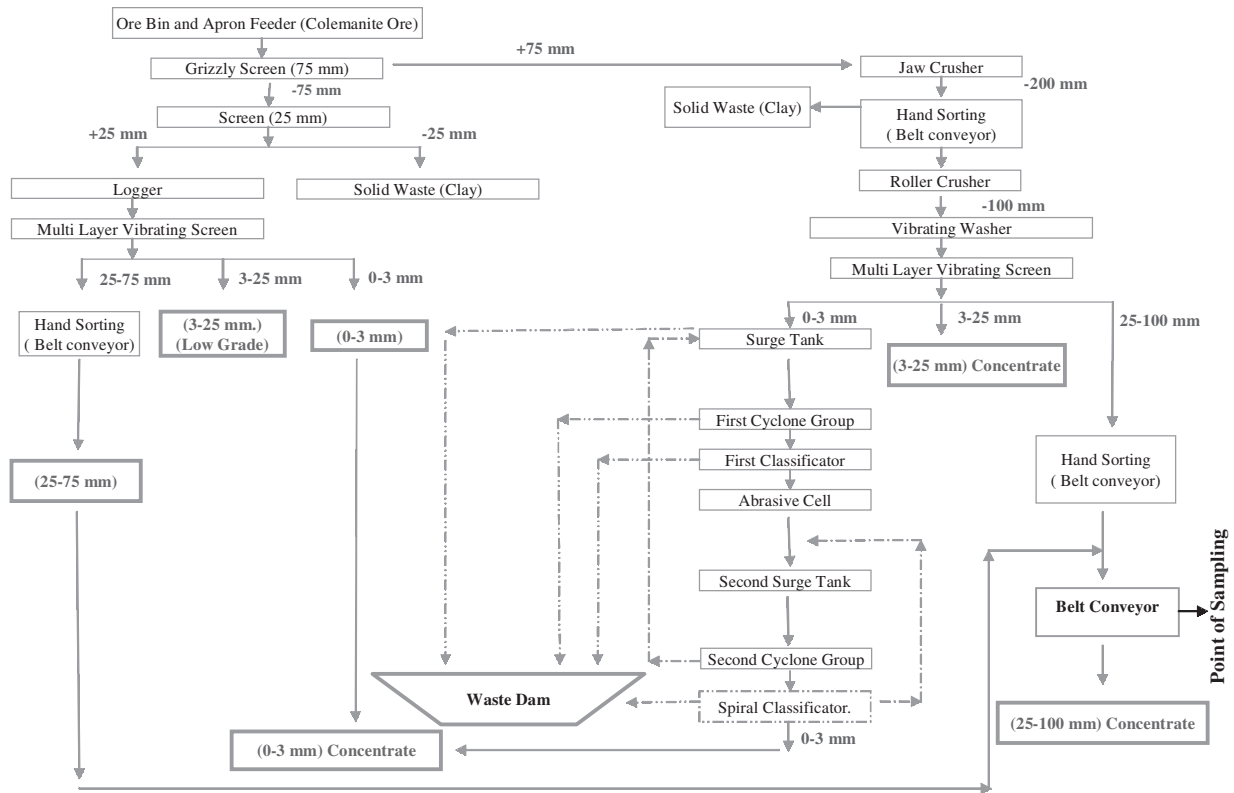


Fig. 1— Flowsheet of Hisarcik concentrator

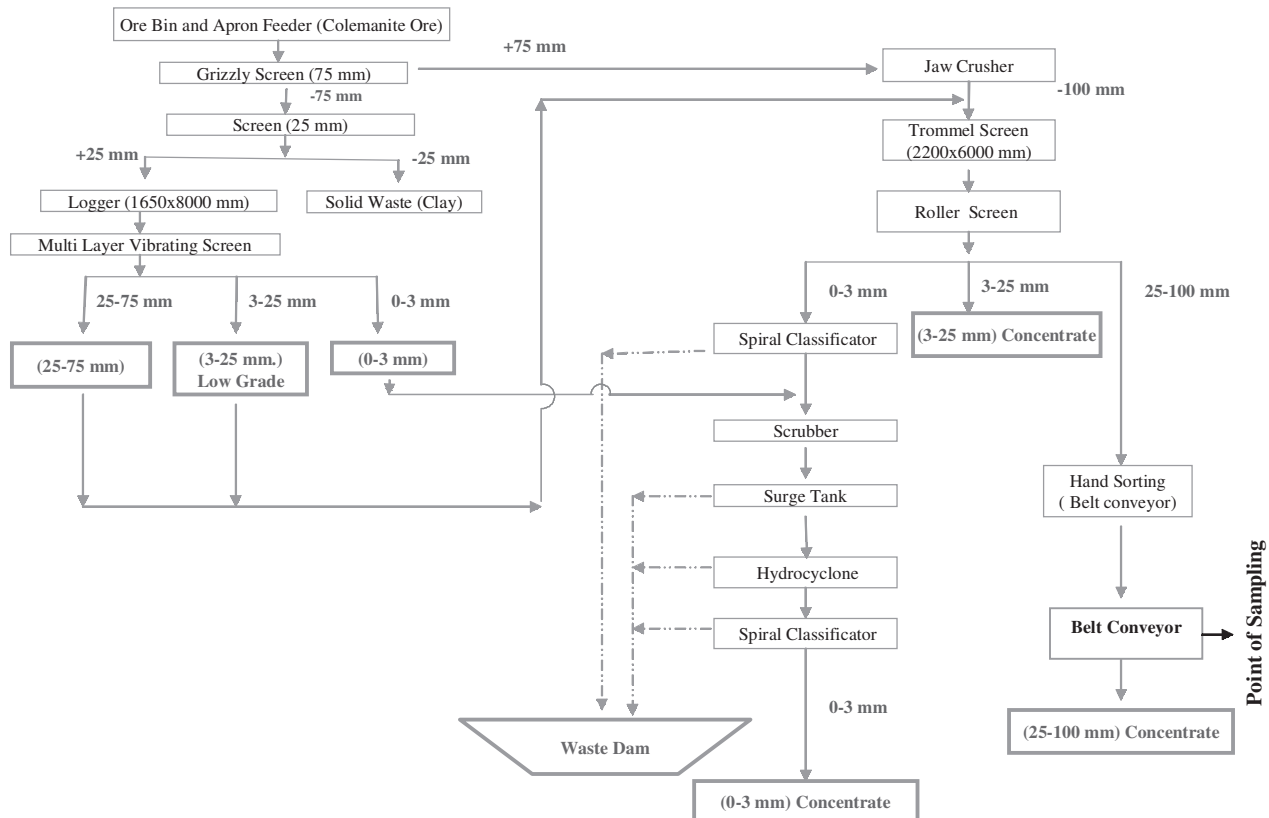


