

Along track dynamical topography by Geosat, ERS-1 and Topex/Poseidon altimeters with a new geoid model

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A new gravitational equipotential (geoid) model estimates that has been developed up to spherical harmonic expansion to 360 degree have been described. These model coefficients help in defining improved representations of the earth's gravitational potential beyond that available from just satellite or terrestrial data and are the current state of art potential coefficients. The old and new geoid model estimates are compared and assessed with the along track sea surface height measurement by Geosat, ERS-1 and Topex/Poseidon (T/P) altimeters. Dynamic topography with reference to old and new geoid are also estimated and analysed. Assessment of all the altimeters is aimed for futuristic goal of estimating the climatic dynamic topographic signal over the Indian Ocean with a single standard geoid reference datum.

Worldwide increasing interest in the fluctuations of the ocean circulation and data resources results to monitor the dynamic topography regularly. Wyrski¹ mapped and investigated the mean annual and random variability caused due to fluctuations in the position of currents, using 66,600 hydrographic data points from different stations. The satellite determination of dynamic topography, needs more accurate description of the Earth's gravitational field. Before the development of the satellite technique the observation technique was restricted to terrestrial gravimetry. Satellite tracking technique opened the potential of geopotential modeling by addition of new types of efficient data collection techniques with global coverage capabilities. Satellite altimetric missions with repeat ground tracks play a significant role in the study of mesoscale variabilities of the ocean surface with the knowledge of geopotential (geoid) models. Both Seasat and Geosat were planned in orbits that repeated approximately in 17 days. The T/P has 10 days repeat cycle. For a number of studies^{2,3} "collinear" passes are examined to assess

ocean variability or systematic changes in sea level. The present study describes the data base from all the three altimetric systems and their correction procedures. It follows with the algorithm for geoid model estimates based on the coefficients of 360 degree and order⁴ and finally the analysis of the dynamic topography based on the present geoid estimate versus old geoid model estimates which are supplied in the Geophysical Data Records (GDRs) of the respective altimeters have been carried out. Thus, with the improvement in qualities and distribution of satellite tracking data, as well as increase in coverage, gradual refinements in resolution of geoid models and improvement in accuracy of surface data the geopotential model showed significant progress⁵⁻⁷.

Materials and Methods

The ERS-1 and T/P measurements have the benefit of having a measured wet tropospheric correction from the onboard Along Track Scanning Radiometer (ATSR-M) and three channel microwave radiometer respectively. The

Geophysical Data Records of Geosat which did not have any microwave radiometer on board, supplied the wet tropospheric range correction based on the Fleet Numeric Oceanographic Center (FNOC) model derived humidity field which is relatively a big source of error. Geophysical corrections such as the solid Earth tides and the ocean tides⁸, were applied to the altimeter measurements and the final corrected product is used in the analysis. All the altimeter data tracks were selected in such a way that data flagged as over the coasts or otherwise suspect (pointing error etc.) are not used. Hence there was no need either to edit the data by any of the criteria. Also no smoothing and filtering was done as mostly spike free tracks were taken for analyses. Mostly all the altimeter data are selected in the months of April and May for consistency and inter comparison of their performance. The system specifications of all the three altimeters are given in Table 1.

The Geosat data used is based on the GDR's provided by US National Oceanic and Atmospheric Administration. NASA Goddard Space Flight Center's (GSFC) GEM-T2 model is used to compute Geosat dynamic topography. The ERS-1 satellite launched in July 1991 into an orbit of inclination 98° by the European Space Agency (ESA), was first placed on 3-day repeat orbit. Further the Topex/Poseidon carries two altimeters, the NASA radar altimeter (NRA) and CNES solid-state experimental altimeter (SSALT) with 10 days repeat cycle. Both instruments share the same antenna to the extent that only one can be operational at any one time. The merged geophysical data records from T/P altimeters were provided by the Jet Propulsion Laboratory - Physical Oceanography Distributed Active Archive Center (PO.DAAC), on CD-ROM. They were corrected for transmission and ocean surface variations as in case of ERS-1 and Geosat altimeters. The improving accuracy of gravity field models has offered to compute the along track dynamic topography with more accuracy. The primary purpose of satellite altimetry is to measure the sea surface elevation resulting from the dynamic ocean currents.

Following the general formulations⁹, the gravitational potential V follows the following

Table 1—System specifications of Geosat, ERS-1 and T/P altimeters.

