

An analytical investigation of wave-induced oscillations of water in a rectangular harbour

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The wave-induced oscillations of water in a rectangular harbour have been studied on the basis of the linear potential wave theory. Mathematical expressions for the problem of oscillations of water and the amplification factor for the rectangular harbour have been derived. Based on these expressions, a number of computer programmes have been developed in FORTRAN 77 and executed on the IBM 4332 LO2 System at the Computer Centre of Bangladesh University of Engineering and Technology. The computational results have been plotted and then physically interpreted with a view to predicting the hydrodynamic response characteristics of the rectangular harbour.

The water contained in a harbour has certain natural frequencies of oscillation, depending upon the geometry of the water boundaries. Oscillations of unacceptable magnitude are caused in certain harbours by resonance between one or more of the natural periods and rhythmical disturbances in the sea adjacent to the harbour entrance.

The harbour resonance problem is of very great practical importance in coastal engineering. This is particularly so in connection with ship mooring problems. It is well-known that harbour oscillations of only a few inches may excite large motions in ship-mooring systems causing mooring lines to break and ships to collide with adjacent structures. To minimise such events is the goal of harbour and breakwater design and for that purpose one must be able to determine harbour response characteristics.

Many investigators¹⁻¹⁴ have devoted considerable efforts with a view to achieving a good understanding of the phenomena of harbour resonance. As the first but an important step to approach the practical situation, McNown⁹ studied the forced oscillation in a circular harbour which is connected to the open sea through a narrow mouth. A similar approach was applied by Kravtchenko and McNown⁷ to the rectangular harbour. Ippen and Goda⁵ also studied the problem of a rectangular harbour connected to the open sea. In that analysis the waves radiated from the harbour entrance to the open sea were evaluated using the Fourier transformation method,

while the solution inside the harbour was obtained by the method of separation of variables.

The aim of the present work is to study analytically the wave-induced oscillations of water in a rectangular harbour. The findings of this study may be useful in understanding the response characteristics of the above-mentioned harbour.

Mathematical Formulation

Oscillations of water in a rectangular harbour due to breaking of the primary wave system—Let us consider a rectangular harbour connected to the open sea as shown in Fig. 1. The present analysis is based upon the superposition of various incident and reflected waves using the theory of complex variables. In addition to the usual small-amplitude wave assumptions, the limitation is imposed that the width of the domain seaward of the entrance is less than one-half the wavelength.

The incident wave coming from the open sea arrives at the mouth of this basin with an ampli-

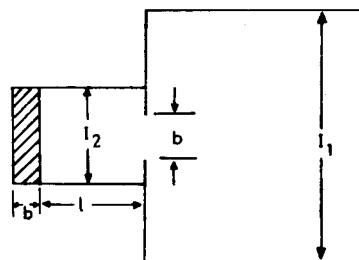


Fig. 1—Definition sketch of a rectangular harbour

tude A_1 and a phase $\hat{A}_1$ at a given time. This wave is considered to be defined by the complex number $\bar{A}_1$ of modulus $|\bar{A}_1| = A_1$ and argument $\hat{A}_1$. Similarly, the wave reflected by the overall layout is defined by a complex number $\bar{B}_1$ of modulus $|\bar{B}_1| = B_1$ and argument $\hat{B}_1$. If there is no loss of energy at the discontinuity and inside the harbour $|\bar{A}_1| = |\bar{B}_1|$.

Consider the following incident wave approaching the harbour entrance

$$\bar{A}_1 = A_1 e^{i\hat{A}_1} \quad \dots (1)$$

The wave $\bar{A}_1$ is partially transmitted and partially reflected. The portion of $\bar{A}_1$ transmitted into the harbour is

$$\bar{\alpha}_1 \bar{A}_1$$

and the portion which is reflected from the seaward boundary of the entrance is

$$\bar{\beta}_1 \bar{A}_1$$

The quantities $\bar{\alpha}_1$ and $\bar{\beta}_1$ refer to the complex representation of the transmission and reflection coefficient.

$$\bar{\alpha}_1 = \alpha_1 e^{i\hat{\alpha}_1} \quad \dots (2)$$

$$\bar{\beta}_1 = \beta_1 e^{i\hat{\beta}_1} \quad \dots (3)$$

The moduli α_1 and β_1 are the transmission and reflection coefficients, while the arguments $\hat{\alpha}_1$ and $\hat{\beta}_1$ represent the change in phase due to the discontinuity. Therefore,

$$\bar{\alpha}_1 \bar{A}_1 = \alpha_1 A_1 e^{i(\hat{\alpha}_1 + \hat{A}_1)} \quad \dots (4)$$

$$\bar{\beta}_1 \bar{A}_1 = \beta_1 A_1 e^{i(\hat{\beta}_1 + \hat{A}_1)} \quad \dots (5)$$

Inside the harbour the wave $\bar{C}_1$ progresses toward the beach. Due to friction effects this wave is attenuated in travelling the length of the harbour and at the beach it is

$$\bar{C}_0 = \bar{r} \bar{C}_1 \quad \dots (6)$$

where $\bar{r}$ is a complex number whose modulus represents the frictional effects and whose argument compensates for the finite length of the harbour.

At the beach a portion of the wave $\bar{C}_0$ is dissipated and a portion $\bar{D}_0$ reflected. Assuming that this reflection occurs without a change in phase

$$\bar{D}_0 = p \bar{C}_0 \quad \dots (7)$$

where p is a coefficient of reflection which depends upon the dissipative characteristics of the beach.

