

Functional representation of observed correlations for statistical interpolation scheme of objective analysis

S K Sinha, S G Narkhedkar & S Rajamani

Indian Institute of Tropical Meteorology, Pashan, Pune 411 008

Received 29 January 1996; revised 25 March 1997; accepted 10 April 1997

For statistical interpolation scheme of objective analysis, the weights of matrices are determined from the correlation functions of meteorological parameters, which are fitted to a function to facilitate computations. In this regard, the horizontal height correlations of observed height residuals (differences between radiosonde data and the 24 hrs forecast that are used as initial fields) are represented by a sum of two degenerated third-order auto-regressive (TOAR) functions after suitably modifying it for the Indian region. The geostrophic approximation, relating height and wind field is decoupled near the equator. Analyses were made for two different synoptic situations, viz. 4-8 July 1979 and 26-30 July 1991 for 1200 hrs GMT. It was found that the analyses are closer to the actual situations, as the root mean square errors for new scheme were less in both the situations as compared to the analyses made with the Gaussian correlation function (old scheme), representing the height-height correlations.

1 Introduction

It is well known that numerical weather prediction (NWP) is an initial value as well as a boundary value problem. The initial fields representing the atmospheric motion should be very accurate to get better forecasts in short range and medium range. While developing the multivariate statistical interpolation (MSI) scheme Gandin¹ faced some practical difficulties in early sixties because of the high cost of computer time. But, increase in computer power over the past few years has served as a catalyst for optimizing and restructuring the objective analysis schemes. The more efficient and flexible computer codes/programs are possible now because of the availability of improved version of computers. The resolving capability of the analysis depends not only on the grid length of the analysis grid and the data density, but also on the structure functions and other statistics that are basics to a statistical interpolation procedures. Since statistical interpolation for operational NWP works in terms of observed residuals rather than the observations themselves (as residuals are better correlated), the structure functions and other statistics should be recalculated periodically to keep them appropriate to the current forecast system. This has been discussed by Thiebaux *et al.*² Other aspects of correlation structure were examined by Hollingsworth and Lonnberg³, and Lonnberg and Hollingsworth⁴. In their studies, they have focused their attention on the structure

of wind and height observed residuals in winter mainly over North American region between 30°N and 60°N. Following Daley⁵ they derived the kinematical partial differential equations for two-dimensional homogeneous turbulence and then used this technique to examine the wind residuals. Using observed residuals obtained from the European Centre for Medium Range Weather Forecasts (ECMWF) global data assimilation cycle, they examined, for both wind and height, the magnitude of the residuals as a function of position as well as the structure of their horizontal and vertical correlations. In earlier MSI scheme Sinha *et al.*⁶ have fitted a Gaussian function (negative squared exponential) to the observed height-height correlations. Once a height representation has been chosen, the corresponding wind representation would be made by the assumption of geostrophy. We are basically seeking a set of geostrophically consistent correlation functions for geopotential height.

The problem of representing the height-height correlations of the observed residuals has been studied by Julian and Thiebaux⁷. After examining a number of functions, Thiebaux *et al.*² adopted a degenerated second-order auto-regressive (SOAR) function, $(1 + d \cdot r) \exp(-d \cdot r)$, where d and r represent scale parameter and radial distance, respectively and an additive constant to represent the large scale horizontal component for the height-height correlation. Mitchell *et al.*⁸ modified

the above function by replacing the degenerated SOAR function by a degenerated third-order auto-regressive function (TOAR) represented by $(1 + d \cdot r + d^2 \cdot r^2/3) \exp(-d \cdot r)$ and the additive constant has been replaced by a second broader TOAR function.

In this paper, following Mitchell *et al.*⁸, TOAR function has been modified so that the new function represents more closely the observed correlations computed for Indian region. Then, a two-dimensional objective analyses of height and wind have been carried out and the improvements are examined.

2 Methodology

2.1 Formulation of analysis scheme

The analyses of height (z) and wind components (u and v) at some grid point 'g' (z_g^a , u_g^a , v_g^a) at a particular level on the basis of nearby observations are computed as the sum of the linear combinations of the observed corrections $z_i^q (= z_i^o - z_i^f)$, u_i^q , v_i^q added to the first guess values (z_g^f , u_g^f , v_g^f) at the grid point. The analysis equation is

$$\begin{bmatrix} z_g^a \\ u_g^a \\ v_g^a \end{bmatrix} = \begin{bmatrix} z_g^f \\ u_g^f \\ v_g^f \end{bmatrix} + \sum_{i=1}^n A_i \begin{bmatrix} z_i^q \\ u_i^q \\ v_i^q \end{bmatrix} \quad \dots (1)$$

where

$$A_i = \begin{bmatrix} a_i^z & b_i^z & c_i^z \\ a_i^u & b_i^u & c_i^u \\ a_i^v & b_i^v & c_i^v \end{bmatrix} \quad \dots (2)$$

and a^z , b^z , c^z , a^u , b^u , c^u , a^v , b^v and c^v are the elements of the weight A_i , which is a 3×3 matrix.

