

Structural Geometry of False Twist Textured Yarns

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Received 29 April 1978; Accepted 14 May 1978

Using a mathematical expression derived from the general theory of bending and twisting of thin rods, the presence in textured yarns of helices of both *S*- and *Z*-type, reversal points, helices of varying radius, loops and snarls is explained.

In a recent study¹ of the mechanics of latent crimp in false-twist textured yarn, the structural geometry of textured yarn, held at the two ends by tension T , was considered on the basis of Love's theory² of bending and twisting of thin rods. It was shown that at equilibrium, the following expression holds:

$$T + \frac{1}{2}(AK^2 + BK'^2 + C\zeta^2) + \int Zds = \text{constant} \dots (1)$$

where T is the tensile force acting through the cross-section at a point on a filament; A, B are the bending rigidities in the plane of K and K' respectively, K, K' , the two components of the curvature of a filament in its deformed configuration; C is the torsional rigidity of the filament; ζ , the total geometrical twist; and $\int Zds$, the total tangential contact forces up to that point on the filament.

As pointed out earlier¹, Eq. (1) is based on the equilibrium configuration of the textured filament and takes into account the three-dimensional spatial nature of the geometry of the yarn. The equations of equilibrium are developed in complete generality and the various forces which act on the textured filament have been considered in this model.

The purpose of the present paper is to explain some of the known features of the geometry of textured yarns like the presence of helices of both *S*- and *Z*-type, reversal points, helices of varying radius and loops and snarls on the basis of this equation.

Structural Geometry of Textured Yarns

On the basis of Eq.(1), we can explain some phenomena that have been reported in the literature or can be easily observed in the case of textured yarns. The following are the main observations.

(1) In the same equilibrium configuration, ζ can change to $-\zeta$ without affecting relation (1).

Thus, a filament in the form of a regular helix

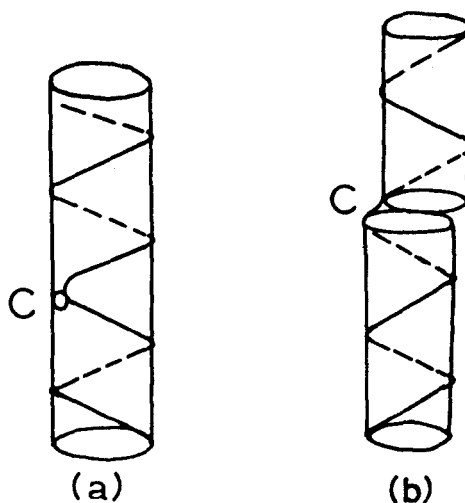


Fig.1—(a) A combination of *S*- and *Z*- helix; *C*, the coupling member. (b) a combination of two segments of the yarn with opposite curvatures; *C*, the coupling member.

and without St. Venant torsion can be in equilibrium with a similar helix but with opposite twist. Thus, we find *S*-helix or *Z*-helix, as shown in Fig. 1(a). At the junction between *S* and *Z* helices, a coupling member can have either a different curvature to satisfy Eq. (1) or St. Venant type twist.

(2) Similarly, K or K' can be replaced by $-K$ or $-K'$ respectively. For example, a single change of K to $-K$ will mean a change of axes of helix, as is obvious from Fig. 1(b). Again, a coupling member can exist.

From (1) and (2), we conclude that although in the natural state or heat-set state, a filament may be a regular helix, yet in its equilibrium state under the action of forces or moments, it may show an entirely different configuration, where twists or curvatures are changing their sign very often.

(3) One can also examine the possibility of a

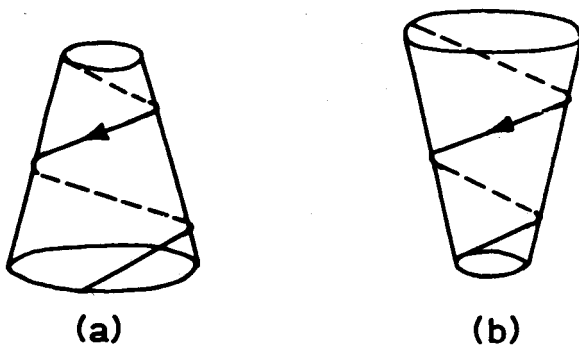


Fig. 2—Helices with radius of curvature (a) increasing, and (b) decreasing, along the filament length.

filament taking up the form of a helix with a changing radius. In order to fix ideas, let us visualize a single filament wound on a conical mandrel, such that the angle of helix α is constant. For this helix, we have

$$K(s) = -\cos\beta \frac{\cos^2\alpha}{r(s)}$$

$$K'(s) = \sin\beta \frac{\sin^2\alpha}{r(s)} \text{ and}$$

$$\zeta(s) = \frac{d\beta}{ds} + \frac{\sin\alpha \cos\alpha}{r(s)},$$

where β is the angle of St. Venant type twist. Substituting in Eq. (1), we see that

$$T + \frac{1}{2} A \cos^2\beta \frac{\cos^4\alpha}{r^2} + \frac{1}{2} B \sin^2\beta \frac{\sin^4\alpha}{r^2} + \frac{C}{2} \left(\frac{d\beta}{ds} + \frac{\sin\alpha \cos\alpha}{r} \right)^2 = \text{constant} \dots (2)$$

The $\int Zds$ term has been omitted, because there is no contact with other filaments and hence the surface forces vanish. From Eq. (2) it is seen that if T is increased, r must also increase. Thus, in the regions where the helix radius is more, the tension is also more and vice versa.

When we consider a multifilament yarn, the sum $(T + \int Zds)$ plays the same role as T alone in Eq. (2). Thus, if the helix is expanding, $(T + \int Zds)$ must be higher. Two special cases, assuming T constant, are illustrated in Fig. 2. If $\int Zds$ is increasing, r must also increase (Fig. 2a). Similarly, if $\int Zds$ is decreasing, then r also decreases (Fig. 2b).

(4) Eq. (1) can be used to study the transformation

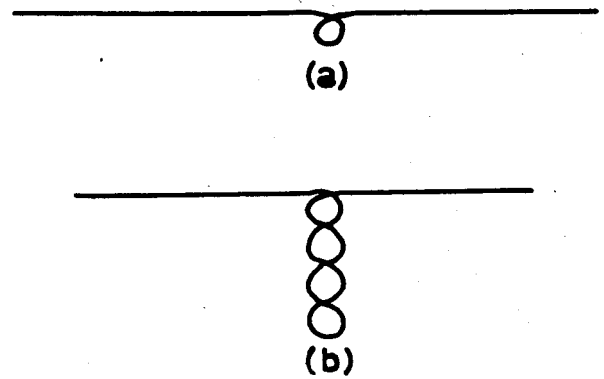


Fig. 3—Representation of snarl formation

from one mode of deformation to another. The important phenomenon of snarling can be explained as follows. When tension T is reduced continuously, the filaments take up a more and more curved or twisted configuration, i.e. K, K' and ζ increase individually or simultaneously. When T is reduced sufficiently or even reversed, K or K' becomes very large and this results in the formation of a loop, as shown in Fig. 3. If the ends are brought still closer together, more loops will form and a typical snarl is developed.

Conclusion

The presence in textured yarns of helices of both S- and Z- type, reversal points, helices of varying radius, loops and snarls can be explained by Love's theory of the bending and twisting of thin rods. The application of this theory to textured yarns leads to the conclusion that although in the natural or heat-set state a filament may be a regular helix, in its equilibrium state under the action of forces or moments, it may show an entirely different configuration where twists or curvatures are changing their sign very often.

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