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On the fractal characterisation of atmospheric velocity field

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The fractal technique, as developed and applied by Grassberger [*Nature (UK)*, 323 (1986) 609] for atmospheric variables, has been used for interpretation of velocity field data near ground during the early monsoon period. It has generally been observed that while both the horizontal components of wind possess low dimensional strange attractors, in the case of the vertical velocity field there is practically no evidence of strange attractor. An explanation of this anomalous behaviour of the vertical velocity field has been provided.

1 Introduction

A dissipative dynamical system which exhibits chaotic behaviour often possesses an attractor in phase space which is strange. Strange attractors are typically characterized by fractal dimension D which is smaller than the number of degrees of freedom. Existence of strange attractors has been observed by Lorenz[1] in finite amplitude convection, by Grassberger[2] in world-wide climatic data, by Tsonis and Elsner[3] in wind data, and by Henderson and Wells[4] in atmospheric pressure data. The subject of deterministic chaos and strange attractors has been excellently reviewed by Hao Bai-Lin[5] and many others.

In the present paper we have examined the tower data of Banaras Hindu University (BHU) and of IIT, Kharagpur, in the rainy season for the evidence of deterministic chaos in the near-ground wind velocity field during the daytime. Three days' BHU data and one day's IIT data have been examined. It has also been investigated if there exists any strange attractor and if so what its dimension is. All the three wind velocities, namely, zonal, meridional and vertical have been separately processed with this objective. Side by side means and standard deviations of the velocity fields and the coefficients of correlation between horizontal components and vertical component of velocity fields have also been calculated with a view to investigating their interdependence or otherwise.

Knowledge of strange attractors enables one to characterize a dynamical system. It is theoretically possible to describe a system, possessing low dimensional strange attractor, with the help of a simple model of fewer degrees of freedom and pave the path of short-term forecasting.

2 Methodology

The method applied in this study is based on a recent development in the analysis of a dynamical system. A dynamical system may be broadly a fully turbulent one, expressible only on statistical level. It may be of random nature with some kind of periodicity. The third possibility is that the dynamical system may be a deterministic one, expressible as the solution of a differential equation. A system in the last category may undergo such development in time that in the long run the solution space asymptotically converges to a fixed space. This fixed space is known as an attractor. If the dimension of an attractor is a fraction and not an integer, it is called a strange attractor.

The concept of fractal dimension was introduced by extending the generalized definition of common dimension given by Hausdorff (vide Mandelbrot[6]). In calculating the dimension of a cube, Hausdorff divided the cube into smaller cubes of sides $1/\varepsilon$ th of the original value. Thus the number of small cubes is given as

$$N = \left(\frac{1}{\varepsilon}\right)^3 \quad \ldots(1)$$

Hence, we get

$$\frac{\log N}{\log\left(\frac{1}{\varepsilon}\right)} = 3 \quad \ldots(2)$$

If this idea is extended to the case of D-dimensional hypercube, we have a similar relation as

$$\frac{\log N(\varepsilon)}{\log\left(\frac{1}{\varepsilon}\right)} = D \quad \ldots(3)$$

In normal situation D is an integer. But if we extend the idea to the case when the bigger cube has a lot of empty space and actual number of smaller cubes needed to fill the bigger cube is less than $N(\varepsilon)$, D becomes a fraction less than the integral dimension of the embedding space. This concept, much exploited by Mandelbrot[6], has been used to analyse chaotic dynamical system with fractal technique.

It is conjectured that if the strange attractor (SA) is so convoluted that it almost fills the volume of the embedding space, the dimension of SA is very near to the dimension of the embedding space. Conversely, nearer the dimension of SA to the dimension of the embedding space, more is the convolution of SA and more complex is the dynamical system. Hence the dimension of SA, if any, may be used as a tool to ascertain the complexity of a dynamical system. The relative complexitiy vis-a-vis intensity of randomness of two dynamical systems may be studied if the dimensions of their strange attractors are known.