Determining the weights A_i 's by solving the above set of equations and subsequently adding the linear combination of the weighted anomalies to the initial guess field at each grid point to get the analysed values, is the main objective of the study.

In order to solve the above equations for weights one has to derive an expression for the mean square interpolation error I_0 , and then make it (I_0) to be minimum with reference to the weights

$$I_0^2 = \left[z_g^a - z_g^f - \sum_{i=1}^n a_i^z z_i^q - \sum_{i=1}^n b_i^z u_i^q - \sum_{i=1}^n c_i^z v_i^q \right]^2 \quad \dots (3)$$

The solution is

$$(A_1 A_2 A_3 \dots A_n) = (C_{g1} C_{g2} C_{g3} \dots C_{gn}) \times \begin{bmatrix} C_{11} & C_{12} & C_{13} & \dots & C_{1n} \\ C_{21} & C_{22} & C_{23} & \dots & C_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & C_{n3} & \dots & C_{nn} \end{bmatrix}^{-1} \quad \dots (4)$$

where C_{ij} is 3×3 covariance matrix

$$C_{ij} = \begin{bmatrix} \overline{z_i' z_j'} & \overline{z_i' u_j'} & \overline{z_i' v_j'} \\ \overline{u_i' z_j'} & \overline{u_i' u_j'} & \overline{u_i' v_j'} \\ \overline{v_i' z_j'} & \overline{v_i' u_j'} & \overline{v_i' v_j'} \end{bmatrix} \quad \dots (5)$$

and

$$C_{gi} = \begin{bmatrix} \overline{z_g' z_i'} & \overline{z_g' u_i'} & \overline{z_g' v_i'} \\ \overline{u_g' z_i'} & \overline{u_g' u_i'} & \overline{u_g' v_i'} \\ \overline{v_g' z_i'} & \overline{v_g' u_i'} & \overline{v_g' v_i'} \end{bmatrix} \quad \dots (6)$$

where, i and j both range from 1 to n .

2.2 Horizontal prediction error correlation function

Daley⁵ introduced correlation functions between three scalar quantities, namely height (z), stream function (ψ) and velocity potential (χ). Assuming nondivergence, homogeneity and isotropy, the scalar covariances involving stream function (ψ) and height (z) can be written as

$$\begin{aligned} \langle \psi_i \psi_j \rangle &= E_\psi^2 F(r_{ij}) \\ \langle z_i \psi_j \rangle &= \langle \psi_i z_j \rangle = E_z E_\psi \mu F(r_{ij}) \\ \langle z_i z_j \rangle &= E_z^2 F(r_{ij}) \end{aligned} \quad \dots (7)$$

where E_ψ and E_z are the standard deviations of the error of stream function and height respectively, F is a correlation function and r_{ij} the scalar distance between two arbitrary points i and j and is given by

$$r_{ij}^2 = r^2 = (x_i - x_j)^2 + (y_i - y_j)^2$$

where x_i , x_j and y_i , y_j are cartesian coordinates of the points and μ the geostrophic coupling coefficient and is given by

$$\mu = 1.0 - \exp(-0.5 |\phi|) \quad \dots (8)$$

with magnitude close to 1 outside of the tropics and equal to zero at the equator, and ϕ is the latitude in degrees.

Following Mitchell *et al.*⁸ we express the horizontal prediction error correlation function for

height over Indian region by introducing an additional constant β (which takes into account the observational error) as

$$F(r) = \beta(1 + \alpha)^{-1}[(1 + d \cdot r + d^2 \cdot r^2 / 3) \exp(-d \cdot r) + \alpha\{1 + (d \cdot r / N) + (d^2 \cdot r^2 / 3N^2)\} \exp(-d \cdot r / N)] \quad \dots (9)$$

$$F'(r) = \beta(1 + \alpha / N^2)^{-1}[(1 + d \cdot r) \exp(-d \cdot r) + (\alpha / N^2)\{1 + (d \cdot r / N)\} \exp(-d \cdot r / N)] \quad \dots (10)$$

$$F''(r) = \beta(1 + \alpha / N^4)^{-1}[\exp(-d \cdot r) + (\alpha / N^4) \exp(-d \cdot r / N)] \quad \dots (11)$$

where prime indicates the normalized logarithmic derivative. Using Helmholtz's theorem

$$u = -\frac{\partial \psi}{\partial y}; \quad v = \frac{\partial \psi}{\partial x}$$

the prediction error covariances are given by

$$\begin{aligned} \langle u_i u_j \rangle &= E_{\psi}^2 \Gamma[F(r)] \\ \langle v_i v_j \rangle &= E_{\psi}^2 \Delta[F(r)] \\ \langle v_i u_j \rangle &= \langle u_i v_j \rangle = E_{\psi}^2 \Theta[F(r)] \\ \langle z_i u_j \rangle &= -\langle u_i z_j \rangle = E_z E_{\psi} \mu \Xi[F(r)] \\ \langle z_i v_j \rangle &= -\langle v_i z_j \rangle = -E_z E_{\psi} \mu \Pi[F(r)] \\ \langle z_i z_j \rangle &= E_z^2 F(r) \end{aligned} \quad \dots (12)$$

where,

$$\begin{aligned} \Gamma &= -\left[\frac{1}{r} \frac{\partial}{\partial r} + (y_i - y_j)^2 \frac{1}{r} \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right] \\ \Delta &= -\left[\frac{1}{r} \frac{\partial}{\partial r} + (x_i - x_j)^2 \frac{1}{r} \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right] \\ \Theta &= \left[(y_i - y_j)(x_i - x_j) \frac{1}{r} \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right] \\ \Xi &= \left[\left(\frac{y_i - y_j}{r} \right) \frac{\partial}{\partial r} \right] \\ \Pi &= \left[\left(\frac{x_i - x_j}{r} \right) \frac{\partial}{\partial r} \right] \end{aligned} \quad \dots (13)$$