	Geosat	ERS-1	T/P
Configuration	Pulse limited	Pulse Limited	Pulse Limited
Frequency	13.5 GHz	13.5 GHz	13.6 and 5.3 GHz
Altitude	800 km	800 km	1336 km
Orbit inclination	103°	98°	66.04°
Repeat period	17 days	3,35,185 days	9.9156 days
Bandwidth	320 MHz	400 MHz	320 MHz
Pulse width	102.4 μ s	20 μ s	100 μ s
Range precision	$\approx$ 5 cm	5-10 cm	$\approx$ 3 cm

Laplace's equation:

$$r^2 \partial^2 V / \partial r^2 + 2r \partial V / \partial r + \partial^2 V / \partial \theta^2 + \cot \theta \partial V / \partial \theta + 1 / \sin^2 \theta \partial^2 V / \partial \lambda^2 = 0 \quad \dots (1)$$

and its general solutions are obtained by separating the variables r, θ, λ by means of the trial substitution

$$V(r, \theta, \lambda) = f(r) Y(\theta, \lambda) \quad \dots (2)$$

where r is the geocentric radius; θ is the geocentric latitude, λ is the polar distance, and f is a function of r only and Y is a function of θ and λ only. The general form of the gravitational potential, V , in spherical harmonics then can be obtained as:

$$V = kM / r \left[1 + \sum_{n=2}^{\infty} (a / r)^n \sum_{m=0}^n (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) P_{nm}(\sin \theta) \right] \quad \dots (3)$$

where kM is the geocentric gravitational constant, r is the geocentric longitude. C_{nm} , S_{nm} are the fully normalized potential coefficients and a is the scaling factor associated with the coefficients.

The disturbing potential, T , is the difference between the actual potential (V) at a point and the "normal" potential at the corresponding point. For our purpose the normal potential will be that associated with an equipotential reference ellipsoid of defined parameters. The geoid undulation height thus can be obtained from

$$N(r, \theta, \lambda) = T(r, \theta, \lambda) / t(r, \theta) \quad \dots (4)$$

where t is the normal gravity.

To implement the equations discussed above a number of additional quantities are needed. Given four parameters defining an equipotential reference ellipsoid all the even degree zonal harmonics are explicitly defined¹⁰.

Results and Discussion

Based on the above algorithm, the geoid undulations are estimated. The 2-D geoid height variations in the Indian oceanic region within the domain, 30°S to 25°N and 40°E to 100°E are shown in Fig.1. Map reveals information with wavelengths greater than 100 km. The long wavelength features are also present in the geoid. Prominent Indian Ocean geoid low (-110m) is marked with densely encircled contour lines. Short and intermediate wavelength features are also expressed. Somali Basin low connecting the Arabian Sea basin low are visible. Areas which are devoid of *in-situ* gravity measurements reflect poor resolution capabilities of the geoid undulations world-over.

The relative difference of the present geoid model and those supplied in GDRs of Geosat and ERS-1 are studied. As is well known that satellite altimeter makes it possible to measure directly the elevation of the satellite over the ocean surface, when combined with precise orbit tracking. The sea surface is not a constant gravitational potential; non gravitational forces keep causing the slight deviations between the geoid and the sea surface. Dynamic topographic accuracy depends on the accuracy of the geoid height estimates.

For the comparison of present geoid model with those provided in the GDRs of the GEOSAT and ERS-1 altimeters, many of the profiles have been studied. Figure 2 a1 shows the GEOSAT track profile of the sea surface height and the geoid heights based on GEM-T1 model and the present model. The proximity of the new geoid height with the sea surface height compared to that of GEM-T1 is presented. Most of the bottom topographic features are delineated by the present model along the track when comparing the sea surface height. The accuracy of geopotential at varying sea surface height locations clearly seems to have been improved compared to the earlier representation by GEM-T1 model as a basis.

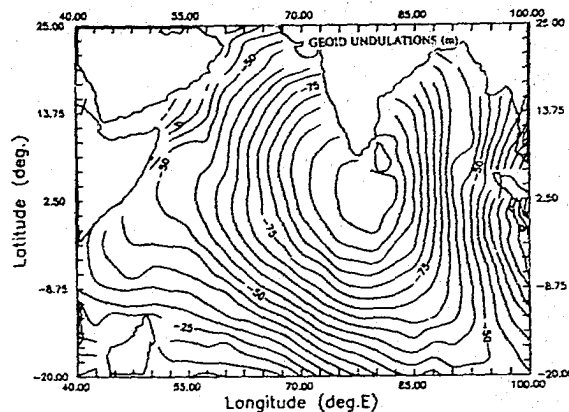


Fig. 1—Geoid undulations in the Indian Ocean.