We shall now give in brief how the dimension of SA can be calculated from data. Let us take the time series of a sample of size N, representing a dynamical system. In this time series let x be the fluctuations of instantaneous values from the ensemble mean. We now construct vectors $x(t+\tau)$, ..., $x[t+(p-1)\tau]$ with lag time τ. According to Schuster[7], the best choice of τ is that for which correlation coefficient is about 0.5. In our case, we have selected the minimum value of τ for which the value of correlation coefficient falls in the range 0.4-0.6. Sets of vectors are constructed for different values of p ranging from 2 to 12. We thus have vector spaces of dimensions 2 and onwards, each vector representing a point in the respective space. We may observe that the number of vectors in the p-vector space are $N-(p-1)\tau$.

Having constructed the p-vector spaces we may now calculate the Euclidean distances between all pairs of points in each space. Then these distances are categorized into classes. The cumulative frequencies $C(r)$, determined by dividing cumulative population by square of N, the total population, are then plotted against class (r) in log-log plot. Thus log $C(r)$-log (r)curve is obtained for each value of p.

It is observed that some portion of log (r)-log (r) is straight with slope d. For each p there is different value of d. d is then plotted against p. In case an SA exists this curve reaches a plateau at a value D, the maximum value of d. D is then the dimension of SA of the system.

3 Case studied

Monsoon in the eastern part of India extending up to the southern part of Uttar Pradesh including Varanasi generally starts in the second week of June. We have taken three sets (7, 8 and 13 June 1990) of wind velocities from the tower data of Banaras Hindu University and one set (10 June 1988) from that of IIT, Kharagpur. These data were realized at 8 m from ground level by a sonic anemometer. Each set comprises data pertaining to five different times of each day, namely, 0546, 0833, 1130, 1430 and 2130 hrs. Every time 840 zonal, 840 meridional and 840 vertical velocities have been realized, each spanning over 1½ min. The realized sonic data are fast data which would enable us to monitor rapid fluctuations of velocity at 8 m level. These data have been used to analyse the turbulent characteristics of atmospheric velocity fields with the help of fractal dimension.

Since the four days under study fall in the early monsoon period, it is expected that rain is occasional but not rare and ground is generally wet. As a result both the updraft and the downdraft velocities are likely to be weak. Simultaneously, vertical temperature gradient may also be small.

Another aspect of the early monsoon is that the direction of the horizontal wind may be consistent over the whole of a particular day but may vary from one day to the other.

The four sets of sonic data are analysed by the fractal algorithm, discussed earlier. The interpretation and analysis of the results are given in the subsequent sections.

4 Results

Mean zonal velocities $\overline{U}$ and mean meridional velocities $\overline{V}$ at 0546, 0833, 1130, 1430 and 2130 hrs of each day have been calculated. The BHU data correspond to inversion, development of thermal convection and peak convection periods at Varanasi, as observed by Tripathi *et al.*[8] Mean zonal velocities $\overline{U}$ and mean meridional velocities $\overline{V}$ of the IIT data have also been calculated. It has been observed that the IIT data are of the same type as the BHU data. Standard deviations have also been calculated for all the sets of data. It has been found that standard deviations at a particular time of day, normalized by corresponding means, remain almost same on all the four days. Those are different at different periods of day, from which we would like to assume that there is some similarity in the shapes of distribution curves at a fixed time of a day on all the four days. Thus detailed diurnal analysis of wind field covering *inter alia* the development of mixing layer in the early monsoon has been investigated for all the four days. For reasons of simplicity we would assign subscripts 1, 2, 3 and 4 respectively to the values pertaining to the 7, 8, 13 June experiments at BHU and the 10 June experiment at IIT, Kharagpur.