Substituting $F(r)$ from Eq. (9) into Eq. (12) and expressing the correlation as

$$\text{Corr}(a, b) = \frac{\langle a, b \rangle}{[\langle a, a \rangle \langle b, b \rangle]^{1/2}}$$

the expressions for the horizontal zz , uu and vv correlations and the related cross-correlations can be written as

$$\text{Corr}(u_i, u_j) = F'(r) - (y_i - y_j)^2 d^2 \frac{(1 + \alpha / N^4)}{(1 + \alpha / N^2)} F''(r)$$

$$\text{Corr}(v_i, v_j) = F'(r) - (x_i - x_j)^2 d^2 \frac{(1 + \alpha / N^4)}{(1 + \alpha / N^2)} F''(r)$$

$$\begin{aligned} \text{Corr}(u_i, v_j) &= \text{Corr}(v_i, u_j) \\ &= d^2 (y_i - y_j)(x_i - x_j) \frac{(1 + \alpha / N^4)}{(1 + \alpha / N^2)} F''(r) \end{aligned}$$

$$\begin{aligned} \text{Corr}(z_i, u_j) &= -\text{Corr}(u_i, z_j) \\ &= -\mu \left[\frac{(1 + \alpha / N^2)}{3(1 + \alpha)} \right]^{1/2} d(y_i - y_j) F'(r) \end{aligned}$$

$$\begin{aligned} \text{Corr}(z_i, v_j) &= -\text{Corr}(v_i, z_j) \\ &= \mu \left[\frac{(1 + \alpha / N^2)}{3(1 + \alpha)} \right]^{1/2} d(x_i - x_j) F'(r) \\ \text{Corr}(z_i, z_j) &= F(r) \end{aligned} \quad \dots (14)$$

where, $F(r)$, $F'(r)$ and $F''(r)$ are already defined.

3 Analysis experiments

3.1 Synoptic situation

In order to examine how this new scheme with revised prediction error correlation function performs, we have chosen the period from 4 to 8 July 1979 and 26 to 30 July 1991 for 1200 hrs GMT. In the first case during this period, a low pressure was formed over the head of the Bay of Bengal with its central region near 20°N and 90°E on 4 July and moved north-westward. It moved slowly at the beginning and intensified into a depression on 7 July and crossed the Indian coast on 8 July. In the second situation, a cyclonic circulation was observed on 25 July 1991. Under the influence of this cyclonic circulation, a low pressure was formed over north-west Bay on 26 July, became depression on 27 July and intensified into a deep depression on 28 July, crossed the coast near Paradeep on 29 July and then moved almost in westerly direction and was over Madhya Pradesh on 30 July. The track of the depression in the case of first situation is shown in Fig. 1.