Fig. 2— Flowsheet of Espy concentrator

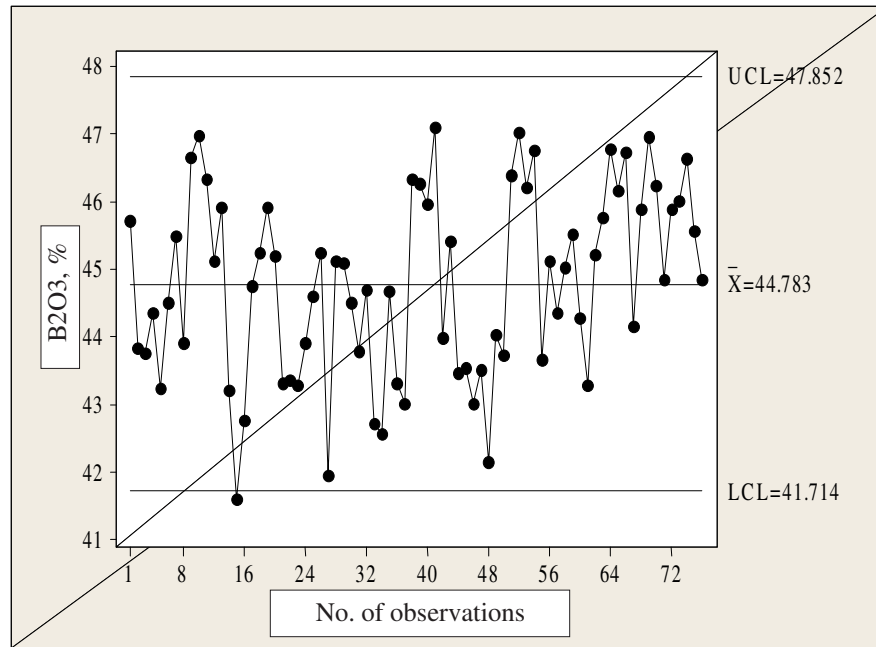


Fig. 3— Control chart for individuals (Espey)

