



Magic Square

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MAGIC squares form a part of recreational mathematics and have been a source of fascination for aeons of time. A magic square is a square arrangement of distinct positive numbers (usually integers) such that the sum of each row, each column and each of the diagonals works out to be the same. A square consisting of consecutive integers starting with 1 is sometimes known as a "normal" magic square.

One of the methods of construction of magic square, of order n , where n is an odd integer, is as follows:

- (i) Begin with placing 1 in the center square of the top row.
- (ii) For placing the next integer, move one row above and one column to the left.
- (iii) If a move takes to a position such that the row is above the first row and column is beyond the last column, then fill in the next integer just below the last filled-in number.
- (iv) If a move takes to a position which is already filled, then fill in the next integer just below the last filled-in number.
- (v) If a move takes to a position above the first row, but the column is within the valid range, then move to the last row (column remains the same).
- (vi) If a move takes to a position beyond the last column, but the row is within the valid range, then move to the first column (row remains the same).

[N.B. In step (i), instead of beginning with the top row, i.e. first row, one can also begin with the last row. Remaining steps remain the same]

A step-by-step illustration for filling a 3×3 square is given below:

Start with the middle position of the first row. Therefore in row 1, column 2, we place 1.

	Col 1	Col 2	Col 3
Row-1		1	
Row-2			
Row-3			

Now move one row above and one column to the right, i.e. row 0 and column 3. But row 0 does not exist, hence we move to the last row i.e. row 3.

	1	
		2

Now move one row above and one column to the right, i.e. row 2 and column 4. But column 4 does not exist, hence we move to column 1.

	1	
3		
		2

Now move one row above and one column to the right, i.e. row 1 and column

2. That is an already filled position. So we fill in the next integer just below the integer last filled, i.e. we place the integer 4 in row 3 and column 1:

	1	
3		
4		2

Now for placing 5, move one row above and one column to the right, i.e. row 2 and column 2.

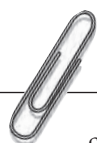
	1	
3	5	
4		2

Now move one row above and one column to the right, i.e. row 1 and column 3.

	1	6
3	5	
4		2

Now move one row above and one column to the right, i.e. row 0 and column 4. This move takes us to a position such that the row is above the first row and column is beyond the last column, so we fill in the next element just below the last filled-in number, i.e. the next integer will be placed below 6; i.e. at row 2 and column 3:

	1	6
3	5	7
4		2



FEATURE ARTICLE

Now move one row above and one column to the right, i.e. row 1 and column 4. But column 4 does not exist, hence we move to column 1.

8	1	6
3	5	7
4		2

Now move one row above and one column to the right, i.e. row 0 and column 2. But row 0 does not exist, hence we move to the last row i.e. row 3.

8	1	6
3	5	7
4	9	2

The sum of each row, each column and each diagonal works out to 15, which can be found by the formula

$n(n^2 + 1)/2$, where n is the number of rows or the number of columns in the square, i.e. n indicates the order of the square. [Note: the formula $n(n^2 + 1)/2$ is to be used only when consecutive numbers starting from 1 upto n^2 are used to populate the magic square.]

There are different algorithms for constructing a magic square having even number of rows and columns. A simple method for constructing a 4-by-4 magic square is given below:

Fill in the numbers 1 to 16 sequentially as depicted below:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Now perform interchanges as shown below:

The element in row 1 column 1 with row 4 column 4

The element in row 2 column 2 with row 3 column 3

The element in row 3 column 2 with row 2 column 3

The element in row 4 column 1 with row 1 column 4. On performing the

indicated interchanges we get a 4-by-4 magic square:

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

There are 880 distinct magic squares (excluding those obtained by reflection and rotation) of order 4×4 . Due to space and time constraints all the 880 magic squares cannot be shown herein.

On subtracting every number in the magic square from $n^2 + 1$, another magic square is obtained which is known as "complementary" magic square. For instance, on subtracting every member of the magic square above from $4^2 + 1$, i.e. 17, we get the following magic square:

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

A special 4-by-4 magic square is given below:

12	6	15	1
13	3	10	8
2	16	5	11
7	9	4	14

The sum of elements of each row is 34, the sum of elements of each column is 34, the sum of each of the diagonals is 34, also the sum of elements of the 2-by-2 squares which can be formed, without changing the order of the numbers, is 34.

A magic square which can be formed using Prime numbers is given below:

43	67	109
139	73	7
37	79	103

A square that fails to be a magic square only because the sum of the elements in one or both of the diagonals does not equal the magic constant is known as a semi-magic square. Till now we have considered magic squares under the addition operation only. Squares which are magic under multiplication can also be constructed and are known as multiplication magic squares. Also squares which are magic under addition as well as multiplication can also be constructed and are known as addition-multiplication magic squares.