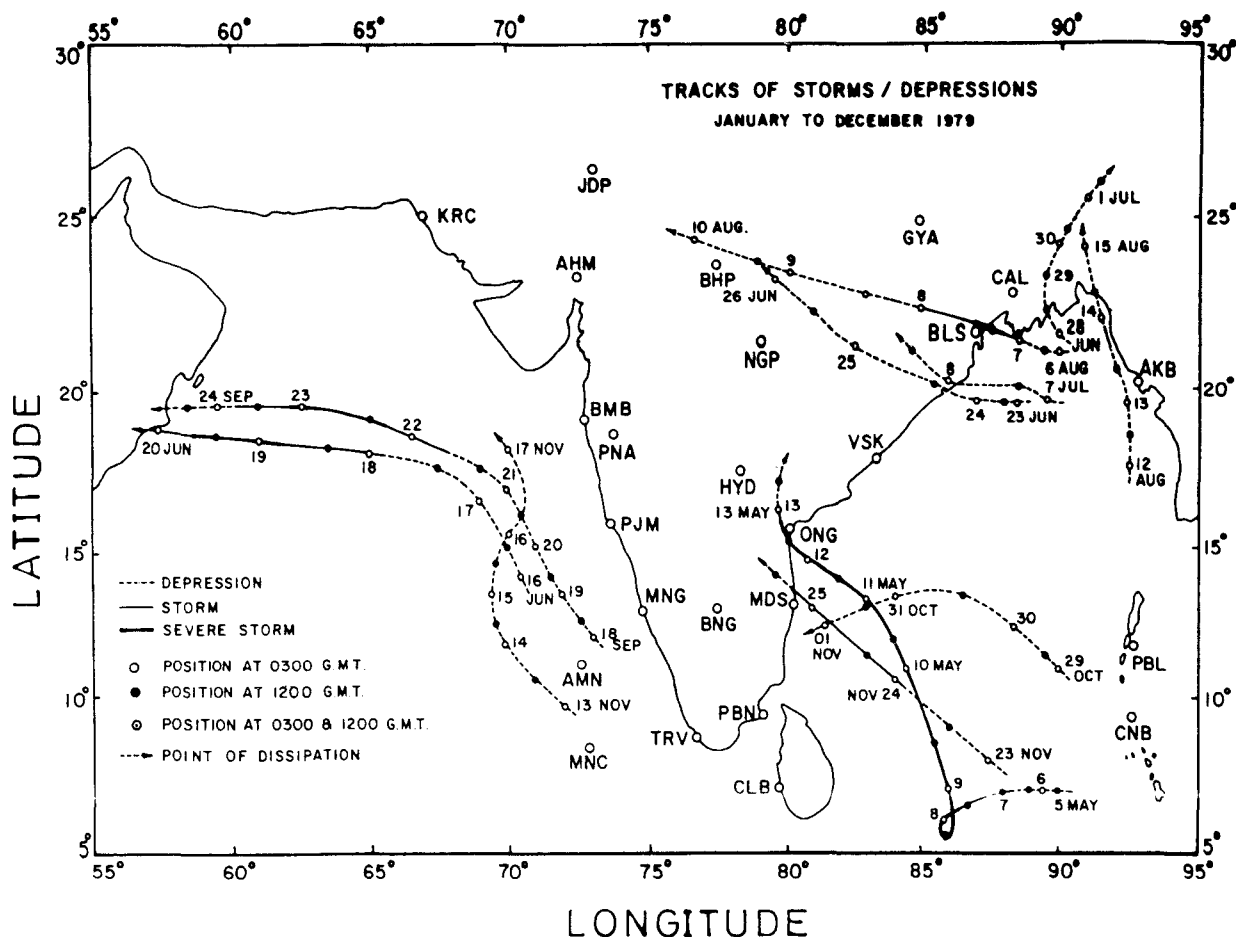


Fig. 1—Tracks of storms/depressions of January-December 1979

3.2 Correlation computation and its fitting

The correlations for the level 700 hPa for the height residuals was computed for every station with respect to every other station over India and the adjoining region based on four July months from 1976 to 1979. These residuals were normalized before plotting against distance. There was scatter of points and hence points within two degree segment were averaged. To these averaged correlations, the revised function [Eq. (9)] was fitted. We have used the International Mathematical and Statistical Libraries (IMSL) routine to evaluate the zero-intercept value β and other constants involved in the function given by Eq. (9), by minimizing the sum of the squared deviations of the observed correlations and the computed correlations. The value of the constants obtained are $\beta=0.61$, $d=18$, $\alpha=0.62$ and $N=4.36$. These values produced a good fit to the observed correlations. The observed correlations after averaging, the revised function with the best value of the constants and the Gaussian function fitted to an observed height-height correlations are shown in Fig. 2.

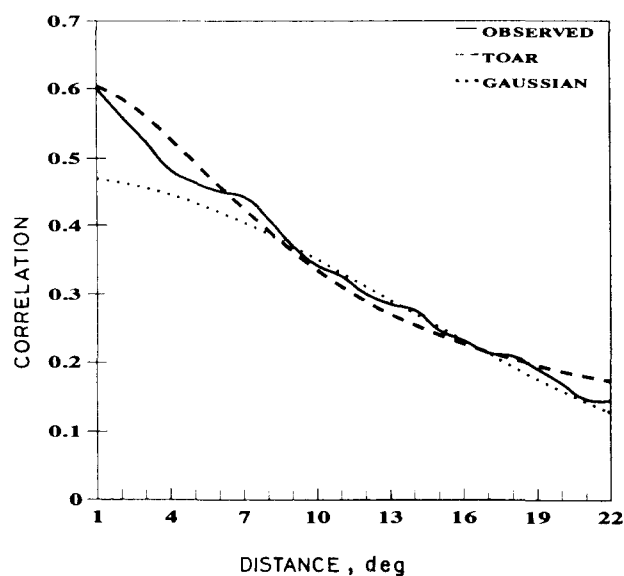


Fig. 2—Observed interval-averaged correlation versus separation (in deg.) of height residuals at 700 hPa

3.3 Analysis of height and wind

One of the significant advantages in the use of the multivariate objective analysis is the ability to allow observations of one variable (e.g. winds) to influence directly the analysis of another variable (e.g. height). The analyses of height and wind fields using this multivariate scheme were made for 850, 700, 500, 300 and 200 hPa levels for the period 4-8 July 1979 for 1200 hrs GMT over a region bounded by 1.875°N to 39.375°N and 41.250°E to 108.750°E and also for 26-30 July 1991 at 850, 700 and 200 hPa levels over a region

bounded by 2.5°N to 40°N and 40°E to 110°E. Analyses in both the situations were found to be fairly representative of the synoptic conditions prevailed. Analyses for typical cases of 7 July 1979 and 28 July 1991 at 700 hPa level are shown in Fig. 3[(a), (b)] and Fig. 4[(a), (b)]. The wind speeds and directions are indicated by the arrows. The magnitude of the wind is indicated in the upper right hand corner which shows a wind arrow and the corresponding speed in m/s. From Figs 3-5, one can infer that the monsoon low has been depicted well in all the analyses and in both

