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Transient temperature distribution in homogeneous and composite materials

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The paper deals with the time temperature history in semi-infinite plates of different materials for the same exposed temperature and initial temperature. Theoretical study is based on Gauss's Integral formula. A comparison has been made between the results of theoretical and experimental study. It becomes easy to select the proper material to get required time or temperature by the curve plotted between RC-value and response factor, depending on situations. This method can be used widely for measurement of steady and transient thermal properties of building and insulating materials and may find extensive application in the prediction of thermal response subjected to high temperature, exposure due to fire and for extremely low temperature like cryogenics also. RC values for different single section and multi-layered composite sections have also been discussed here.

When one of the surfaces of a semi-infinite plate is kept, in contact with high temperature or very low temperature, the time-temperature history inside the plate becomes important. In a heat conduction body, variations in the surface conditions are always affected at the interior points. On the contrary, heat flux histories can be obtained from other surface or interior temperature data taken at a sub surface location. The exact data and effect of moisture on heat flow[1] for the composite, multi-layered plate may be useful for transient heat flow problems as moisture absorbed materials have different RC values.

An accurate knowledge of temperature variation is often necessary in engineering design. The success of design depends upon the accurate prediction of temperature level in some critical part. In most of the engineering applications, transient systems are encountered. For instance, heat flow to boiler tubes cooling the IC engine cylinders and instantaneous heat gain or loss in buildings are taken as transient heat flow processes. Whenever there is a sudden change in temperature, whether it rises or falls and remain constant thereafter, the transient heat flow technique is useful for prediction of temperature distribution inside the plates.

Most of the heat transfer measurements consists of monitoring of the temperature of a body at selected points and then relating that temperature history to one of the heat transfer rates. If the temperature of a slab does not change during a process, the temperature distribution near the surface is identical to that in an infinitely thick slab. This type of plate fall into the category of semi-infinite plate.

Study on transient heat flow in semi-infinite plate has been made by some researchers previously. By this study Taler[2] has determined the heat transfer coefficient of the surface of semi-infinite bodies. The R-C value is an important factor on which prediction of temperature or time duration are fully dependent. The computation technique of temperature history inside the semi-infinite body is simple and less time consuming, although the data analysis is relatively complicated. But with the help of RC value of some sections and illustrations, it has been made further easier.

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Numerical Formulation

The exact data for the problem are generated using the analytical solution[3] given as,

$$T_{(x.t)} = T_i + (T_s - T_i)\{1\text{-}F(\mu)\} \quad \ldots (1)$$

of one dimensional heat conduction equation.

$$\frac{\delta^2 T}{dx^2} = \frac{1}{\alpha}\frac{\delta T}{\delta t} \quad \ldots (2)$$

with initial and boundary conditions

$$\begin{aligned} &T = T_i, \text{ at } t = 0 && 0 \le x \le \infty \\ &T = T_s, \text{ at } t = 0 && t \le 0 \\ &T = T_{(x,t)}, \text{ at } x = L && t > 0 \end{aligned} \quad \ldots (3)$$

In Eq. (1), $F(\mu)$ is Error function and is computed[4] by the series equation,

$$F(\mu) = \frac{2}{\pi}\left\{\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{\mu^{2n-1}}{(n-1)!(2n-1)}\right\} \quad \ldots (4)$$

$$\text{where } \mu = \frac{x}{2\sqrt{\alpha t}} \quad \ldots (5)$$

Eq. (5) is written in the form of RC, as

$$u^2 = \frac{x^2}{4\alpha t}$$

$$= \frac{1}{4t}RC$$

Therefore,

$$\mu = \frac{1}{2}\sqrt{\left[\frac{RC}{t}\right]} = \frac{1}{2}(RC/t)^{1\backslash 2} \quad \ldots (6)$$

The method of computing RC value for composite bodies is given by Suman *et al*[5].

Experimental Procedure

The time temperature history in semi- infinite plates of single or composite section, is determined as follows—there is a hot plate system in which hot fluid of desired temperature is circulated so that plate reaches up to that temperature. Thermocouples are installed at three points on each surface of semi-finite plate sample which is clamped with the hot plate system as shown in Fig. 1. The exposed temperature of surface is taken when hot plate temperature is stable. Exposed surface temperature is lower than the hot plate temperature. Temperatures at centre and two other points are recorded with help of copper constantan thermocouples connected with micro voltmeter through selector switches. Half hourly data were recorded at the desired thickness. In Fig. 1 the composite plate comprising 50 mm thick gypsum, 4 mm thickness of AC sheet and 4.5 mm thick of fibre cement plate is clamped with hot plate. For getting the accurate result, the system of plates should be kept in an insulated box. The temperature observations are recorded in terms of micro volt which is converted into degree celsius with the help of standard table of the corresponding thermocouples.