The difference between the corrected sea surface height and the geoid height yields the dynamic topography. The dynamic topographies with respect to both geoids are presented in Fig. 2 b1. It shows that except at some positions in the central track points, the old and new geoid model estimates does not coincide. The dynamic topographic magnitudes at most of the distant locations are different. The separation of these estimates is quite large in the same track. As the number of observations are more for generating higher degree and order of spherical harmonic representation of geoid, it improve significantly the geopotential representation of the ocean floor. This is one of the main objectives of the present study, i.e. to obtain accurate along track dynamic height profiles for interpretation of mesoscale to synoptic scale oceanic features. At some places the geoid of lower and higher resolutions are in perfect match in the same altimetric profile. With two such dynamic topographies inferred from a common along track satellite observations does make it clear about the importance of a suitable reference surface that needs to be known before processing data for dynamics of the oceans.

Another such example from same Geosat altimetric profile is shown in Fig. 2 a2 to strengthen the argument in the above section. This shows the increasing difference in two geoid estimates with successive track observation points. The difference is attributable again to the accuracy of higher resolution (degree and order) geoid model compared to lower one to capture the

geopotential signatures of the respective areas of observations of oceans. The peak at about 300 km observation has been depicted much better by new geoid than earlier one which is supplied in the Geosat GDR. From 180 to 320 km distance observation points the highs and lows of the sea level are properly captured by new geoid. The respective dynamic topographic estimates in Fig. 2 b2 shows that the older geoid model supplied with Geosat over estimates the dynamic topography compared to new geoid as a reference. The slope of the sea level with reference to new geoid model does represent the dynamic features with more realistic wavelength components of ocean circulation features.

The ground track profile of ERS-1 sea surface height and the old and new model based geoid heights are shown in Figure 3 a1. Here again the new geoid represents much more true replica of

ocean bottom features than old geoid supplied in ERS-1 GDRs. The old geoid height variations supplied in ERS-1 GDRs seems to over and under estimate drastically the dynamic heights at two or three places, within a very small segment. It does not seem to represent the true dynamics of the ocean surface. Figure 3 b1 shows the dynamic topography based on the reference geoid supplied in GDR (marked 1) and new geoid estimates (marked 2) in Fig. 3(a1). The locations corresponding to 300 to 400 km point observations makes it further clear, how much to rely on new geoid estimates for dynamic topographic estimates and hence its interpretations.

One more example for ERS-1 altimetric profile of sea surface height and geoid estimates based on new and old models is shown in Fig. 3 a2. The marked improvement is evident from this figure itself and thus the Fig. 3 b2 also shows the

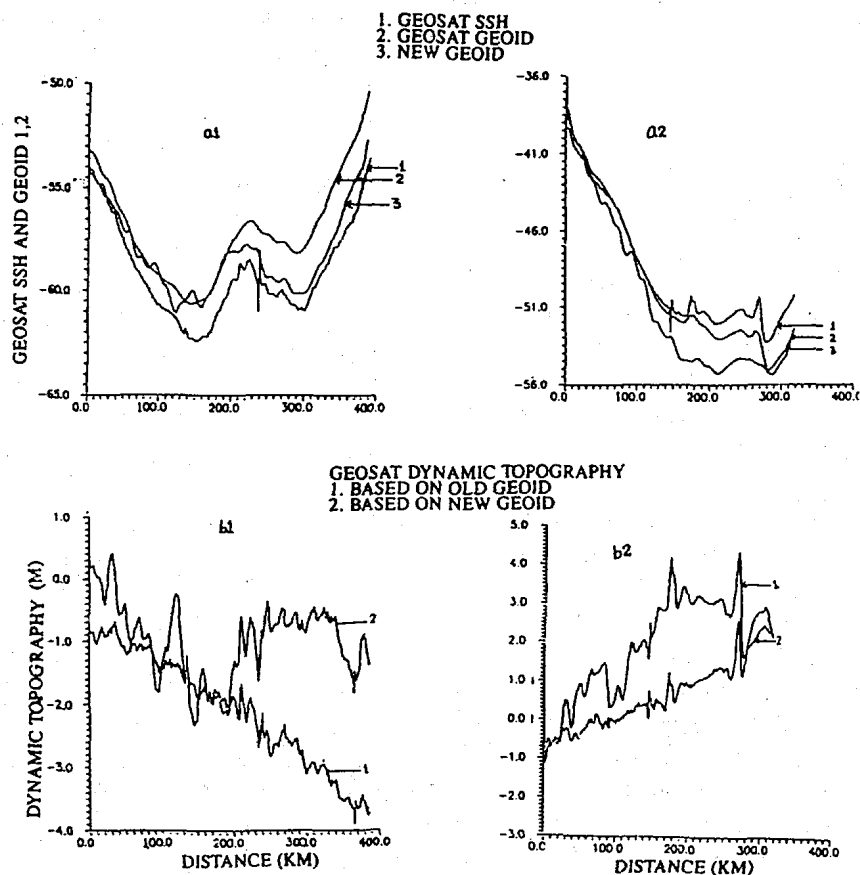


Fig. 2—The Geosat observed SSH and geoid a1,a2 undulations due to old and present geoid and respective dynamic topography b1,b2 estimated for two different collinear tracks.