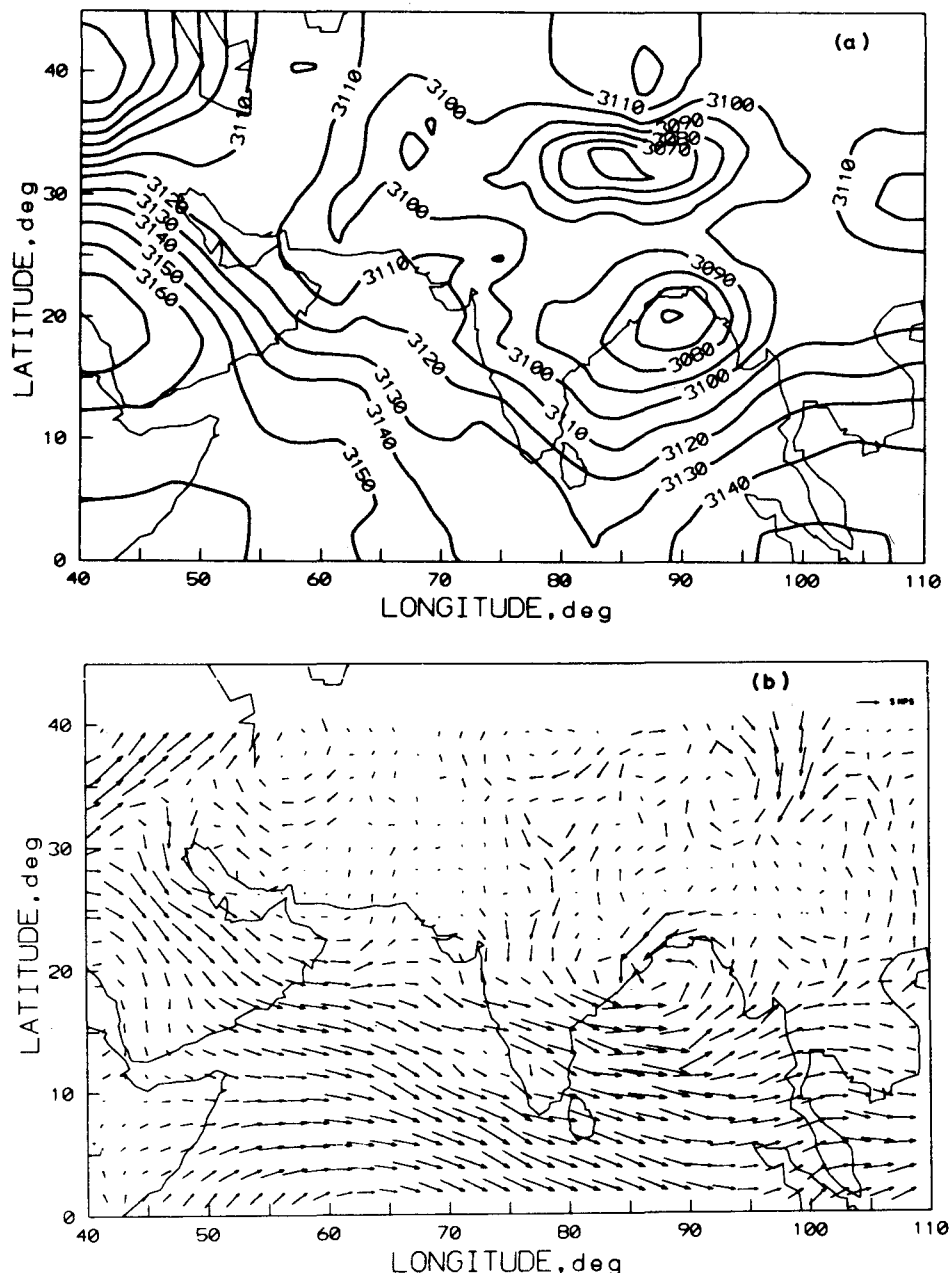


Fig. 3—Contour plots of 700 hPa height analysis of 7 July 1979 for 1200 hrs GMT with (a) TOAR function and (b) for wind field