This wave is attenuated as it travels toward the entrance in a way identical to the change in $\bar{C}_1$ as it travelled toward the beach. Therefore, the wave reaching the entrance can be expressed as

$$\bar{D}_1 = \bar{r} \bar{D}_0 \quad \dots (8)$$

A portion $\bar{\beta}_2 \bar{D}_1$ of this wave is reflected back toward the beach and a portion $\bar{\alpha}_2 \bar{D}_1$ is transmitted through the entrance into the main wave basin. Therefore, the resultant waves $\bar{C}_1$ and $\bar{B}_1$ can be expressed as

$$\begin{aligned} \bar{C}_1 &= \bar{\alpha}_1 \bar{A}_1 + \bar{\beta}_2 \bar{D}_1 \\ \text{or,} \\ \bar{C}_1 &= \bar{\alpha}_1 \bar{A}_1 + \bar{\beta}_2 \bar{r} p \bar{C}_1 \end{aligned} \quad \dots (9)$$

$$\begin{aligned} \text{or,} \\ \bar{C}_1 &= \bar{\alpha}_1 \bar{A}_1 + \bar{\beta}_2 \bar{r} p \bar{r} \bar{C}_1 \\ \bar{B}_1 &= \bar{\beta}_1 \bar{A}_1 + \bar{\alpha}_2 \bar{D}_1 \end{aligned} \quad \dots (10)$$

From Eqs (6)-(10), all of the primary components can be obtained in terms of the incident wave.

$$\bar{C}_1 = \frac{\bar{\alpha}_1}{i - \bar{\beta}_2 \bar{r}^2 p} \bar{A}_1 \quad \dots (11)$$

$$\bar{C}_0 = \frac{\bar{r} \bar{\alpha}_1}{i - \bar{\beta}_2 \bar{r}^2 p} \bar{A}_1 \quad \dots (12)$$

$$\bar{D}_0 = \frac{p \bar{r} \bar{\alpha}_1}{i - \bar{\beta}_2 \bar{r}^2 p} \bar{A}_1 \quad \dots (13)$$

$$\bar{D}_1 = \frac{p \bar{r}^2 \bar{\alpha}_1}{i - \bar{\beta}_2 \bar{r}^2 p} \bar{A}_1 \quad \dots (14)$$

$$\bar{B}_1 = \left[\bar{\beta}_1 + \frac{p \bar{r}^2 \bar{\alpha}_1 \bar{\alpha}_2}{i - \bar{\beta}_2 \bar{r}^2 p} \right] \bar{A}_1 \quad \dots (15)$$

At the entrance, the total value of the oscillation of water in the basin is given by the equation

$$(\bar{C}_1 + \bar{D}_1) = \frac{(1 + p \bar{r}^2) \bar{\alpha}_1}{i - \bar{\beta}_2 \bar{r}^2 p} \bar{A}_1 \quad \dots (16)$$

and at the back by the equation

$$(\bar{C}_0 + \bar{D}_0) = \frac{(1 + p) \bar{r} \bar{\alpha}_1}{i - \bar{\beta}_2 \bar{r}^2 p} \bar{A}_1 \quad \dots (17)$$

The value in the open sea at the mouth is

$$(\bar{A}_1 + \bar{B}_1) = \left[\bar{i} + \bar{\beta}_1 + \frac{p\bar{r}^2 \bar{\alpha}_1 \bar{\alpha}_2}{\bar{i} - \bar{\beta}_2 \bar{r}^2 p} \right] \bar{A}_1 \quad \dots (18)$$

The magnitude of the oscillation at the rear of the harbour is given by the absolute value $|\bar{C}_0 + \bar{D}_0|$; at the entrance in the basin given by the absolute value $|\bar{C}_1 + \bar{D}_1|$; and at the entrance in the open sea by the absolute value $|\bar{A}_1 + \bar{B}_1|$.

Defining the amplification factor as the ratio of this amplitude to the incident wave amplitude and after some manipulations, magnification factor at the rear of the harbour is obtained as

$$\frac{|\bar{C}_0 + \bar{D}_0|}{A_1} = \left| \frac{(1+p) \bar{r} \bar{\alpha}_1}{\bar{i} - \bar{\beta}_2 \bar{r}^2 p} \right| e^{i\bar{A}_1}$$

or,

$$\frac{|\bar{C}_0 + \bar{D}_0|}{A_1} = \left| \frac{(1+p) r e^{i\bar{r}} \alpha_1 e^{i(\bar{A}_1 + \bar{A}_1)}}{1 - \beta_2 e^{i\bar{\beta}_2} r^2 e^{i2\bar{r}} p} \right| \quad \dots (19)$$

or,

$$\frac{|\bar{C}_0 + \bar{D}_0|}{A_1} = \left| \frac{(1+p) r \alpha_1 e^{i(\bar{r} + \bar{A}_1 + \bar{A}_1)}}{1 - \beta_2 r^2 p e^{i(2\bar{r} + \bar{\beta}_2)}} \right|$$

$$\text{If } r_1 e^{i\theta_1} + r_2 e^{i\theta_2} = r_3 e^{i\theta_3}$$

$$\text{then } r_3 = [r_1^2 + r_2^2 + 2r_1 r_2 \cos(\theta_1 - \theta_2)]^{1/2} \quad (\text{see ref. 13}).$$