respective dynamic heights. To select dynamic topography based on new geoid model estimates (marked 2 in Fig. 3 b2) thus has the strong theoretical basis same as in all other above cases. With new geoid model this accuracy of dynamic topography seems to have been improved very significantly as seen in Fig. 3 b1, b2 and can be used to estimate the along track instant dynamic features based on the relative elevations of the sea surface.

A final example is presented here using T/P altimetric profile. The dynamic topography thus obtained is shown in Fig. 4 a. The T/P measures the sea surface height up to an unprecedented

accuracy of 3 cm. Thus the dynamic topography has the dynamic range of ± 50 cm as seen in the Fig. 4a and even some times it may be difficult to delineate the signal from the noise. A solid line showing the polynomial fitting to the dynamic topography is also shown. As the exact repeat mission of T/P is of about 10 days, the same polynomials excluding the topographic variations is shown in Fig. 4 b, for successive collinear tracks. The changes in dynamic topography along the track in successive 10 days is marked as 1 to 4. The mesoscale features are thus detectable from these profiles. The process of detection of such ocean surface and bottom topographic features as

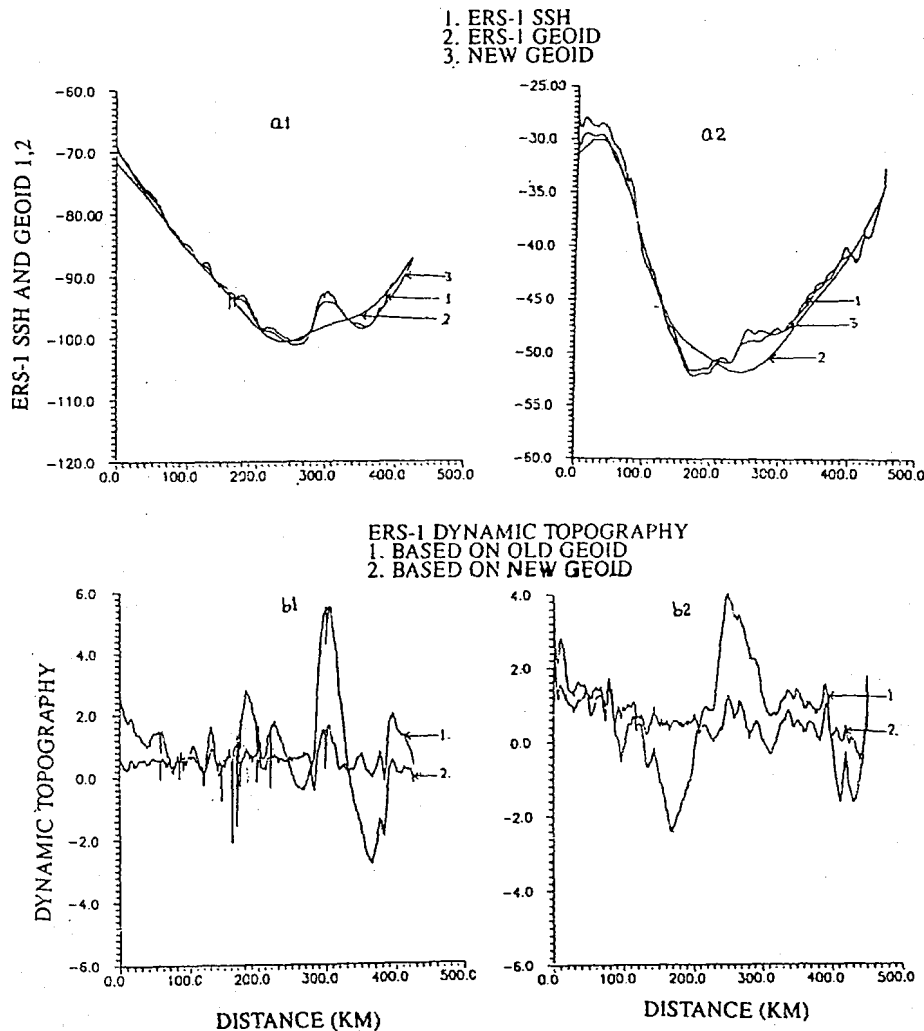


Fig. 3—The ERS-1 observed sea surface height and geoid undulations a1, a2 due to old and present geoid and respective dynamic topography b1, b2 estimated for two different collinear tracks.