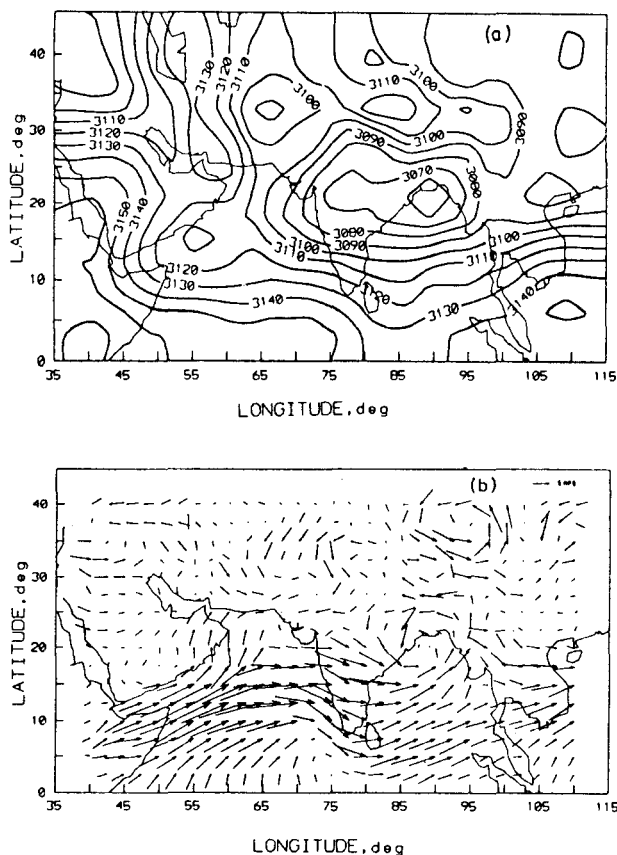


Fig. 4—Contours plots of 700 hPa height analysis of 28 July 1991 for 1200 hrs GMT with (a) TOAR function and (b) for wind field

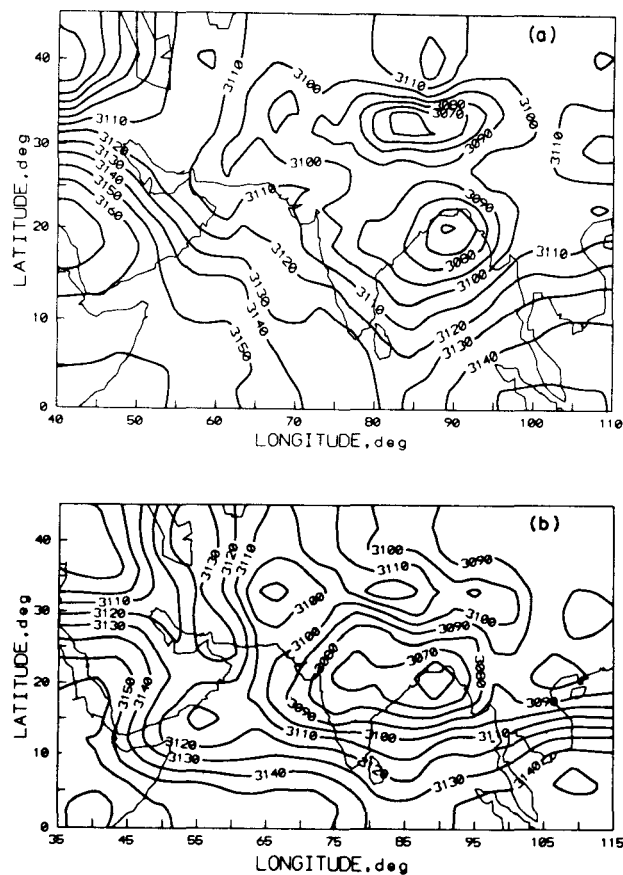


Fig. 5—Contour plots of 700 hPa height analysis of (a) 7 July 1979 and (b) 28 July 1991 for 1200 hrs GMT with Gaussian function

Table 1—Root mean square error statistics of height (m) and wind (m/s) fields for new (TOAR) and old (Gaussian) schemes
Error statistics on

Levels (hPa)	4 July 79		5 July 79		6 July 79		7 July 79		8 July 79	
	New	Old	New	Old	New	Old	New	Old	New	Old
z	11.6	13.5	10.6	11.9	9.8	11.0	7.2	8.8	8.4	10.0
850 u	0.6	3.0	0.3	2.6	0.7	3.2	0.3	2.9	0.5	2.7
v	0.6	3.2	0.3	2.9	0.6	2.7	0.4	2.7	0.7	4.0
z	13.8	14.4	12.4	13.2	10.9	11.6	13.0	13.8	14.8	15.6
700 u	0.6	3.0	0.4	2.9	0.6	2.4	0.6	3.6	0.3	3.4
v	0.7	3.4	0.3	2.6	0.8	3.1	0.7	2.8	0.3	2.5
z	15.2	16.3	15.0	16.2	14.3	15.6	15.6	16.8	16.5	17.6
500 u	0.5	3.5	0.6	3.2	0.3	3.2	0.4	3.8	0.2	3.8
v	0.3	3.0	0.5	3.5	0.4	2.8	0.3	2.6	0.2	2.6
z	26.2	28.0	20.9	22.2	21.6	23.6	21.2	22.3	15.9	18.4
300 u	0.4	3.3	0.5	3.8	0.4	4.5	0.5	4.5	0.3	3.6
v	0.4	5.0	0.7	3.4	0.6	3.4	0.5	4.4	0.2	3.6
z	22.2	24.6	26.6	28.8	18.5	19.7	25.3	27.1	24.1	26.4
200 u	0.5	4.7	0.6	4.7	0.4	3.8	0.5	5.7	0.7	4.4
v	0.6	4.5	0.5	4.4	0.4	3.2	0.4	4.5	0.3	3.9