Applying this formula, one obtains

$$\frac{|\bar{C}_0 + \bar{D}_0|}{A_1} = \frac{r(1+p) \alpha_1}{[1 + p^2 r^4 \beta_2^2 - 2pr^2 \beta_2 \cos(2\bar{r} + \bar{\beta}_2)]^{1/2}} \quad \dots (20)$$

In a similar manner it can be shown that at the entrance in the basin

$$\frac{|\bar{C}_1 + \bar{D}_1|}{A_1} = \left[\frac{\alpha_1^2 + 2pr^2 \alpha_1^2 + p^2 r^4 \alpha_1^2}{1 + p^2 r^4 \beta_2^2 - 2pr^2 \beta_2 \cos(2\bar{r} + \bar{\beta}_2)} \right]^{1/2} \quad \dots (21)$$

It is assumed that the discharge of incident and reflected waves in domain (1) is equal to the discharge of the transmitted wave in domain (2) on the discontinuity at any time (Fig. 2). It is natural to make this assumption, for if this were not the case, a quantity of water would successively be accumulated near the discontinuity and would issue from it. The discharge is the product of the

velocity times the cross-sectional area. The velocity u is a function of the vertical abscissa. Hence the discharge could be written as

$$I \int_0^h u(z) dz \quad \dots (22)$$

in which I is a constant width and h the depth of the considered cross section.

Let us assume that the domains (1) and (2) have constant width I_1 and I_2 and constant depths h_1 and h_2 respectively. According to the principle of the continuity one equates the value of the discharge in the two domains at the discontinuity. That is

$$I_1 \int_0^{h_1} u_1(z) dz = I_2 \int_0^{h_2} u_2(z) dz \quad \dots (23)$$

Assuming that the motions in domains (1) and (2) are defined by the potential functions ϕ_1 and ϕ_2 respectively and that the horizontal x-axis origin coincides with the discontinuity, $u_1(z)$ and $u_2(z)$ could be expressed as a function of ϕ_1 and ϕ_2

$$u_1(z) = - \left(\frac{\partial \phi_1}{\partial x} \right)_{x=0} \quad \dots (24)$$

$$u_2(z) = - \left(\frac{\partial \phi_2}{\partial x} \right)_{x=0} \quad \dots (25)$$

Therefore, the continuity Eq. (23) becomes

$$I_1 \int_0^{h_1} \left(\frac{\partial \phi_1}{\partial x} \right)_{x=0} dz = I_2 \int_0^{h_2} \left(\frac{\partial \phi_2}{\partial x} \right)_{x=0} dz \quad \dots (26)$$

The potential function ϕ_1 is given by the sum of the potential function of an incident periodical gravity wave of amplitude a travelling towards the

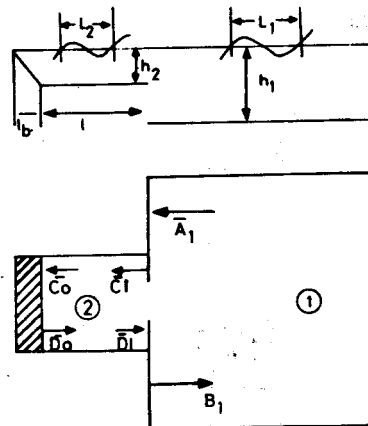


Fig. 2—Notation of primary waves

discontinuity and the potential function of a reflected wave of amplitude $a\beta_1$ travelling seawards and having a difference of phase β at the discontinuity.

So ϕ_1 may be written as

$$\begin{aligned} \phi_1 = & \frac{a\sigma \cosh k_1(h_1+z)}{k_1 \sinh k_1 h_1} \sin(k_1 x - \sigma t) \\ & - a\beta_1 \frac{\sigma \cosh k_1(h_1+z)}{k_1 \sinh k_1 h_1} \\ & \times \sin(k_1 x + \sigma t + \beta_1) \end{aligned} \quad \dots (27)$$

The potential function ϕ_2 is that of a wave amplitude $a\alpha_1$ travelling in the same direction of the incident wave and having a phase difference $\hat{\alpha}_1$. One has the incident wave at the discontinuity.

$$\begin{aligned} \phi_2 = & a\alpha_1 \frac{\sigma \cosh k_2(h_2+z)}{k_2 \sinh k_2 h_2} \sin(k_2 x - \sigma t - \hat{\alpha}_1) \\ & \dots (28) \end{aligned}$$

Differentiating Eqs (27) and (28) with respect to x , one obtains

$$\begin{aligned} \frac{\partial \phi_1}{\partial x} = & a\sigma \frac{\cosh k_1(h_1+z)}{\sinh k_1 h_1} \\ & \times [\cos(k_1 x - \sigma t) - \beta_1 \cos(k_1 x + \sigma t + \beta_1)] \\ & \dots (29) \end{aligned}$$

$$\begin{aligned} \frac{\partial \phi_2}{\partial x} = & a\alpha_1 \sigma \frac{\cosh k_2(h_2+z)}{\sinh k_2 h_2} \cos(k_2 x - \sigma t - \hat{\alpha}_1) \\ & \dots (30) \end{aligned}$$

and

$$\int_0^h \frac{\cosh k(h+z)}{\sinh kh} dz = \frac{1}{k} = \frac{L}{2\pi} \quad \dots (31)$$