Material of this study is $B_2O_3\%$ contents of colemanite concentrates produced in concentrators of EBW-Kütahya. Colemanite concentrates (size: 25-100 mm, 3-25 mm and 0-3 mm) are obtained from Espey and Hisarcik concentrators by crushing-washing-screening-handsorting. Minimum required $B_2O_3\%$ value for saleable concentrates are: 25-100 mm, 42%; 3-25 mm, 38%; and 0-3 mm, 36%. In present study, $B_2O_3\%$ data for concentrates (25-100 mm) obtained from Espey and Hisarcik concentrators have been used. Sampling points are indicated in flow sheets of concentrators given in Fig. 1 and Fig. 2 respectively. The data gathered regularly on daily basis cover the second quarter of 2006. Sampling method consists collection of sample from belt conveyor 24 times a day, mixing, size reduction (crushing and grinding), coning and quartering, and analyzing.

Espey Concentrate

In order to analyze variation in $B_2O_3\%$ content, control chart for individuals has been firstly established since sample size used for process monitoring is $n=1$. This type of chart uses moving range of two successive observations to estimate process variability. Process is out of statistical control since there is a point outside lower control limit and fluctuations of points around centerline are irregular and excessive (Fig. 3).

Control chart (Fig. 3) is based on the assumption that there is no correlation between successive observations.

However, since assumption of independence of observations is questionable in minerals industry, existence of AC should be investigated in the first place. ACF (Fig. 4) and PACF (Fig. 5) plots indicate positively correlated relation in observations. Since a serious amount of AC is present in data, an ARIMA model should be fitted to them. Prior to fitting ARIMA model, the data should be examined for the presence of any trend since identification process for AR and MA components requires stationary series. ACF and PACF plots revealed that there is no indication for the presence of trend of observations since ACF and PACF rapidly decay to zero after first few lags. Therefore, series is stationary and ARIMA (p,0,q) model should be fitted to data.

Patterns of ACF and PACF plots indicate that ARIMA (1,0,0) or AR (1) model is suitable for data set. Parameters (Table 1) of this model were estimated by using Melard's algorithm. The t statistics of parameters and their associated p values revealed that parameters are statistically significant. When time-series model is correctly specified, ACF and PACF of error series should not be significantly different from zero. Figs 6 and 7 show that residuals are uncorrelated and random. Thus, model (Table 1) is proper for $B_2O_3\%$ data. SCC chart based on residuals of ARIMA (1,0,0) model (Fig. 8) shows that most of the plotted points occur near centerline in a random manner with no clustering or trending, and hence the process is in control statistically.

Table 1— Parameter estimates (approx. signature, 0) of ARIMA (1,0,0) model

Parameter estimates	Espey concentrator		Hisarc1k concentrator	
	Non- seasonal lags AR1	Consantant	Non- seasonal lags AR1	Consantant
Estimates*	0.448	44.793	0.609	44.776
Standard error	0.103	0.255	0.105	0.356
t	4.334	175.956	5.794	125.725