Table 2—Root mean square error statistics of height (m) and wind (m/s) fields for new (TOAR) and old (Gaussian) schemes

Error statistics on

Levels (hPa)	26 July 91		27 July 91		28 July 91		29 July 91		30 July 91	
	New	Old	New	Old	New	Old	New	Old	New	Old
z	10.2	11.0	7.1	11.8	11.6	12.5	16.0	17.5	10.1	10.8
850 u	0.9	3.1	1.2	4.9	0.6	3.1	0.6	3.8	0.7	4.2
v	0.7	2.3	0.6	2.7	0.8	7.1	0.6	4.1	1.1	6.7
700 z	10.7	11.2	11.3	12.0	12.4	13.5	14.1	15.1	10.2	10.7
u	1.4	2.9	1.0	3.4	1.0	3.4	1.0	3.6	1.3	3.3
v	1.3	2.6	1.1	2.8	1.1	3.1	1.1	3.1	1.2	3.5
200 z	21.0	22.8	28.0	29.3	25.7	28.2	24.9	26.3	25.2	25.8
u	0.9	4.1	1.5	4.5	1.1	5.8	1.6	5.4	1.0	6.2
v	1.2	3.8	1.1	3.5	0.9	4.0	1.2	4.6	0.9	5.5

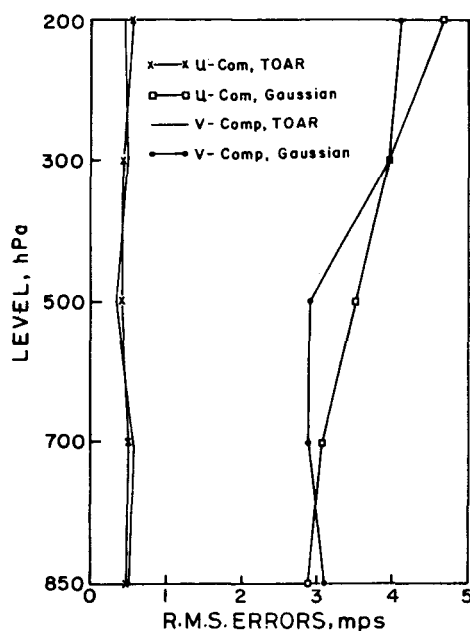


Fig. 6—Vertical profiles of rms differences of (O-A) for different components of wind (m/s) for 1979 situation

the situations. In order to examine how the analyzed fields fit the observations, we computed observed minus analysis, (O-A), root mean square (rms) statistics. Tables 1 and 2 show the (O-A) rms differences for height and wind fields for both the situations.

Figs 6 and 7 bring out the (O-A) rms differences for wind and height fields, respectively, for both the schemes (TOAR and Gaussian functions) for the 5-day period, 4-8 July 1979. This rms differences indicate a closer fit of the analyses to the observations. On comparing the new analyses

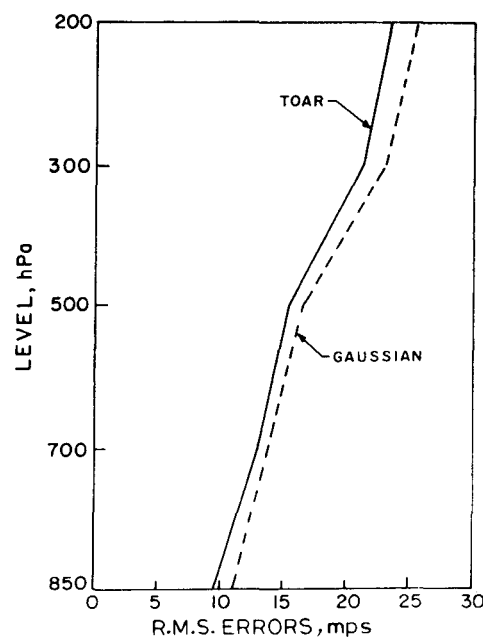


Fig. 7—Same as Fig. 6 but for height field (m)

(TOAR) with the analyses made using Gaussian correlation function, we found (Table 3) specifically that for the wind field, (O-A) rms differences drop from values nearly 3.61 m/s to 0.47 m/s for *u*-component and from 3.39 m/s to 0.47 m/s for *v*-component for the 1979 situation. For 1991 case, values for *u*-component dropped from 4.1 m/s to 1.05 m/s and from 3.96 m/s to 0.95 m/s for *v*-component. In the case of height fields also there have been reduction in rms errors on all the days and at all the levels. This suggests that analyses with TOAR function are more closer to the observations than the corresponding analyses with Gaussian function. Figure 8 shows the differences

