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Periodic and transient heat flow through building section

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Periodic heat flow through building section is practically being used for air-conditioning applications. An effort has been made to combine the non-periodic transient heat flow due to instantaneous rise in outdoor temperature with periodic heat flow. The study becomes useful to predict thermal performance of building section during heat wave in summer and snow fall in winter season. In order to solve nonperiodic heat flow problem, transcendental equation is required to be solved. Approximate values of unknown roots of transcendental equation for different ranges of Biot number have been given for reference. The solution has been combined alongwith the periodic heat flow. The response factor and temperature distribution in single and multilayered building section have also been determined.

The transient heat transfer through infinite building section is important in thermal engineering and it plays a significant role in indoor air temperature variation. This variation depends upon outside temperature profile and thermophysical properties of the material used in building sections. Building is influenced by both periodic and transient heat flow whenever there is sudden out door air temperature deviation due to heat wave or cold wave. Analytical solution by various techniques including separation of variables, green function or Fourier transform have been employed for the transient heat conduction problem in homogeneous or, composite building section. In these specified methods only periodic transient heat conduction has been taken into account for air-conditioning load calculation. This may lead to considerable inaccuracies in the case of significant temporary deviation of outdoor air temperature from periodicity. Thus, a number of methods are available to compute periodic heat flow into the building but for non-periodic transient heat flow computation technique is far lagging behind. Time Domain Response Functions[1] is suitable method to solve linear problems involving transient and periodic heat flow. However, an attempt has been made to develop a correlation[2] for computation of maximum non-periodic indoor wall heat flow caused by a temporary temperature rise in typical wall construction. The solution is based on Finite difference method. An approximate technique was employed[3] to arrive at weighing function. Weighing function takes into account both periodic and non-periodic transient heat flow. Similarly Chaudhary and Warsi[4,5] described a function to embody the complex nature of response. The function was defined as the enclosure response to an outdoor impulsive temperature variation and was the characteristic of the homogeneous and composite sections of enclosure. This is a good method for non-periodic heat transfer study but is too complicated.

An investigation is carried out to solve the problem of non-periodic heat flow through infinite slab. This requires involvement of the solution of a transcendental equation. This equation has been solved earlier by Heisler chart method[6], and Finite-difference method[7]. In the present study, this equation has been solved by Newton-Raphson iteration method by eliminating the usage of charts or graphs. Periodic heat flow has been computed by using R-C (time constant) method. Periodic heat flow along with transient heat flow will be applicable only when temperature deviation is not too large. In the present study the response factor

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and temperature distribution in single and multilayered building section have been determined by taking two examples.

Governing Equation

Let l is the thickness of roof composed of a single homogeneous layer with density ρ, specific heat C and thermal conductivity K. Then one dimensional heat conduction equation may be expressed as,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}, \frac{\partial T}{\partial t} \quad \ldots (1)$$

The solution is usually in the form,

$T(x,t)=\exp(-\lambda^2\ \alpha t)\ (A\ \mathrm{Cos}\ \lambda x + B\ \mathrm{Sin}\ \lambda x)$ ... (2)

where, ($\alpha = K/\rho C$) (diffusivity), $x=l$; A, B and λ are constants and can be derived by the boundary conditions.

(i) Periodic transient heat flow.

at $x=0$, $T_0=A.\ \exp(-\lambda^2\ \alpha t)$ ... (3)

and

$x=L$, $T_L=\exp(-\lambda^2\ \alpha t)\ (A\ \mathrm{Cos}\ \lambda L + B\ \mathrm{Sin}\ \lambda L)$ ... (4)

It can be put in the form of

$T_L=A_1\ [\exp(-\lambda^2\ \alpha t)]\ (\mathrm{Cos}\ wt\ 2+\phi)$, by considering Cos wave heat flow

where, ϕ is a function of x,

$w=2\pi/t$

(ii) Non-periodic transient heat flow
Initial condition,

at, $t=0$, $x=0$, $T=T_0$ ... (5)

at, $x=0$, $\partial T/\partial x=0$ ... (6)

and final boundary condition,

at, $x=L$, $-\partial T/\partial x = hT/k_{(x=L)}$ ... (7)

Eqs (2) and (6) leads to,

$\exp(-\lambda^2 \alpha t)\ B\lambda\ 0 \Rightarrow B=0$ ... (8)

Therefore,

$T_{(x,t)}=A\ \mathrm{Cos}\ \lambda x\ [\exp(-\lambda^2.\alpha t)]$ ... (9)