*Melard's algorithm was used for estimation

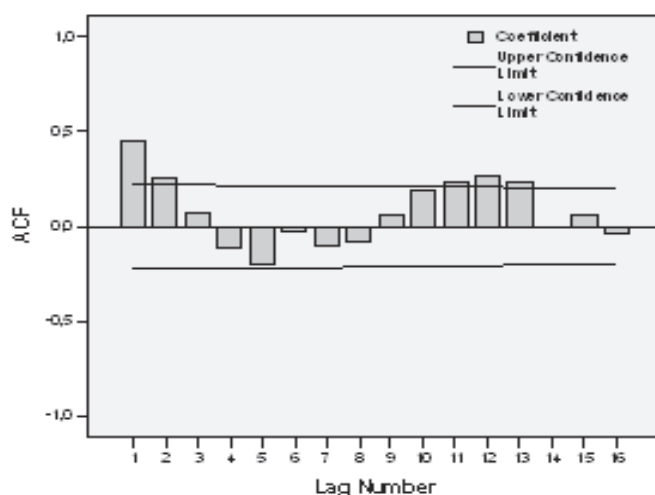


Fig. 4— ACF for Espey

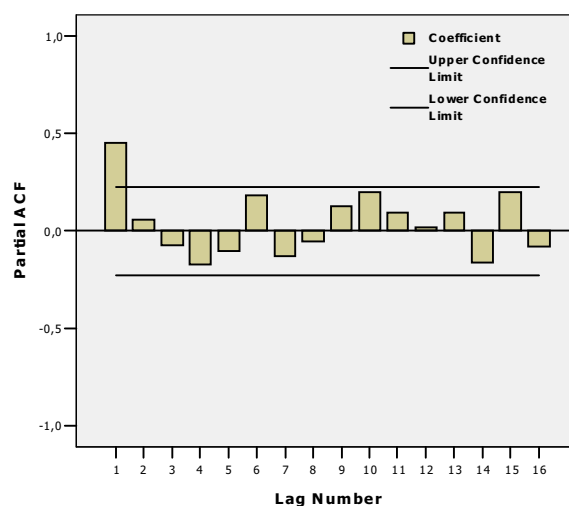


Fig. 5— PACF for Espey

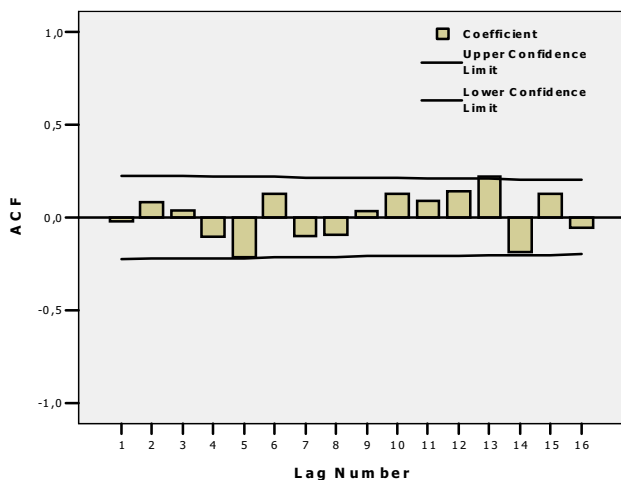


Fig. 6— ACF for errors from ARIMA

It means only random-cause variation is affecting process. Upper and lower control limits of this chart are wider than individual chart since it considers the existence of AC in data.

Hisarc1k Concentrate

Individual chart of $B_2O_3\%$ content for Hisarc1k concentrate (Fig. 9) shows that distribution of points around centerline is considerably irregular and there are 4 points outside lower control limit. This indicates unstable or out-of-control process. ACF (Fig. 10) and PACF (Fig. 11) plots indicate statistically significant ACs present in the data. Rapid decay apparent at graphs of ACF and PACF means that process is stationary. Similar to Espey, ARIMA (1,0,0) or AR (1) has been found to be the best model. Parameters of the model (Table 1) are statistically significant at 1% level. Using ACF (Fig. 12) and PACF (Fig. 13) plots, residuals are found

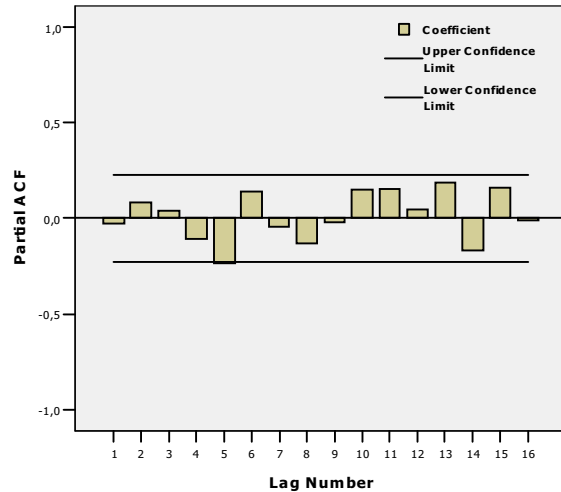


Fig. 7— PACF for errors from ARIMA (1,0,0) (Espey)

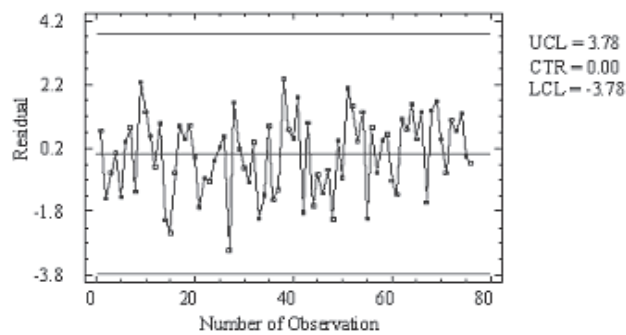


Fig.8— Special cause control chart (Espey)