Putting these expressions into Eq. (26), one obtains

$$\begin{aligned} I_1 a\sigma \frac{L_1}{2\pi} [\cos \sigma t - \beta_1 \cos(\sigma t + \beta_1)] \\ = a\sigma I_2 \frac{L_2}{2\pi} \alpha_1 \cos(\sigma t + \hat{\alpha}_1) \end{aligned} \quad \dots (32)$$

$$\begin{aligned} \dots I_1 L_1 [\cos \sigma t - \beta_1 \cos(\sigma t + \beta_1)] \\ = I_2 L_2 \alpha_1 \cos(\sigma t + \hat{\alpha}_1) \end{aligned} \quad \dots (33)$$

Assuming that $\sigma t = 0$ and $\sigma t = \pi/2$ successively, one gets

$$I_1 L_1 [1 - \beta_1 \cos \beta_1] = I_2 L_2 \alpha_1 \cos \hat{\alpha}_1 \quad \dots (34)$$

$$I_1 L_1 [0 - \beta_1 \sin \beta_1] = I_2 L_2 \alpha_1 \sin \hat{\alpha}_1 \quad \dots (35)$$

Multiplying Eq. (34) by i and then adding with Eq. (35), one obtains

$$\frac{I_2 L_2}{I_1 L_1} \alpha_1 e^{i\hat{\alpha}_1} + \beta_1 e^{i\beta_1} = 1 \quad \dots (36)$$

Introducing

$$Z = \frac{I_2 L_2}{I_1 L_1} \quad \dots (37)$$

$$Z \alpha_1 e^{i\hat{\alpha}_1} + \beta_1 e^{i\beta_1} = 1 \quad \dots (38)$$

$$Z \bar{\alpha}_1 + \bar{\beta}_1 = \bar{i} \quad \dots (39)$$

Similarly, considering an incident wave from the opposite direction, one can find

$$\frac{1}{Z} \bar{\alpha}_2 + \bar{\beta}_2 = \bar{i} \quad \dots (40)$$

The transmitted energy of a periodical gravity wave of amplitude a and with a length of crest equal to L is given⁴ by as

$$P = \frac{1}{2} \rho g \frac{L}{T} a^2 I \left(1 + \frac{2kh}{\sinh 2kh} \right) \quad \dots (41)$$

On the other hand, the most important loss of energy at the discontinuity is due to the convective inertia. Since this force is negligible, one may neglect the loss of energy and equate the incident wave energy to the sum of the reflected energy and the transmitted energy. For the sake of simplicity let us introduce the new symbols:

$$A_1 = \left(1 + \frac{2k_1 h_1}{\sinh 2k_1 h_1} \right) \quad \dots (42)$$

$$A_2 = \left(1 + \frac{2k_2 h_2}{\sinh 2k_2 h_2} \right) \quad \dots (43)$$

The principle of conservation of energy may be written as follows:

$$\begin{aligned} \text{Incident energy} &= \text{Reflected energy} \\ &+ \text{Transmitted energy} \end{aligned}$$

$$\frac{1}{2} \rho g \frac{L_1}{T} a^2 I_1 A_1 = \frac{1}{2} \rho g \frac{L_1}{T} (a\beta_1)^2 I_1 A_1 + \frac{1}{2} \rho g \frac{L_2}{T} (a\alpha_1)^2 I_2 A_2 \quad \dots (44)$$

Therefore,

$$A_1 I_1 L_1 a^2 = A_1 I_1 L_1 (a\beta_1)^2 + A_2 I_2 L_2 (a\alpha_1)^2 \quad \dots (45)$$

Dividing the above expression by $a^2 I_1 L_1 A_1$ and introducing $Z = (I_2 L_2) / (I_1 L_1)$ and $A = A_2 / A_1$ one gets

$$AZ\alpha_1^2 + \beta_1^2 = 1 \quad \dots (46)$$

Similarly for a wave in the opposite direction

$$\frac{1}{AZ} \alpha_2^2 + \beta_2^2 = 1 \quad \dots (47)$$

Now, if one considers two waves of the same period arriving from two opposite directions with different heights and with any phase difference; the principle of conservation of energy takes the form

$$\begin{aligned} & A_1 I_1 L_1 |\bar{a}_1|^2 + A_2 I_2 L_2 |\bar{a}_2|^2 \\ &= A_1 I_1 L_1 |\bar{a}_1 \bar{\beta}_1 + \bar{a}_2 \bar{\alpha}_2|^2 + A_2 I_2 L_2 |a_1 \bar{a}_1 + \bar{a}_2 \bar{\beta}_2|^2 \\ &= A_1 I_1 L_1 |a_1 \beta_1 e^{i(\hat{a}_1 + \hat{\beta}_1)} + a_2 \alpha_2 e^{i(\hat{a}_2 + \hat{\alpha}_2)}|^2 \\ &+ A_2 I_2 L_2 |a_1 \alpha_1 e^{i(\hat{a}_1 + \hat{\alpha}_1)} + a_2 \beta_2 e^{i(\hat{a}_2 + \hat{\beta}_2)}|^2 \quad \dots (48) \end{aligned}$$

If

$$r_1 e^{i\theta_1} + r_2 e^{i\theta_2} = r_3 e^{i\theta_3}$$

then

$$r_3 = [r_1^2 + r_2^2 + 2r_1 r_2 \cos(\theta_1 - \theta_2)]^{1/2} \quad (\text{see ref. 13})$$