After evaluation of A, λ and at $x=L$, the equation has been put in the form[8] of,

$$\frac{T_{(x,t)}}{T_0} = [\exp\{(-\delta/L)^2 \alpha t\}]\left(\frac{1}{1+\dfrac{2\delta}{\mathrm{Sin}2\delta}}\right) \quad \ldots (10)$$

δ depends upon thickness L and Biot number BI for $x\#L$,

$$\frac{T_{(x,t)}}{T_0} = 2[\exp\{(-\delta/L)^2 \alpha t\}]\left(\frac{\mathrm{Sin}\delta\ \mathrm{Cos}\delta\ x/L}{\delta + \mathrm{Sin}\delta \mathrm{Cos}\delta}\right) \quad \ldots (11)$$

Numerical Solution

Application of computation of surface temperature and heat flow to buildings

(i) *Periodic heat flow*—The solution of periodic heat flow equation for computation of inner surface temperature, T_{is} is expressed as,

$$T_{is}=T_{ia}+\frac{U}{h_i}(\overline{T}_{soA}-\overline{T}_{ia})+\sum_{n=1}^{\infty} A_n \lambda_n \mathrm{Cos}(nwt-\gamma_n-\phi_n) \quad \ldots (12)$$

Heat flow from outside to inside the envelope at any time can be expressed by the equation,

$$Q=U(\widetilde{T}_{soA}-\widetilde{T}_{is}) \quad \ldots (13)$$

(ii) *Non periodic transient heat flow*—Non periodic heat flow through infinite building section and with effect from this increased temperature of unexposed surface is given by Eq. (10) as,

$$\frac{T_e - T}{T_e - T_i} = 2[\exp\{-(\delta/x)^2.\alpha.t\}][1/\{1+(2\delta/\mathrm{Sin}2\delta)\}]$$

where, left hand side of this equation is known as response factor.

Now the value of δ is computed from the transcendental Eq. (8)

$BI=\delta\ \tan\ \delta$ ... (14)

where, BI is called Biot number and is a dimensionless quantity. From the infinite section, BI can be calculated as,

$$BI=\frac{hx}{k} \quad \ldots (15)$$

Eq. (13) can be expressed in the form of a function of δ, as

$F(\delta)=\delta\ \tan(\delta)-BI$ ... (16)

Differentiation leads to,

$F'(\delta)=\delta\ \mathrm{Sec}^2\ \delta+\tan\ \delta$

In order to achieve the roots of transcendental equation (14), the approximate value of δ is chosen for a particular range of BI. Table 1 contains such approximate values of δ for different ranges of BI.

Exact value of δ has been computed by utilizing Newton-Raphson[9] Iteration process.

This iteration method gives the relation of the form,

Table 1—Approximate value of δ for different range of Biot number

Biot number	δ-value	Biot number	δ-value
0.2	0.45	6.0	1.35
0.5	0.65	8.0	1.40
0.8	0.80	9.4	1.42
1.0	0.87	12.0	1.45
1.14	0.90	16.0	1.48
1.56	1.00	25.0	1.51
2.20	1.10	74.0	1.55
3.0	1.20	100.0	1.55

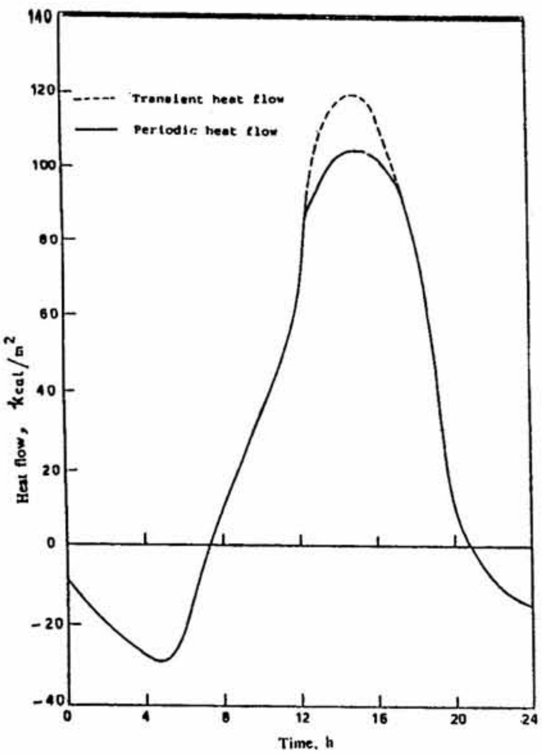

Fig. 1—Transient and periodic heat flow through 10 cm RCC roof section

$$\delta_{n+1} = \delta_n - [f(\delta n)/f'(\delta n)] \quad \ldots (17)$$

By putting n=0 in Eq. (17), it is inferred that,

$$\delta_1 = \delta_0 - [f(\delta_0)/f'(\delta_0)] \quad \ldots (18)$$

where $\delta_0 = \delta_{app}$ is initial value and can be chosen from Table 1.