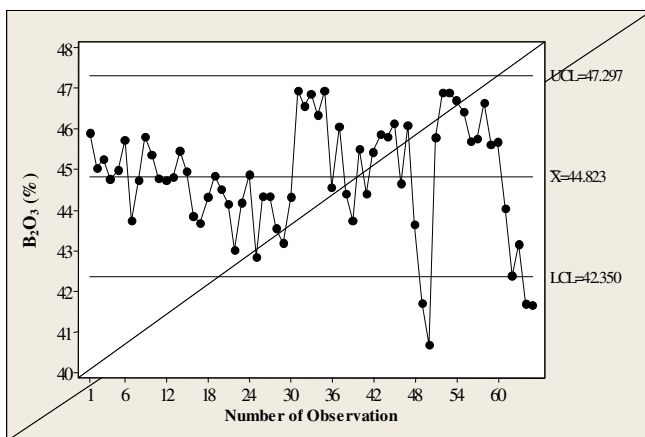


Fig. 9— Control chart for individuals (Hisarcik)

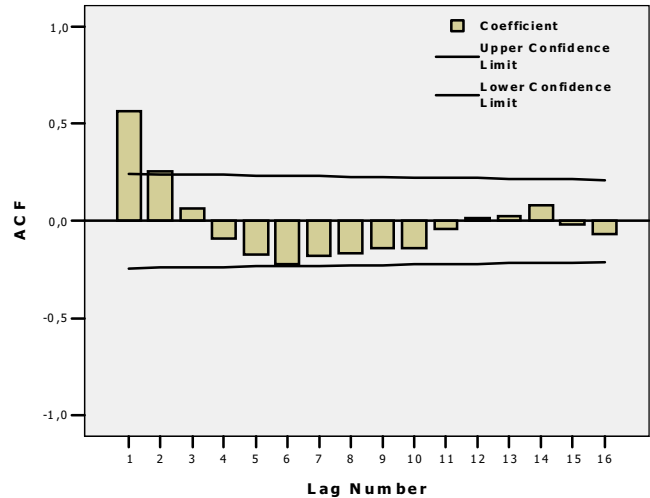


Fig. 10— ACF for Hisarcik

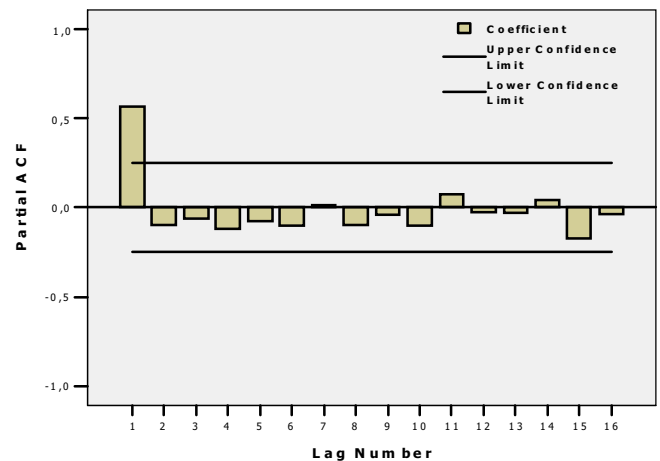


Fig. 11— PACF for Hisarcik

not significantly different from zero. Thus, model (Table 1) is proper for the data.

SCC chart for Hisarcik concentrate (Fig. 14) shows that there is a point beyond control limits. This means that process is not in control and source of variation should be investigated. Because of the violation of independence assumption, individual chart (Fig. 9) has generated more out-of-control points than SCC chart. It can be concluded that upper and lower control limits of individual chart are tighter than residual chart.

Results and Discussion

Traditional control charts require observations from the process to be independent of one another. Failure to meet this requirement causes traditional control charts to give a huge number of false alarms. AC in observations has substantial effect on the performance of IC charts. SCC charts based on ARIMA models are more