Applying this formula, one obtains,

$$\begin{aligned} & A_1 I_1 L_1 |\bar{a}_1|^2 + A_2 I_2 L_2 |\bar{a}_2|^2 \\ &= a_1^2 (A_1 I_1 L_1 \beta_1^2 + A_2 I_2 L_2 \alpha_1^2) + a_2^2 (A_1 I_1 L_1 \alpha_2^2 \\ &+ A_2 I_2 L_2 \beta_2^2) + 2A_1 I_1 L_1 a_1 a_2 \alpha_2 \beta_1 \cos(\hat{a}_1 + \hat{\beta}_1 \\ &- \hat{a}_2 - \hat{\alpha}_2) + 2A_2 I_2 L_2 a_1 a_2 \alpha_1 \beta_2 \\ &\times \cos(\hat{a}_1 + \hat{\alpha}_1 - \hat{a}_2 - \hat{\beta}_2) \quad \dots (49) \end{aligned}$$

Introducing the value of Z , Eq. (49) can be written as

$$\begin{aligned} & A_1 I_1 L_1 a_1^2 + A_2 I_2 L_2 a_2^2 \\ &= a_1^2 A_1 I_1 L_1 (AZ\alpha_1^2 + \beta_1^2) \\ &+ A_2 I_2 L_2 a_2^2 [(1/AZ) \alpha_2^2 + \beta_2^2] \\ &+ 2a_1 a_2 A_1 I_1 L_1 [\alpha_2 \beta_1 \cos(\hat{a}_1 - \hat{a}_2 + \hat{\beta}_1 - \hat{\alpha}_2) \\ &+ AZ\alpha_1 \beta_2 \cos(\hat{a}_1 - \hat{a}_2 + \hat{\alpha}_1 - \hat{\beta}_2)] \quad \dots (50) \end{aligned}$$

combining Eqs (46), (47) and (50), one obtains

$$\alpha_2 \beta_1 \cos(\hat{a}_1 - \hat{a}_2 + \hat{\beta}_1 - \hat{\alpha}_2) + AZ\alpha_1 \beta_2 \cos(\hat{a}_1 - \hat{a}_2 + \hat{\alpha}_1 - \hat{\beta}_2) = 0 \quad \dots (51)$$

Assuming that $(\hat{a}_1 - \hat{a}_2) = 0$ and $(\hat{a}_1 - \hat{a}_2) = \pi/2$ successively, one obtains

$$\alpha_2 \beta_1 \cos(\hat{\beta}_1 - \hat{\alpha}_2) + AZ\alpha_1 \beta_2 \cos(\hat{\alpha}_1 - \hat{\beta}_2) = 0 \quad \dots (52)$$

$$\alpha_2 \beta_1 \sin(\hat{\beta}_1 - \hat{\alpha}_2) + AZ\alpha_1 \beta_2 \sin(\hat{\alpha}_1 - \hat{\beta}_2) = 0 \quad \dots (53)$$

Combining Eqs (52) and (53)

$$\frac{\cos(\hat{\beta}_1 - \hat{\alpha}_2)}{\sin(\hat{\beta}_1 - \hat{\alpha}_2)} = \frac{\cos(\hat{\alpha}_1 - \hat{\beta}_2)}{\sin(\hat{\alpha}_1 - \hat{\beta}_2)} \quad \dots (54)$$

$$\therefore \cos(\hat{\beta}_1 - \hat{\alpha}_2) \sin(\hat{\alpha}_1 - \hat{\beta}_2) - \cos(\hat{\alpha}_1 - \hat{\beta}_2) \sin(\hat{\beta}_1 - \hat{\alpha}_2) = 0 \quad \dots (55)$$

After simplification, one obtains

$$\sin\left(\frac{\hat{\beta}_1 + \hat{\beta}_2 - \hat{\alpha}_1 - \hat{\alpha}_2}{2}\right) = 0 \quad \dots (56)$$

$$\sin\left(\frac{\hat{\beta}_1 + \hat{\beta}_2 - \hat{\alpha}_1 - \hat{\alpha}_2}{2}\right) = \sin n\pi \quad \therefore \hat{\beta}_1 - \hat{\alpha}_2 = \hat{\alpha}_1 - \hat{\beta}_2 + 2n\pi \quad \dots (57)$$

From Eqs (52), (53) and (57), one gets

$$\alpha_2 \beta_1 = AZ\alpha_1 \beta_2 \quad \dots (58)$$

$$AZ\alpha_1^2 + \beta_1^2 = \frac{1}{AZ} \alpha_2^2 + \beta_2^2 = 1 \quad \dots (59)$$

Squaring both sides of Eq. (58) and then using Eq. (59), one obtains

$$\alpha_2^2 = (AZ\alpha_1)^2 \left(\frac{\beta_2}{\beta_1}\right)^2$$

or

$$\alpha_2^2 = (AZ\alpha_1)^2 \frac{1 - \frac{1}{AZ} \alpha_2^2}{1 - AZ\alpha_1^2}$$

or

$$\alpha_2^2 - AZ\alpha_1^2 \alpha_2^2 = (AZ\alpha_1)^2 - AZ\alpha_1^2 \alpha_2^2 \quad \therefore \alpha_2 = AZ\alpha_1 \quad \dots (60)$$