eddies and seamounts respectively are based on the matched filtering approach. The source of data was same for both except that the signal for one acts as a noise for the other and vice versa. That is why the accuracy of a geoid is of paramount importance for study of the dynamic topographic processes. The true dynamic topographic height is possible with the best possible geoid height as a reference. The same dynamic portion should be properly subtracted from the sea surface height measurements from the altimeters for looking at the ocean bottom features. This makes the satellite altimetry an exciting tool for oceanographic studies.

Verification of dynamic topographic features

The dynamic topographic features have further been verified using Geosat, ERS-1 and T/P altimetric along track profiles using an advanced signal processing technique called matched filtering. Despite various corrections applied, still because of the presence of errors like orbit error, residual error due to atmospheric and oceanic effects, uncertainty in the knowledge of the marine geoid etc, the data are noisy. The task therefore is to extract the typical sea surface height signature due to mesoscale eddy from this noisy data set. This is accomplished by applying the matched filtering algorithm followed by a threshold detector

(Block diagram in Fig. 5). The approach consists of the steps as described briefly below:

The data series $x(t)$, $t=1, \dots, n$ is treated as an autoregressive sequence of a finite order p in the following manner:

$$x(t) = \sum_{k=1}^p C_k x(t-k) + W_p(t), \quad t=p+1, \dots, n \quad \dots(5)$$

where $W_p(t)$ is zero mean white noise of variance $\sigma^2(p)$ and $C_1, \dots, C_p$ are autoregressive coefficients. The standard autoregressive (AR) problem is next solved by modified covariance approach, wherein one minimizes (for each order $p=0, 1, \dots, p_{\max}$) the averages of the estimated forward and backward prediction error powers

$$\sigma^2(p) = 1/2 (\tau^f + \tau^b) \quad \dots(6)$$

with

$$\tau^f = 1/(n-p_{\max}) [Z(j) - \sum C_k Z(j-k)]^2$$

$$\tau^b = 1/(n-p_{\max}) \sum_{j=1}^{p_{\max}+1} [Z(j) - \sum C_k Z(j-k)]^2$$

The minimization is done with respect to the coefficients (C_k) and the resulting equations are solved by Cholesky decomposition technique as described in detail by Basu *et al.*¹¹. Further the matched filter is designed¹², which is an algorithm, optimum in terms of detecting a given signal in the noisy data. Once the signal and noise PSD are

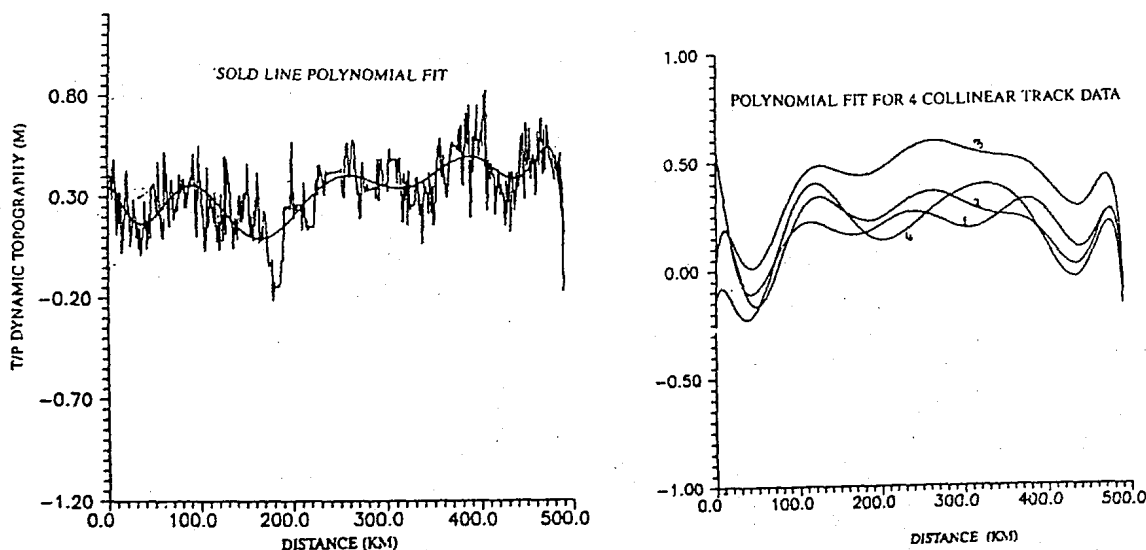


Fig. 4—The T/P estimated dynamic topography for a single (a) and four (b) successive collinear tracks with a polynomial fit.