Theoretical estimation of time—In problems when exposed temperatue (T_{exp}) is known along with thickness x and temperature at this point, $T_{(x.t)}$ then function, $F(\mu)$ is computed with the help of Eq. 1. With this, $F(\mu)$. μ is determined with the help of a table given by Gupta and Prakash[6]. Thermophysical properties like k, ρ, c of the material should be known to get the value of t.

Estimation of temperature—Temperature of unexposed surface of desired thickness at given time may be computed by determining μ from Eq. (6). Then $F(\mu)$ may be computed by Eq. (4) or with the

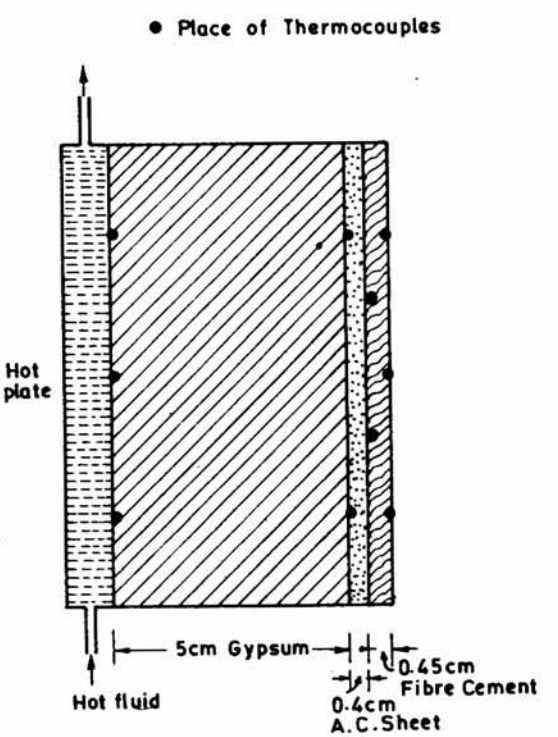

Fig. 1—Experimental set up of transient heat conduction in semi-infinite plate.

help of the table reported earlier[5]. Then $T_{(x,t)}$ may be estimated with the help of Eq. (1).

Illustration—If the temperature of outer surface of wall of a plane is suddenly changed from 20 to 800°C and remains constant thereafter then for determining the temperature of inside surface of wall of the plane after half an hour when thickness of wall and thermal diffusivity are 18 cm and 2.78×10^{-3} m²/h respectively, μ is computed as,

$$\mu = x / 2\sqrt{\alpha t} = 2.4139$$

$F(\mu) = 0.9697$ as obtained from Eq. (4) and $T_{(x,t)} = 44.24$°C as computed from Eq. (1).

The temperature of inside surface of the wall of plane will be 44.24°C just after half an hour time.

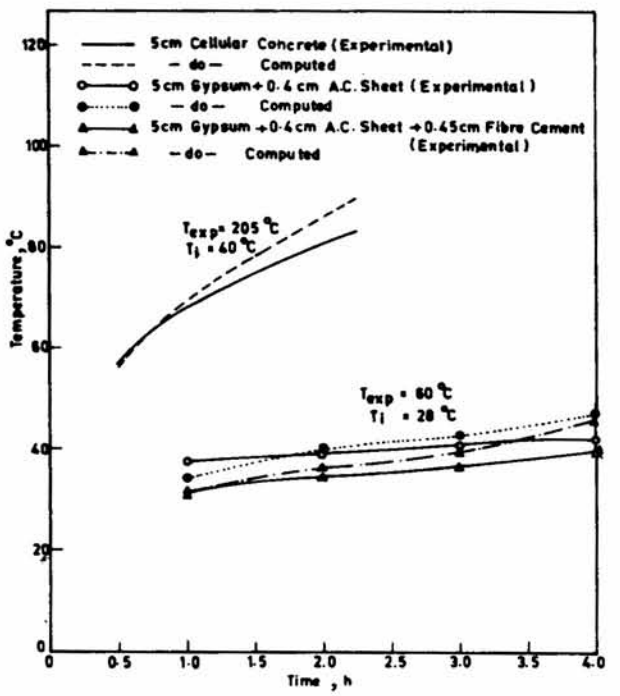

Fig. 2—Comparison between experimental and computed time-temperature history in single and composite plates.