Substituting this expression into Eq. (59), one obtains

$$\beta_1 = \beta_2 = (1 - AZ\alpha_1^2)^{1/2} \quad \dots (61)$$

From the continuity relationship, one finds successively for the first incident wave (Fig. 3)

$$Z^2\alpha_1^2 = 1 + \beta_1^2 - 2\beta_1 \cos \beta_1 \quad \dots (62)$$

Putting $\beta_1 = (1 - AZ\alpha_1^2)^{1/2}$ in Eq. (62), one gets

$$\cos \beta_1 = \frac{2 - Z\alpha_1^2(A + Z)}{2(1 - AZ\alpha_1^2)^{1/2}} \quad \dots (63)$$

On the other hand,

$$\beta_1^2 = 1 + Z^2\alpha_1^2 - 2Z\alpha_1 \cos \hat{\alpha}_1 \quad \dots (64)$$

This equation gives

$$\cos \hat{\alpha}_1 = \frac{\alpha_1}{2}(A + Z) \quad \dots (65)$$

Similarly for the second incident wave, one can find

$$\cos \beta_2 = \frac{2 - Z\alpha_1^2(A + Z)}{2(1 - AZ\alpha_1^2)^{1/2}} \quad \dots (66)$$

$$\cos \hat{\alpha}_2 = \frac{\alpha_1}{2}(A + Z) \quad \dots (67)$$

One can see that

$$\cos \hat{\alpha}_1 = \cos \hat{\alpha}_2$$

$$\therefore \hat{\alpha}_1 = \hat{\alpha}_2 + 2n\pi$$

From Eq. (57), one obtain

$$\hat{\alpha}_1 = \hat{\alpha}_2 = \frac{\beta_1 + \beta_2}{2} + \frac{n}{2}\pi$$

In order to obtain the response curve of the harbour from Eq. (20) the following quantities r , p , $\hat{r}$ and α_1 must be evaluated. The attenuation parameter r is described by an exponential damping term

$$r = e^{-(k_1/L_2)} \quad \dots (68)$$

where k is a friction coefficient.

The coefficient of reflection p depends on the slope of the beach, the wave steepness, the roughness and porosity of the beach, etc. The expression given by Mehaute is

$$p = \left(1 - \frac{l_b}{L_2}\right)^{1/2} \quad \dots (69)$$

where l_b is the beach length.

The wave $\bar{C}_1$ travelling from the harbour entrance to the beach with a phase angle $\hat{r}$ relative to the original phase angle. This angle is a function of the relative length of the harbour compared to the wave length

$$\hat{r} = -\frac{2\pi l}{L_2} \quad \dots (70)$$

The transmission coefficient is described by the empirical expression

$$\alpha_1 = \left(\frac{b}{L_2}\right)^{1/2} \left(\frac{L_1}{L_2}\right)^{1/4} \frac{2}{1 + A \frac{L_2}{L_1}} \quad \dots (71)$$

From Eq. (20) the amplification factor will be maximum, when

$$\cos(2\hat{r} + \beta) = 1$$

$$\therefore (2\hat{r} + \beta) = 2n\pi$$

The maximum amplification factor (at resonance) at the rear of the harbour is

$$R = \frac{r(1+p)\alpha_1}{1 - pr^2\beta_2} \quad \dots (72)$$

The length of the basin can be obtained from Eq. (70) as

$$l = \frac{L_2}{2} \left(n + \frac{\beta_2}{2\pi}\right)$$

Results and Discussion

Mathematical analyses for the problem of oscillations of water in a rectangular harbour have been presented in the preceding section. Based on the aforesaid theoretical treatment, a number of computer programmes have been developed in FORTRAN 77 and executed on the IBM 4332 LO2 System at the Computer Centre of Bangladesh University of Engineering and Technology. Exhaustive computational experiments have been carried out with a view to predicting the salient

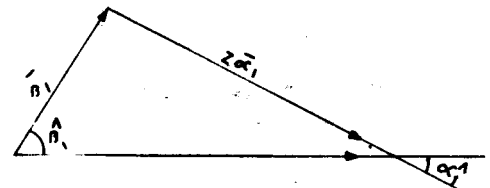


Fig. 3—Vectorial representation of the continuity principle

features of oscillations of water in a harbour of rectangular shape.

Fig. 4 shows the variation of the amplification factor (IR) with the ratio of the length of the entrance to the length of the basin (b/l). It is observed that the wider is the harbour mouth the smaller is the amplitude of the resonant oscillation, i.e., narrowing the harbour entrance does not diminish resonant oscillation which is contradictory to the common sense based on the conventional reasoning for a non-resonant harbour where less wave energy is expected to transmit into the harbour through a smaller opening.

Fig. 5 illustrates the variation of the amplification factor with the ratio of the wavelength in the harbour to that in the open sea (L_2/L_1). It is observed that the amplification factor decreases with increase of the above-mentioned ratio (L_2/L_1).

Fig. 6 shows how the amplification factor changes with the ratio of the width of the open sea to that of the harbour (I_1/I_2). It is observed that the amplification factor increases linearly with the increase of the ratio of the width of open sea to that of the harbour (I_1/I_2).

Fig. 7 shows the variation of the amplification factor with the ratio of the depth of the open sea to that of the harbour (h_1/h_2). It is observed that the amplification factor decreases with the increase of the above-mentioned ratio (h_1/h_2).