known the transfer function of the filter can be written as

$$H'(f) = M'(-f)/S(f) \quad \dots (7)$$

where $M'(f)$ is the signal Fourier transform and $S(f)$ is the PSD as estimated above. The role of the signal is played by a generic signature due to a warm core anticyclonic eddy. This signature is modelled as¹³:

$$m(x) = D \exp[-9.21(x/W)^2] \quad \dots (8)$$

where x is the distance along the satellite ground track, D maximum signature height and W , the signature width. Processing the data series $x(t)$ with matched filter amounts to taking the convolution

$$z(t) = \sum_{K=-\infty}^{+\infty} h(k) x(t-k) \quad \dots (9)$$

where $h(k)$ denotes the impulse response of the filter. The output of the filter is conveniently scaled by the rms signal-to-noise ratio (SNR) as

$$Z(t) = \text{scaled output} = z(t)/\text{SNR} \quad \dots (10)$$

Finally a threshold detector is applied. The output $Z(t)$ is compared with the predetermined threshold detector (TH), chosen to produce a specified false alarm rate (FAR).

$$\text{TH} = [2 \ln (F_m/\text{FAR})]^{1/2} \quad \dots (11)$$

where $F_m = (2\pi)^{-1} \cos^{-1}(X/\text{SNR}^2)$ is the rms bandwidth of the filter output. The maximum likelihood estimate of the location of the detected eddy signature is the value of the t for which $Z(t)$ achieves a local maximum above the threshold TH.

The above sketched approach is applied to numerous GEOSAT, ERS-1 and T/P altimetric profiles passing over the Arabian Sea region but only some of the important track results of both the satellite altimeters are discussed here for brevity. The sensor configurations of both the altimeters are discussed in some earlier section.

The data adaptive signal processing approach is applied to some of the Geosat, ERS-1 and T/P altimeter tracks passing over the region of study from 5°S to 15°N and 40°E to 65°E during July to Sept. in 1988 and 1993 respectively. The data consisted of dynamic topography, corrected for various atmospheric and oceanographic effects. Some of the tracks were collinear.

Geosat-altimeter

The first example involves a descending Geosat pass on 17th August from 15.8°N, 53.90°E to 4.87°S, 45.54°E. Figure 6 B1 depicts the dynamic sea surface height (measured relative to marine geoid) along this track. Upper panel A1 of the same figure shows the scaled output from the matched filter. The horizontal dashed line in this and subsequent figures indicates the detection threshold corresponding to a chosen false alarm rate (FAR) of 0.1 alarms/megameter. This value has been used by Brammer¹⁴ for detecting Gulf stream eddies and is found to be suitable here. The output of the filter has been truncated at the ends so as to avoid edge effects arising out of numerical convolution. The number of points to be detected at both the ends vary from case to case and is decided by algorithm itself. The output shows two detections at 6.62°N,

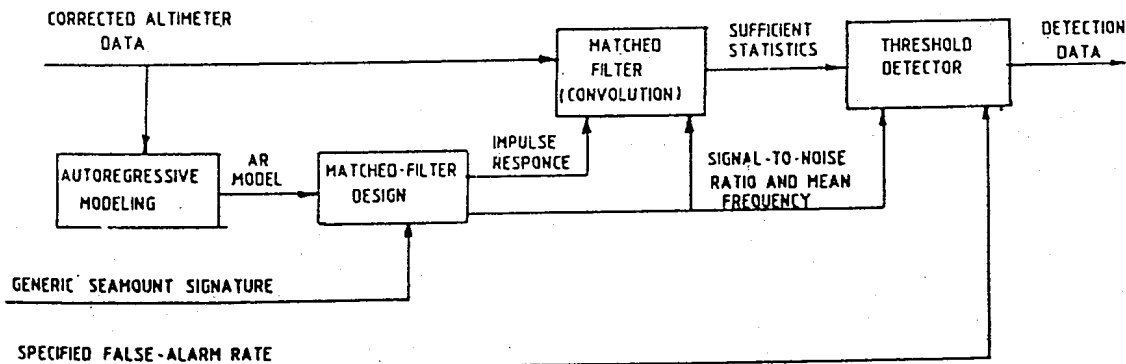


Fig. 5—Structure of data adaptive seamount detection algorithm.