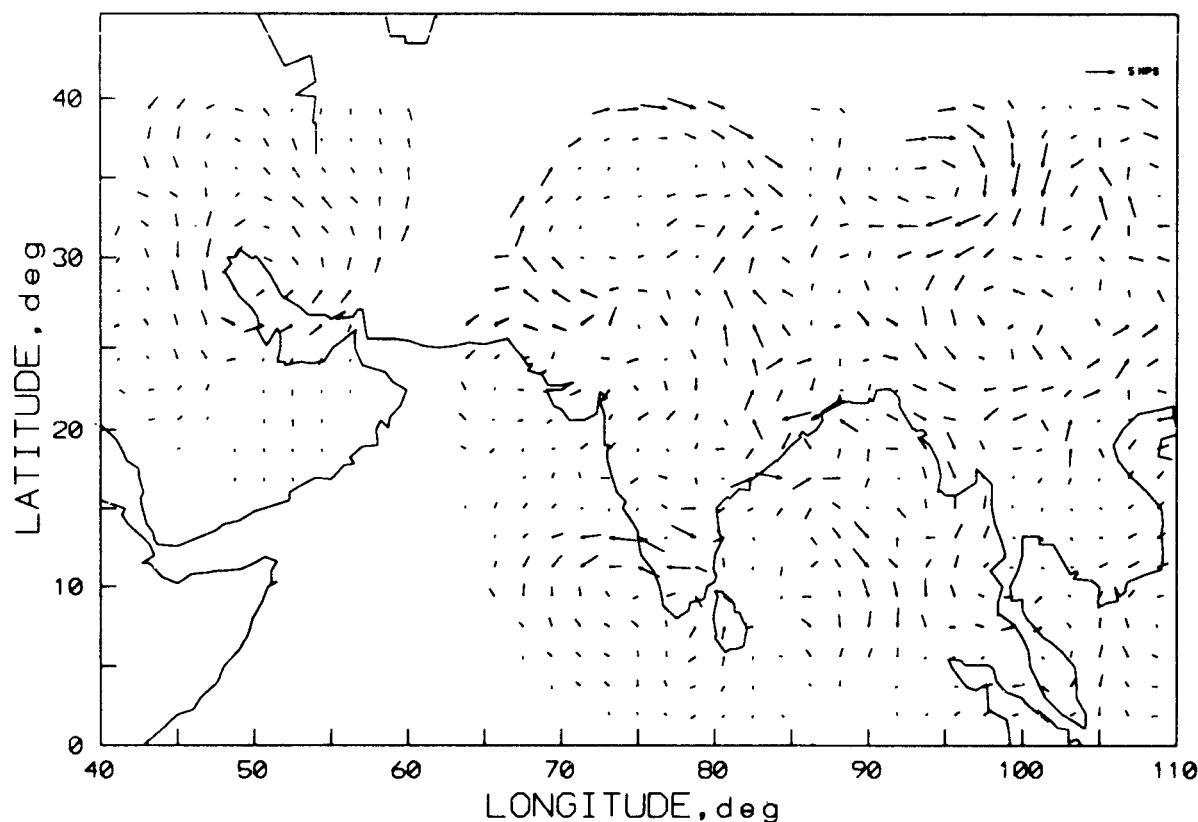


Fig. 8—Wind differences between TOAR scheme and Gaussian scheme at 700 hPa on 7 July 1979

Table 3—Average rms errors in height (m) and wind (m/s)

		Errors in		
		<i>z</i>	<i>u</i>	<i>v</i>
1979	New	16.46	0.47	0.47
	Old	1989	3.62	3.39
1991	New	15.90	1.05	0.95
	Old	17.23	4.11	3.96

between the two types of wind analyses (TOAR scheme-Gaussian scheme) for 7 July 1979 at 700 hPa level.

4 Conclusions

In earlier study Sinha *et al.*⁶ used Gaussian correlation function to fit the height-height correlations. In the present case the Gaussian function has been replaced by a degenerated TOAR function and the additive constant has been replaced by a second, broader TOAR function.

The TOAR function with the constants obtained for India and neighbourhood fits the observed height-height correlations better than the usual Gaussian function. Partly because of this, the rms errors also decreased suggesting that the synoptic situations are better represented now.

Acknowledgements

The authors would like to thank Prof. R N Keshavamurty, Director, Indian Institute of Tropical Meteorology, Pune, for his interest in this study and providing suggestions for improvement. Sincere thanks are due to Dr S S Singh, Head, FRD, for his encouragement. Authors are also thankful to the anonymous reviewers for their insightful remarks, which contributed to a clear and concise paper.

References

- 1 Gandin L S, *In Objective Analysis of Meteorological Fields*, Gidrometeoizdat, Leningrad, Israel Programme for Scientific Translation, 1963, p 242.
- 2 Thiebaux H J, Mitchell H L & Shantz D W, *Mon Weather Rev(USA)*, 114 (1986) 1048.
- 3 Hollingsworth A & Lonnberg P, *Tellus (Sweden)*, 38 (1986) 111.
- 4 Lonnberg P & Hollingsworth A, *Tellus (Sweden)*, 38 (1986) 137.
- 5 Daley R, *Mon Weather Rev(USA)*, 113 (1985) 1066.
- 6 Sinha S K, Narkhedkar S G, Talwalkar D R & Rajamani S, *Adv Atmos Sci (China)*, 9 (1992) 431.
- 7 Julian P R & Thiebaux H J, *Mon Weather Rev(USA)*, 103 (1975) 605.
- 8 Mitchell H L, Charette C, Chouinard C & Brasnett B, *Mon Weather Rev(USA)*, 118 (1990) 1591.