It has been observed that the mean directions of wind flow $\overline{U}_1$, $\overline{U}_2$ and $\overline{U}_4$ are E-W, whereas $\overline{V}_1$, $\overline{V}_2$ and $\overline{V}_4$ are N-S. The directions remain unchanged almost all through these three days. On the other hand on 13 June these directions have been reversed, i.e. $\overline{U}_3$ is W-E and $\overline{V}_3$ is S-N, which again persist for the whole day. On all the four days, however, there is no appreciable change in the wind speed. The abrupt change in the direction of mean wind on the 13th is in conformity with the development of climate in the monsoon, discussed earlier. Vertical velocities $\overline{W}$'s have been found to be significantly small on all the four days. Since almost similar results have been reported at BHU and IIT with two different sets of instruments over wide range of time, we would like to assume that the experimental error is insignificant.

Step by step results up to the determination of fractal dimensions of the strange attractors have been presented only for the zontal velocity at 0546 and 1130 hrs of 7 June in Fig. 1(a) and (b). It may

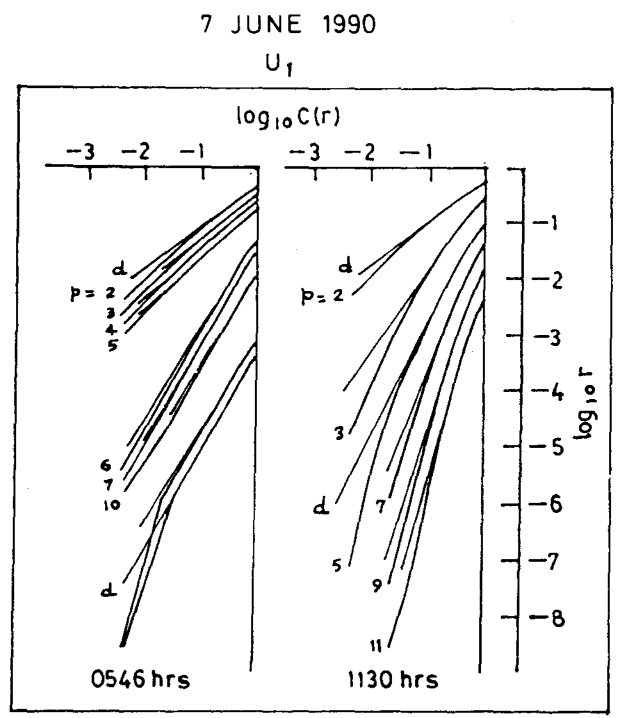

Fig. 1(a)—Log $C(r)$ versus log r for zonal velocity of 7 June 1990 for 0546 and 1130 hrs. Straight lines are drawn to measure the slopes of the straight portions of the curves.

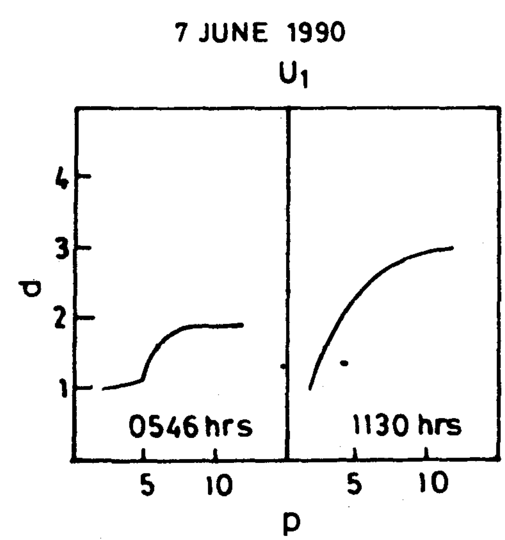

Fig. 1(b)—p versus d for zonal velocity of 7 June 1990 for 0546 and 1130 hrs.

be noted in this context that the values of τ varied from 4 to 7. In each case we have started with a set of 840 velocity data and built the corresponding vector space. We get the smallest vector space wih 763 vectors when p = 12 and τ = 7. This even is not a small number for our purpose.