50.12°E and at 3.88°N, 49.10°E. The first one is interpreted as signature of the section of the southern gyre since these locations seem to agree with results of model simulations by Luther & O'Brien¹⁵. Maximum likelyhood estimates of the amplitudes of both the signatures are 1.37 m. Figure 6 A2,B2 depicting a repeat-track on the 3rd September shows that great whirl is again detected at the same location. However the southern gyre signature is no more detected. This may, of course, signify the weakening of southern gyre in September.

ERS-1 - altimeter

On the similar line but with reference to concurrent geoid model estimates, one example concerns the processing of a descending ERS-1 track on 19th August 1993. This is a relatively longer track, ≈ 2000 km in its extent. In Fig. 7 b1,a1 detection of three positions are seen at (14.97°N, 61.70°E), (11.71°N, 60.96°E) and (10.72°N, 60.73°E). The later two positions seems to be the same eddy with larger horizontal dimensions of well known Somali eddy. This location is quite different from that in the previous example of eddy detection by Geosat altimeter

data using the same matched filter. There is, however, no contradiction here since the horizontal extent of the great whirl in Somalia basin is known to be of the order of 400-600 km.

Topex/Poseidon - altimeter

Similar data of dynamic topography were prepared for variety of sea state conditions using T/P altimeter. One track is being analysed here in continuation with the above Geosat and ERS-1 altimetric profile analysis for eddy detection. For the T/P altimetric profile in Figure 7, the lower panel b2 shows the dynamic height and upper one a2, its respective filter output. Two of the prominent locations of the eddies of the horizontal dimensions of about 200 km are above threshold at (7.14°N, 51.23°E) and (10.12°N, 52.2°E) respectively. The first one correspond to the great whirl and the second one is the extension of the circulation comprising of Somali gyre system.

With the precise orbit determination and with variety of space and ground based tracking systems in case of T/P, the orbit accuracies were significantly better than the GEOSAT and ERS-1 and even original mission requirement of T/P itself. Hence these orbit error considerations

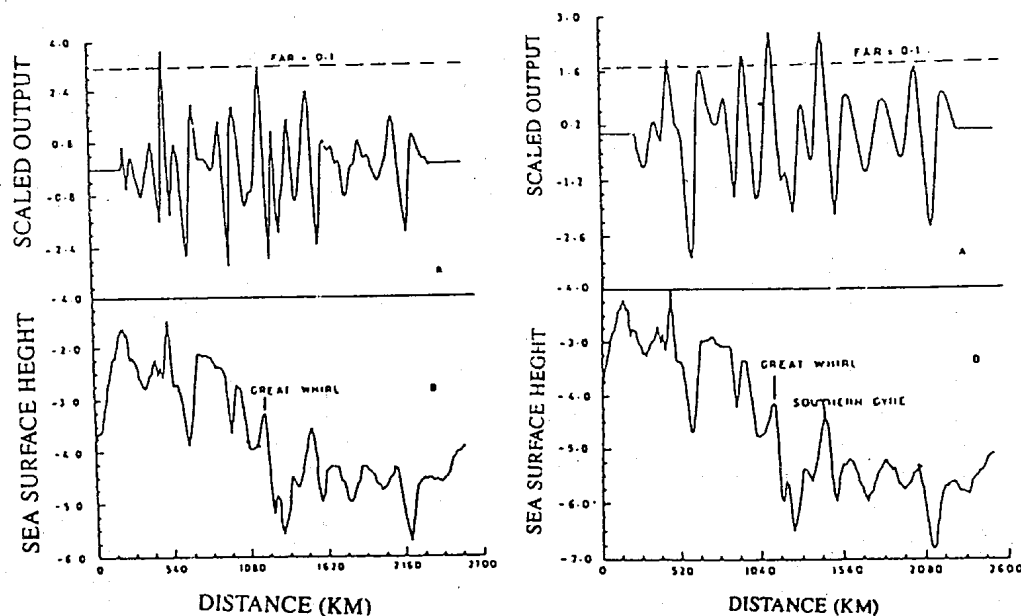


Fig. 6—Detection of eddy signature by matched filtering, in two collinear Geosat tracks, with lower panel B1,B2 the dynamic topography and the upper panel A1,A2 their filter output respectively.