Thus, first iteration leads to,

$$\delta_1 = \delta_{app} - \frac{\delta_{app}\tan(\delta_{app}) - BI}{\delta_{app}\,\mathrm{Sec}^2(\delta_{app}) + \tan(\delta_{app})} \quad \ldots (19)$$

The iteration process will continue for (n+1) times until, $\delta_{n+1} = \delta_n$ is achieved.

Thus δ_n will be the accurate root of the transcendental equation.

Example (i) Temperature of exposed surface of 10 cm thick RCC roof instantaneously rises due to heat wave from 38°C to 48°C. For computing the non-periodic transient heat flow and temperature distribution due to this increment in 1 h along with response factor, the procedure used is,

when K=1.2 Kcal/h n^2 °C, h=10 Kcal/h m °C and C=0.2 Kcal/Kg are also given, the computation of Biot number gives,

$BI=(hL)/K$

=0.833

For applying Newton-Raphson iteration method, the required value of δ_{app} is taken from Table 1 for the range of BI=0.800<0.833<10.

By taking lower side, δ_{app}=0.800

Thus from Eq. (18),

$$\delta_1 = 0.8 - \frac{0.8\tan(0.8) - 0.833}{0.8\mathrm{Sec}^2(0.8) - \tan(0.8)}$$

=0.8036

For second iteration, $\delta_1 = \delta_2$=0.8036 is obtained. Thus 0.8036 is the accurate value of δ.

The application of Eq. (10) leads to,

$$\frac{T_{(x,t)}}{T_0} = \{\exp[(-\delta/L)^2 \alpha t]\}\left(\frac{1}{1+\dfrac{2\delta}{\mathrm{Sin}2\delta}}\right)$$

=3.73

The rise in the temperature of unexposed surface of the roof due to 10°C rise in the temperature of exposed surface is 3.73°C.

Heat flow at this hour=$U(T_{oi} - T_{is})$

=13.45 Kcal/m^2 h

For example (i) Periodic heat flow for 24 h has been computed with the help of Eqs (12) & (13), transient heat flow has been combined with periodic heat flow and plotted in Fig. 1. Lower curve is shown for periodic heat flow where as upper dotted curve has been shown as an increase due to transient heat flow. Temperature distribution and response factor at various x/L in 1h is given in Table 2.

Similarly, by computing the non-periodic transient heat flow of peak temperature hour and on being added to the respective hour's periodic heat flow, the true value of peak heat flow for the computation of the air-conditioning load of the enclosure is obtained.

Table 2—Temperature distribution in single layered building section when there is 10°C rise in outer surface temperature

Depth at x/L	Rise in temperature at x/L	Temperature at x/L	Response factor at x/L
0.2	8.99	46.94	0.89
0.4	8.64	46.94	0.86
0.5	8.38	46.38	0.84
0.6	8.07	46.07	0.81
0.8	7.29	45.29	0.73
1.0	6.27	44.27	0.63

x Temperature predicted at $x=L$, $0<x<L$
L Total thickness

Table 3—Temperature distribution in multilayered building section when there is 15.6°C drop in outer surface temperature

Depth at x/L	Rise in temperature at x/L	Temperature at x/L	Response factor at x/L
0.2	15.05	22.55	0.96
0.4	13.59	24.01	0.87
0.5	12.53	25.07	0.87
0.6	11.25	26.35	0.72
0.8	8.19	29.41	0.52
1.0	4.70	32.90	0.29

x Temperature predicted at $x=L$, $0<x<L$
L Total thickness

Example (*ii*) For determining inside surface temperature of 11.5 cm brick, 5 cm Mudphuska and 11.5 cm brick building section due to periodic heat flow and computing the temperature distribution in the section when there is drop of temperature from 37.6 to 22°C due to snowfall with the assumption that dropped temperature remains constant for next two hours, the procedure adopted is,

Given,	k	ρ	c
Brick	0.75	1760	0.2
Mudphuska	0.49	1600	0.2

For the computation of inner surface temperature, Eq. (12) has been employed.
Average t_{ia} is taken as 26.7°C
U=1.66 and h_i=10.0 kcal/h m °C.