Fractal analysis gives some more statistical detail of the weather of these four days. Figure 1(a) Shows the relation between log C (r) and log r for p=2, 3, 4, ... 12 and also the slopes d of the straight portions of each graph. In Fig. 1(b) d has been plotted against p. It is seen in each of these cases that d reaches a maximum value D, which is the fractal dimension of the strange attractor of the respective case. While calculating D's, it has been observed that in some cases d is monotonically increasing and a maximum value of D is unlikely to realise. It may be concluded that there is no strange attractor in the aforesaid cases.

Figure 2(a)-(d) exhibits D's of U, V and W respectively for 7, 8 and 13 June 1990 of BHU and 10 June 1988 of IIT. D's of U and V are almost the same at 1130 hrs, indicating that the randomness of these two velocities are statistically alike at the noon time even if their means are different.

Evidence of low dimension strange attractors of the horizontal velocity fields is there almost everyday, whereas the vertical velocity field is quite different. On 7 and 8 June vertical velocity field generally does not exhibit presence of strange attractor. On 13 June the field is turbulent almost all through the day as indicated by almost complete absence of strange attractor. In the IIT experiment on 10 June we observe almost the repetition of the same phenomenon.

5 Conclusions

We have observed from the above results that while the horizontal velocity components have shown evidence of strange attractors in all the