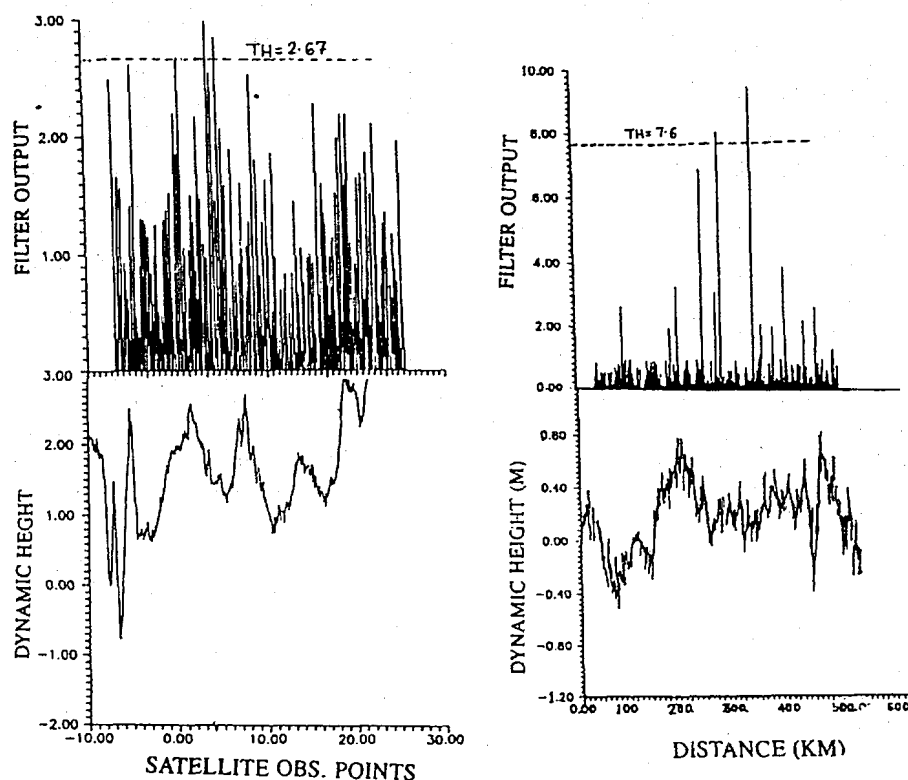


Fig. 7—Example of eddy detection from ERS-1 (a1,a2) and T/P (b1,b2) altimeter dynamic topography profiles.

remarkably improved the sea surface measurements by T/P altimeter. The three frequency passive radiometers onboard the T/P highly improved the wet tropospheric range correction measurements. The total error budget of T/P was highly evaluated and was provided in the Geophysical Data Records. Mostly the signal is merged with the noises of measurements, still the data adaptive signal processing applied here seems to detect the desired signal of circulation features with a great success.

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References

1 Wyrki K, *J Phys Oceanogr*, (1975) 450.

- 2 Cheney R E, Marsh J G, & Backley, *J Geophys Res*, 88 (1983) 4343.
- 3 Cheney R E, Miller L, Agreen R W, Doyle N S & Douglas B, *John Hopkins APL Technical Digest*, (1989), 362.
- 4 Rapp R H, Y M Wang & Palvis N K, *Rep 410*, Dep Geod Sci and Surv, (Ohio State Univ, Columbus), (1991) 25.
- 5 Rapp R H, *Rep 278*, Dep Geod Sci and Surv, (Ohio State Univ, Columbus), (1978) 17.
- 6 Rapp R H, *Bull Geod*, 54 (1980) 149.
- 7 Rapp R H, *J Geophys Res*, 88(C3) (1983) 1552.
- 8 Cartwright D E & R D Ray, *J Geophys Res*, 95(C3) (1990) 3069.
- 9 Heiskanen W, & H Moritz, *Physical Geodesy*, (W H Freeman and Comp. San Francisco) 1967, 19.
- 10 Rapp R H, *Rep 334*, Dep Geod Sci and Surv, (Ohio State Univ, Columbus) 1982.
- 11 Kay S M, *Modern Spectrum analysis: Theory and applications*, (Prentice Hall, New Jersey), 1988, 543.
- 12 Basu S, R M Gairola, Varma A K, N Gautam & P C Pandey, *Indian J Mar Sci*, 22 (1993) 188.
- 13 White J V, R V Sailor, A R Lazarewicz & A R Leschak, *J Geophys Res*, 88 (1983) 1541.
- 14 Brammer R F, R P Pass & J V White, *IEEE Trans Automat Control*, AC-28, (1983), 263.
- 15 Luther M E & J O'Brien, *Prog Oceanogr*, 14 (1985) 353.