For computation of transient heat flow and temperature drop, the value of BI is determined as,

$$BI = \frac{h.L}{k} = h\left(\frac{L_1}{k_1} + \frac{L_2}{k_2}\right)$$
$$=4.09$$

from table corresponding δ_{app}=1.2

Then after third and fourth iteration, we get, $\delta_3=\delta_4$=1.27. Thus correct value of δ=1.27.

Temperature distribution in the section is computed by using Eq. (11).

Temperature of inner surface with corresponding drop and transient temperature drop have been given in Tables 3 and 4 respectively.

Discussion

The effort of this study is to involve both the periodic and instantaneous heat flow in the computation of rise in temperature. The method of computation for periodic is well known, and for non-periodic heat flow, computational method has been made easier. With the help of chosen δ_{app} from Table 1 and iteration method, value of δ is computed accurately. Care should be taken for choosing δ_{app} which is preferably towards the lower side for the particular range of BI as has been selected in the given example. Antonopoulos *et al.*[2] have conducted parametric study and they characterize any temporary deviation of the outdoor air temperature from periodicity. Due to this outside temperature deviation, indoor heat flow deviation has been defined. The present study is useful to compute the peak temperature and periodic heat flow of a particular hour and corresponding non-periodic heat flow of the same hour. It may be added for the computation of heating or air-conditioning load. Computation of non-periodic heat flow through all the sections of building may be made by the given method.

Fig. 1 consists of the heat flow due to periodic heat flow for 24 h into the building through 10 cm RCC roof. For 3 to 4 pm transient heat flow is given by dotted curve. Thus, peak heat flow is 118.8 Kcal/h and after adding the non-periodic heat flow for that particular hour, we get a rise in total peak heat flow.

Tables 2 and 3 consists of response factor and temperature distribution due to transient heat flow. From these tables it may be inferred that for higher temperature deviation, the change in resultant temperature at unexposed surface is higher after same time of interval. In Table 2 drop is 10°C and after 1 h at the middle point of the section the change in temperature is only 8.83°C where as in

Table 4—Inside surface temperature of multilayered building section

Time, h	Temperature °C	Temperature drop at outer surface, °C	Correspon-ding drop at inner surface after 2 h, °C
2	37.77		
4	36.70		
6	35.30		
8	35.40		
10	35.66		
12	36.31		
14	37.05		
16	37.60	15.60*	0.0*
18	38.14	—	8.26*
20	38.80		
22	39.16		
24	38.77		

*Temperature drop due to transient heat flow from inner surface to outer surface

Table 3 drop is 15.6°C and change in temperature after same interval and at middle point is 12.53°C.

Temperature drop in two hours as can be seen in Table 4 showing the inner surface temperature due to periodic heat flow. But due to out side snow fall, outdoor air temperature drops by 15.6°C and as a result inner surface temperature decreases by 8.26°C after two hours.

Conclusion

The presented non-periodic heat flow computation method eliminates the usage of charts and graphs completely. A reference table for required approximate roots of transcendental equation has been given for different ranges of Biot number. The study has been made for both single and multilayered sections to compute periodic and transient heat flow through it. Example (i) and (ii) are given to compute heat flow through single and multilayered building section respectively. Unlike other studies on the calculation of the transient heat flux through building sections considering only periodic heat flow, here the effort has been made to combine the non-periodic transient heat flow along with it. It will be applicable only when the temperature deviation is not too large and thermophysical properties do not vary.

It may be observed from the results of example (i) and (ii) that the rise or fall in the temperature of inner (unexposed) surface is higher for higher temperature deviation of the exposed surface, keeping other factors constant.

The combined method will be useful in estimating the peak heat load and sizing the chillers or air-conditioning equipments.

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Nomenclature

ϕ_n	=	Phase lag for *n*th harmonic
λ_n	=	Decrement factor for *n*th harmonic
R	=	Thermal resistance
C	=	Volumetric Specific heat
T_{ia}	=	Mean indoor air temperature
U	=	Over all thermal transmission coefficient
h_i	=	Inside film heat transfer coefficient
T_{soA}	=	Mean sol air temperature
A_n	=	Fourier amplitude
a_n, b_n	=	Fourier Constants
T_e	=	Outer exposed surface temperature
T_i	=	Initial surface temperature
T	=	Temperature at *x* after time
α	=	Thermal diffusivity
δ	=	λL
$T_{x,t}$	=	$T_e - T$
T_0	=	$T_e - T_i$
Q	=	Heat flow

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