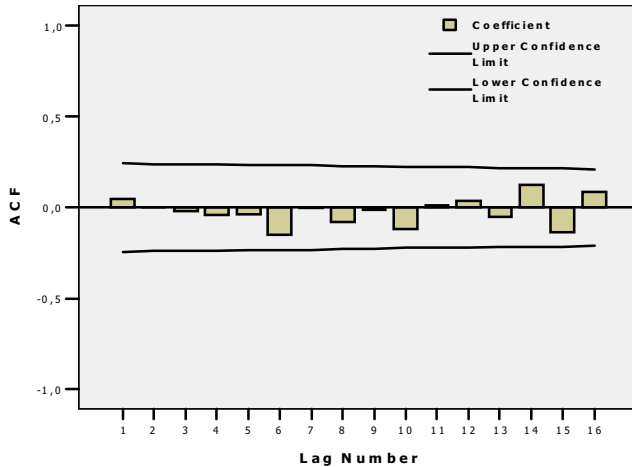


Fig. 12— ACF for errors from ARIMA (1,0,0) (Hisarcik)

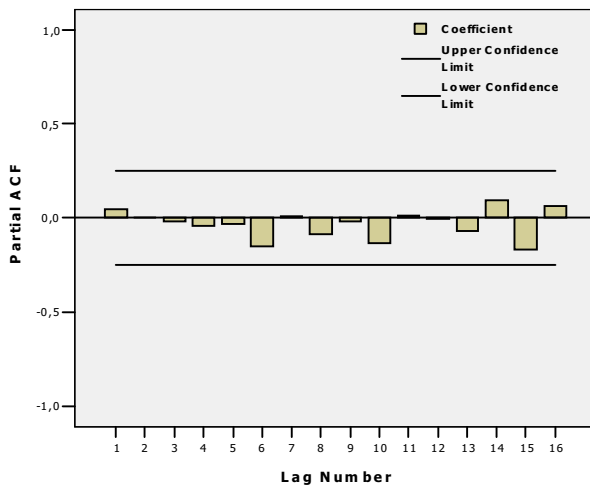


Fig. 13— PACF for errors from ARIMA (1,0,0) (Hisarcik)

appropriate in a correlated environment providing a higher probability of coverage than IC charts.

SCC charts constructed for both concentrators are based on ARIMA (1,0,0) or AR (1) time series models. Samanta¹¹ and Alwan & Roberts⁹ also used same model. On the other hand, ARIMA (1,1,1), ARMA (1,1) and ARIMA (0,1,1) models were used in different studies^{6-8,10}. Different models are results of ACF and PACF plots used during determination of models.

According to SCC chart based on AR (1) model (Fig. 8), Espey concentrator is operating in the presence of random causes. Thus, process of this concentrator is in statistical control. However, process of Hisarcik concentrator is not in control statistically (Fig. 14). It is necessary to investigate assignable causes of process variation for this concentrator. Once assignable causes are identified, it is possible to eliminate them by selecting

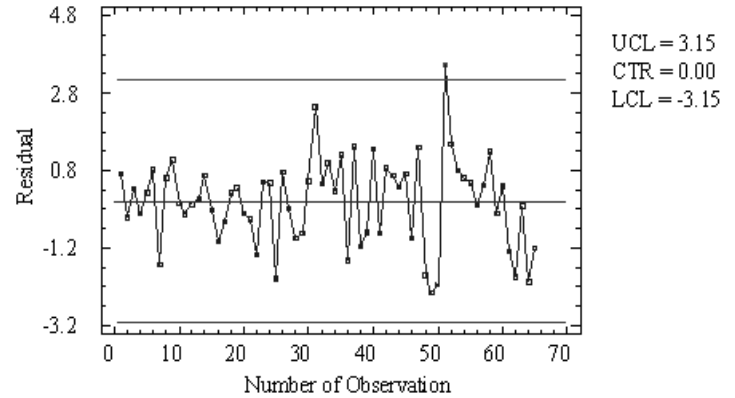


Fig.14— Special cause control chart (Hisarcik)

proper extracting method, accurate production planning, ore blending and subsequent processing of ore etc.

Conclusions

Basic idea of control charts is to test hypothesis that the process is in control, against alternative, that the process is out of control. When a true null hypothesis is rejected, type I error is committed. Existence of AC in the process data causes a substantial increase in average false alarm rate, namely probability of type I error. Therefore, before investigating the stability of process by means of traditional control charts, assumption of independence of data should be verified. If consecutive measurements are highly correlated, SCC chart can be used to capture AC among the measurements instead of traditional control charts. In this study, autocorrelated data obtained from e Espey and Hisarcik concentrators were handled to determine whether processes are in control or not by use of IC chart and SCC chart. IC charts indicated that both concentrators are statistically out of control. However, SCC charts indicated that only Hisarcik concentrator is statistically out of control. In this case, it is clear that type I error will occur if IC chart is used to investigate stability of Espey concentrator.

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