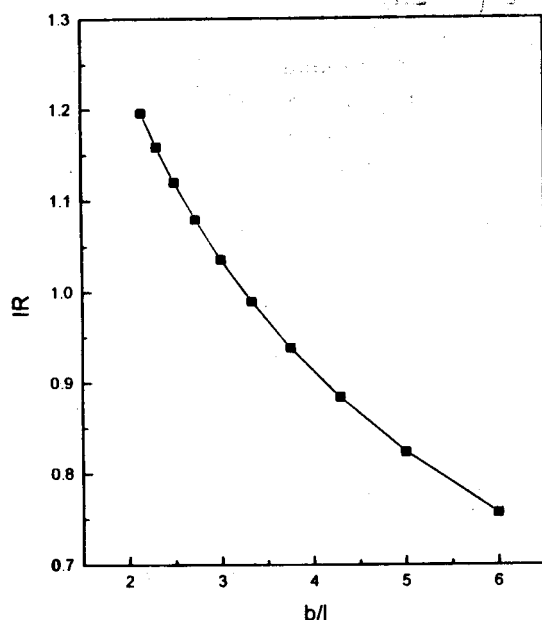


Fig. 4—Variation of amplification factor (IR) with the ratio of the breadth to length of the rectangular harbour (b/l)

Conclusions

This paper presents an analytical investigation of the salient features of oscillations of water in a rectangular harbour. The main interest of this study is to show quantitatively in an idealized case and qualitatively in some more complex cases the importance of a number of parameters such as the size of the entrance, the length of the beach and also the importance of multiple reflections and the number of wave lengths contained in a basin. It has been shown that for the case of resonance, the agitation in a basin could become greater inside the harbour than in the open sea. The following conclusions can be drawn from the present analytical study:

(a) In a rectangular harbour the amplification factor (IR) decreases with the increase of the ratio of the length of the entrance to the length of the basin (b/l).

(b) The amplification factor decreases with increase of the ratio of the wavelength in the harbour to that in the open sea (L_2/L_1).

(c) The amplification factor increases with the increase of the ratio of the width of the open sea to that of the harbour (I_1/I_2).

(d) The amplification factor decreases with the increase of the ratio of the depth of open sea to that of the harbour (h_1/h_2).

(e) The resonance occurs when $(L_2/2) < 1$, i.e., the length of the basin is greater than half of the wavelength in the basin.

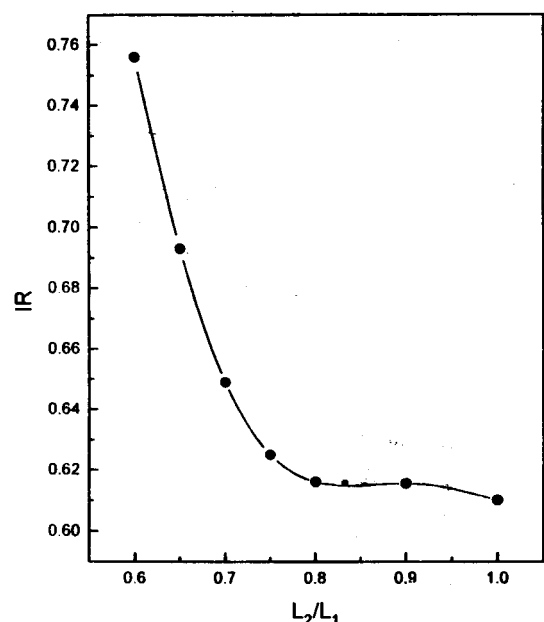


Fig. 5—Variation of amplification factor (IR) with the ratio of the wavelength in the harbour to that of the open sea (L_2/L_1).

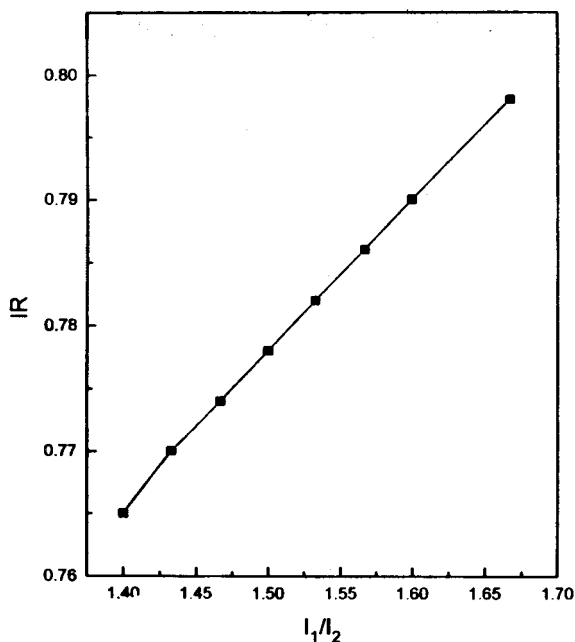


Fig. 6—Variation of amplification factor (IR) with the ratio of the width of the open sea to that of the harbour (I_1/I_2).