An example of a multiplication magic square is given below:

2	9	12
36	6	1
3	4	18

The product of the elements of each diagonal, each row, each column works out to 216.

Alphamagic Squares

Two magic squares are given below:

25	8	12
2	15	28
18	22	5

Magic square 1

10	5	6
3	7	11
8	9	4

Magic square 2

The sum of each row, each column and each diagonal of Magic square 1 is 45.

The sum of each row, each column and each diagonal of Magic square 2 is 21.

Is there any relationship between Magic square 1 and Magic square 2?

No. of letters in twenty five = 10 (the number in first row first column of Magic Square 2)

No of letters in eight = 5 (the number in first row second column of Magic Square 2)



A magic square is a square arrangement of distinct positive numbers such that the sum of each row, each column and each of the diagonals works out to be the same. A square starting with 1 is sometimes known as a “normal” magic square.

No. of letters in twelve = 6 (the number in first row third column of Magic Square 2)

No. of letters in two = 3 (the number in second row first column of Magic Square 2)

No. of letters in fifteen = 7 (the number in second row second column of Magic Square 2)

No. of letters in twenty eight = 11 (the number in second row third column of Magic Square 2)

No. of letters in eighteen = 8 (the number in third row first column of Magic Square 2)

No. of letters in twenty two = 9 (the number in third row second column of Magic Square 2)

No. of letters in five = 4 (the number in third row third column of Magic Square 2)

[Note: The relationship between magic square 1 and magic square 2 was first found by the word game expert by the name of Lee Cecil Fletcher Sallows.]

In short, magic square 2 is generated from the number of letters in the word for each number in magic square 1. Such magic squares are known as alphamagic squares.

Templar Magic Squares

S	A	T	O	R
A	R	E	P	O
T	E	N	E	T
O	P	E	R	A
R	O	T	A	S

The magic square reads the same from top-to-bottom, bottom-to-top, left-to-right, and right-to-left. It is made from a famous Latin sentence sator arepo tenet opera rotas which translates to the “farmer arepo keeps the world rolling”. Such a square is known as templar magic square.

As an exercise the esteemed readers can try to form magic squares of consecutive prime numbers, consecutive members of the Fibonacci series, etc.

Concentric or Bordered magic squares

What are the magic squares for which a magic square results on removing the top and bottom rows and the rightmost and leftmost columns.

4	5	6	43	39	38	40
49	15	16	33	30	31	1
48	37	22	27	26	13	2
47	36	29	25	21	14	3
8	18	24	23	28	32	42
9	19	34	17	20	35	41
10	45	44	7	11	12	46

The sum of the elements of each row, each column, and the diagonals works out to 175. On deleting the first and the seventh rows, and the first and the seventh columns, the magic sum will be 125. Thereafter on deleting the outermost rows and columns (the first and the fifth rows and columns), the magic sum works out to 75.

Antimagic Squares

An antimagic square is a square arrangement of numbers from 1 to n^2 , such that the sum of each row, each column, and the diagonals produces a different sum such that the sums form a sequence of consecutive numbers. An example of a 4-by-4 antimagic square is given below:

15	2	12	4	33
1	14	10	5	30
8	9	3	16	36
11	13	6	7	37
35	38	31	32	

The sums of each row and each column have been highlighted in yellow.

Sum of elements of diagonal from top-left hand corner to bottom-right hand corner = 39.

Sum of elements of diagonal from top-right hand corner to bottom-left hand corner = 34.

A Special Square: Array 57

A 5 X 5 matrix presented below has an interesting property:

19	8	11	25	7
12	1	4	18	0
16	5	8	22	4
21	10	13	27	9
14	3	6	20	2

If we pick any element from each row, so that no two elements lie in the same column and compute the sum of the five elements so chosen, it will always be 57.

Example:

$$8 + 0 + 8 + 21 + 20 = 57$$

The first 8 is taken from 1st row and 2nd column.

0 is taken from 2nd row and 5th column.

The second 8 is taken from 3rd row and 3rd column.

21 extracted from 4th row and 1st column.

20 is extracted from 5th row and 4th column.

There are 120 ways of choosing the numbers which would yield a sum of 57.

A parting thought: Here, the integers to be filled in the 5 x 5 array have been specified and the sum 57 is also known. But a few questions which are worth answering are (i) Can there be any other “sum” i.e. other than 57? (ii) Can other integers be filled in the 5 x 5 array to yield similar results? and (iii) Can we think of another array of different order, say, 7 x 7 array, which exhibits properties similar to the array mentioned above?

One way is to fill in random integers into a given array of order ‘5 x 5’ and then pick any element from each row, so that no two elements lie in the same column and compute the sum of the ‘5’ elements so chosen and check whether it yields a constant sum.

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