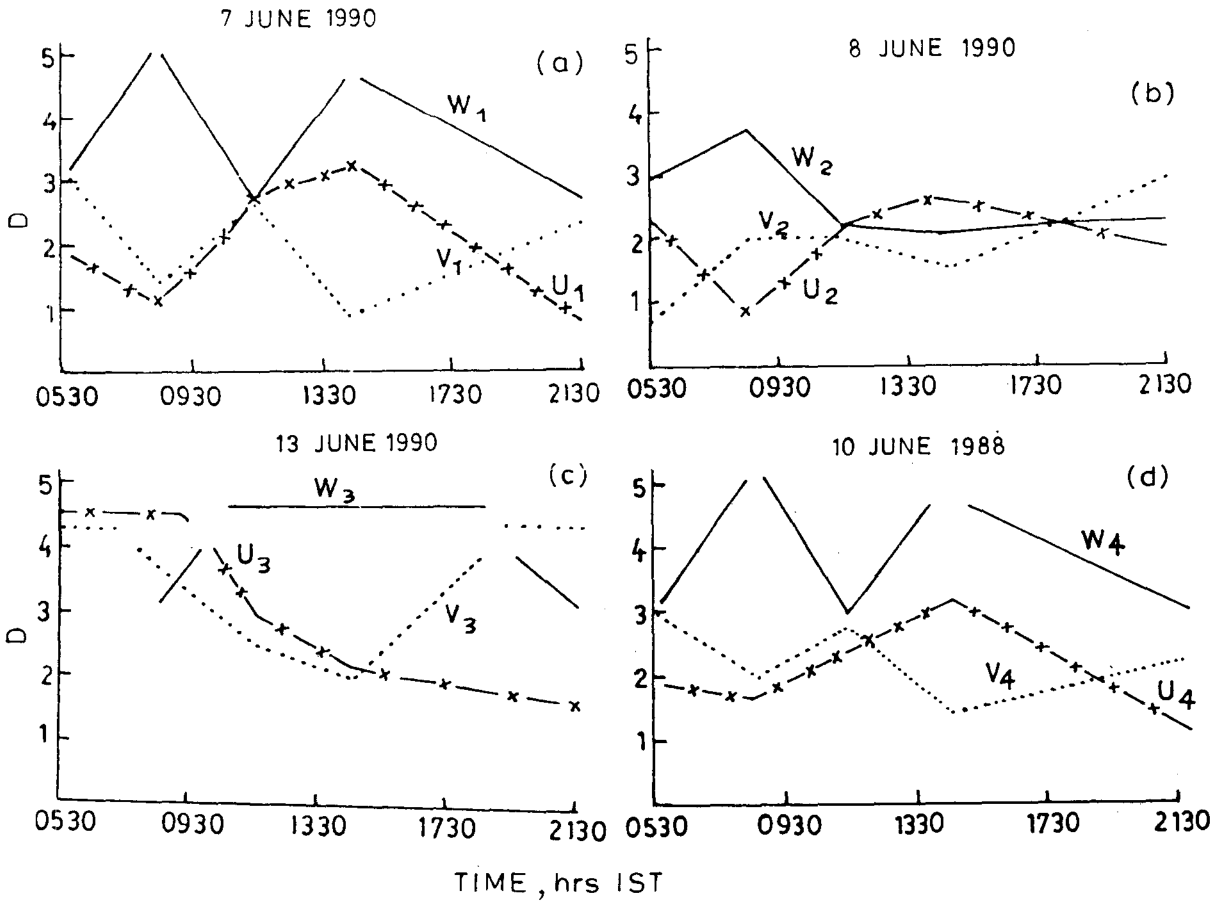

Fig. 2—Fractal dimensions of the three velocity fields calculated for the five periods 0546, 0833, 1130, 1430 and 2130 hrs for all the four days. Lines connecting the values of the consecutive periods have been drawn not with the purpose of showing continuity of fractal values, but with a view to identifying the absence of strange attractor at a particular point of time by breaking the lines [(a) 7 June 1990, (b) 8 June 1990, (c) 13 June 1990, and (d) 10 June 1988; x–x–x, zonal wind; ......, meridional wind; and —, vertical wind].

cases, the vertical velocity field rarely possesses a low dimensional strange attractor. In most of the cases the vertical velocity field does not possess a strange attractor at all.

This finding is very much striking. Being parts of the same dynamical system, different components of the velocity field exhibit different properties as far as existence of low dimensional strange attractor is concerned. In order to understand the underlying cause of this striking feature of the dynamical system, we have examined the dependence of the vertical component on the horizontal components by calculating correlation coefficients R_{UW} and R_{VW} given as

$$R_{UW} = \frac{\overline{UW}}{\sigma_U \sigma_W} \quad \ldots(4)$$

and

$$R_{VW} = \frac{\overline{VW}}{\sigma_V \sigma_W} \quad \ldots(5)$$

Here overbars indicate ensemble average and σ_U, σ_V and σ_W respectively stand for standard deviations of *U*, *V* and *W*.

It has been observed that while $|R_{UW}|<.3$ and $|R_{VW}|<.28$, $|R_{UV}|>.74$. From this statistical analysis we find that the vertical velocity field is poorly correlated with the horizontal velocity field. Or in other words we may say that convection in the early monsoon, which is generally weak in nature, is negligibly affected by the horizontal wind and vice versa. Accordingly, we observe two strikingly different dynamical features in the same dynamical system, indicating that the system which we think to be a single dynamical system may in reality be a superposition of two non-interacting systems, one of the horizontal velocity field and the other of the vertical velocity field.

We have carried out the experiment in two places, one in the coastal region, i.e. Kharagpur, and the other in a mid-continental location, Varanasi, both of course being affected by the same monsoon. In both the cases we have obtained almost similar results.

From the above observations it may be stated that with the help of the fractal analysis of the velocity fields of these four monsoon days at climatically two different places it has been possible to have an insight into the dynamics of the weather in the given early period of monsoon. The existence of low dimensional strange attractor has been observed in the case of zonal and meridional winds. This may offer opportunity for predicting short-term horizontal wind profile during the monsoon. The absence of any strange attractor for the vertical velocity field in this experiment, however, highlights the unpredictable nature of turbulent convection in the boundary layer in the early monsoon due to occasional rainfall followed by bright sun.

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