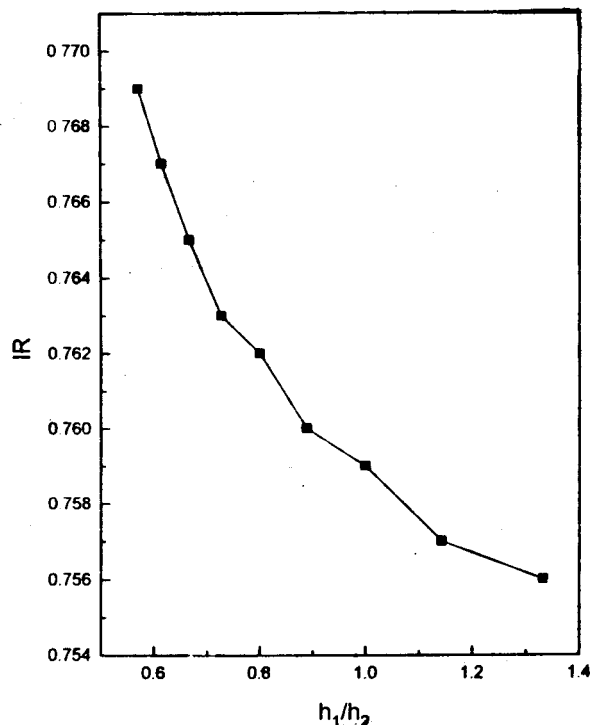


Fig. 7—Variation of amplification factor (IR) with the ratio of the depth of the open sea to that of the harbour (h_1/h_2).

Nomenclature

$\hat{a}_1$	= phase of the incident wave in domain (1)
$\hat{a}_2$	= phase of the incident wave in domain (2)
A	= shoaling co-efficient
A_1	= amplitude of the incident wave
$\hat{A}_1$	= complex number of modulus A_1 and argument $\hat{A}_1$
b	= length of the entrance
B_1	= amplitude of the reflected wave seawards
$\hat{B}_1$	= complex number of modulus B_1 and argument $\hat{B}_1$
C_0	= amplitude of the incident wave at the back of the basin
$\hat{C}_0$	= complex number of modulus C_0 and argument $\hat{C}_0$
C_1	= amplitude of the wave travelling towards the basin at the entrance
$\hat{C}_1$	= complex number of modulus C_1 and argument $\hat{C}_1$
D_0	= wave amplitude of the reflected wave at the back of the basin
$\hat{D}_0$	= complex number of modulus D_0 and argument $\hat{D}_0$
D_1	= amplitude of the wave arriving from the basin to the entrance
$\hat{D}_1$	= complex number of modulus D_1 and argument $\hat{D}_1$
g	= acceleration due to gravity
h_1	= depth of the open sea
h_2	= depth of the basin
IR	= amplification factor
I_1	= width of the open sea
I_2	= width of the basin
k	= coefficient of friction; wave number
l	= length of the basin
l_b	= length of the beach
L_1	= wave-length in the open sea
L_2	= wave-length in the basin
p	= coefficient of reflection at the back of the basin
r	= coefficient of decay of the wave-height in the basin
$\hat{r}$	= difference of phase between the waves at the two extremities of the basin
$\hat{r}$	= complex number of modulus r and argument $\hat{r}$

u	= velocity component in the x-direction
v	= velocity component in the y-direction
Z	= change of domain coefficient
$\hat{\alpha}$	= complex number of modulus α and argument $\hat{\alpha}$
α_1	= coefficient of transmission of the wave $\hat{A}_1$ from the domain (1) to domain (2)
$\hat{\alpha}_1$	= change of phase of the transmitted wave at the entrance towards the basin
$\hat{\alpha}_2$	= change of phase of the transmitted wave at the entrance seawards
α_2	= coefficient of transmission of the wave $\hat{D}_1$ from the domain (2) to domain (1)
$\hat{\beta}$	= complex number of modulus β and argument $\hat{\beta}$
β_1	= coefficient of reflection of the wave $\hat{A}_1$ in the domain (1)
$\hat{\beta}_1$	= change of phase of the reflected wave at the entrance seawards
β_2	= coefficient of reflection of the wave $\hat{D}_1$ in the domain (2)
$\hat{\beta}_2$	= change of phase of the reflected wave towards the basin at the entrance
ϕ_1	= potential function of the motion in the domain (1)
ϕ_2	= potential function of the motion in the domain (2)

References

- 1 Bowers E C, *J Fluid Mech*, 79(1) (1977) 71.
- 2 Claude F N, *J Fluid Mech*, 126 (1983) 219.
- 3 Hwang L S & Tuck E O, *J Fluid Mech*, 42(3) (1970) 447.
- 4 Ippen A T, *Estuary and Coastline Hydrodynamics* (McGraw-Hill Book Company, Inc), 1966.

- 5 Ippen A T & Goda Y, *Wave-induced oscillations in harbors: the solution for a rectangular harbor connected to the open sea*, Rep.no. 59, Hydrodynamics Lab., M.I.T., Cambridge, Mass, 1963.
- 6 Jiang-kang W & Philip L F L, *J Fluid Mech*, 217 (1990) 595.
- 7 Kravtchenko J & McNown J S, *Q Appl Math*, 13 (1955) 19.
- 8 Lee J J, *J Fluid Mech*, 45(2) (1971) 375.
- 9 McNown J S, *Waves and seiche in idealized port, Gravity Wave Symposium* (National Bureau of Standards Circular 521) 1952.
- 10 Mei C C & Agnon Y, *J Fluid Mech*, 208 (1989) 595.
- 11 Miles J & Munk W, *J Waterways and Harbors Division, ASCE*, 87 (1961) 111.
- 12 Raichlen F & Ippen A T, *J Hydraulics Division, ASCE*, 91(2) (1965) 1.
- 13 Spiegel M R, *Theory and problems of complex variables*, SI (Metric) Edition, (Schaum's Outline Series, McGraw Hill Book Company, Singapore), 1981.
- 14 Xiping Y & Chwang A T, *J Waterways, Port, Coastal Ocean Eng, ASCE*, 120 (1994) 125.