Results and Discussion

Two types of curves have been shown in Fig. 2. The first curve is for single layered material and other one for multi layered section. Upper curves are for 5 cm cellular concrete and lower curves are the temperature variation at different layers of composite materials. Comparison between experimental and theoretically computed results have been shown in the figure. Both the results are in agreement with one another initially for several hours but later there is some variation. Difference between exposed temperature and initial temperature is 32°C in case of multi layered section and 165° in case of single layered material. Higher temperature gradient for higher difference of temperature and a lower temperature gradient for lower difference of temperature, was found as can be observed in Fig. 2. The other reason of variation in temperature gradient is thermal insulating property. For lower insulating material it is higher and for high insulating material it is lower. In other

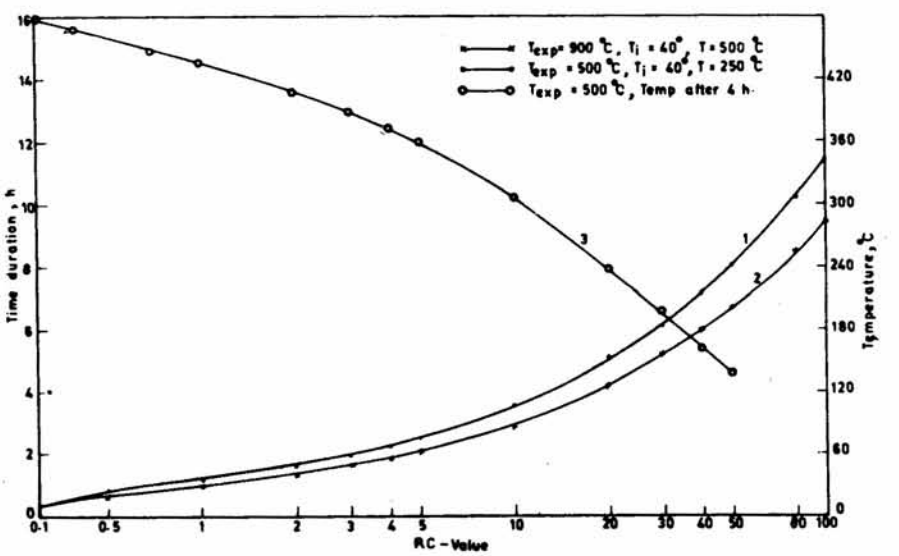

Fig. 3—Variation of RC value with temperature and time for two different response factors.

Table 1—RC Values of some building sections

Section	Value of RC
2.5 cm PUF	1.675
5.0 cm gypsum + 4.0 cm AC sheet	4.589
0.4 cm AC sheet + 0.45 cm fibre cement	0.222
5.0 cm gypsum + 1.5 cm PUF	4.550
1.0 cm fibre cement + 4.0 cm cellular conc	2.288
10.0 cm cement concrete + 2.0 cm mud phuska	12.492
3.0 cm Asbestos + 5.0 cm steel	5.860
3.0 cm Asbestos + 5.0 cm steel + 3.0 cm Asbestos	11.43
0.1 cm Steel + 7.5 PUF + 0.1 cm steel	15.075
5.0 cm mineral wool + 2.0 cm cellular conc	5.732

words, temperature gradient varies directly with difference of exposed and initial temperatures and inversely with RC value of the material, on keeping other factors constant.

Curve 1 and 2 in Fig. 3 denotes the time taken to reach particular temperature verses RC value for response factor of 0.465 and 0.526 respectively. Whereas curve 3 is plotted between RC values and temperature of the surface at given thickness just after 4 h. From curve 1 and 2, it is observed that as RC value increases, the time to reach the particular temperature also increases. The rate of increase is lower up to RC-value of 20, and this rate becomes higher above this RC value. From curve 3, it can be observed that rate of decrease in temperature-rise is sharp above the RC value of 5. Below this value of RC, the rate of decrease in temperature rise with increase of RC value is too slow.

From these curves 1, 2 and 3 it is easier to find out any of the factor, exposed and ambient temperature, initial temperature, fixed time duration and temperature distribution. From the above method, the required RC value may be found for treatment of any problem, then material or combination of materials may be selected depending on their thermophysical properties. This problem has been made further easier by giving RC value of some sections in Table 1. For computation of temperature history in semi-infinite body, it is better to add the actual time taken to raise the required temperature of exposed surface.

Conclusion

The method described gives more accurate results of time temperature history inside semi infinite bodies, provided, system may be kept in an insulated box. Comparison may also be made by experimental and theoretical result for other single or multi layered materials. For same material and exposed temperature, rise in temperature is computed easily after the given time. Similarly, for given required temperature after the fixed time duration for the same exposed temperature, RC-value may be found from Fig. 3. The study of time temperature equation of diffusivity has been changed in RC form. By doing so, it is possible to compute and plot the time temperature curve for other composite materials also.

Selection of the material can be made to avoid damage of indoor conditions. Evaporation problem on refrigerator's outer surface and energy consumption problems may be minimised by the proper utilisation of the study. It can also be utilised for the design of fire resistant problem and equipments. The study may be made more useful by determining the other thermal properties as diffusivity, specific heat and thermal conductivity. Thermal conductance and overall thermal transmittance value may also be determined experimentally by extension of this study.

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Nomenclature

μ = symbol of error function
x = thickness
t = time
T_i = initial temperature
T_s = exposed surface temperature
α = thermal diffusivity
K = thermal conductivity
r = density
c = specific heat
$T_{x,t}$ = temperaturre at x and time t.

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