

Variational data assimilation in ocean model: A simulation experiment

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Variational data assimilation approach using adjoint equations has been used in a reduced gravity model of the Northwestern Indian Ocean. The method minimises a cost function defining the misfit between the model and the data with the model equations acting as constraints. Several numerical experiments have been performed with simulated data generated from a control run of the model with analysed wind forcing data. In the case of perfect data, assimilation procedure is able to recover Sea Surface Elevation and current. The algorithm is found to have good potential and will be very much applicable for assimilation of spaceborne altimeter data.

Ocean models and data assimilation techniques will have much more important role than their meteorological counterpart in successfully extrapolating the surface information into the ocean interior for reconstructing the most realistic possible ocean circulation. Sophisticated data assimilation techniques are required for successfully assimilating asynoptic satellite data into ocean models. The forecast capabilities of ocean models could also be immensely improved by assimilation of real time sea surface height data from satellite altimeter and sea surface wind vectors from satellite scatterometer¹⁻⁴.

Data assimilation is the process by which observations are treated in order to specify the initial conditions of a numerical forecast model. A detailed description⁵⁻⁸ of the variational data assimilation procedure has been provided with a simple wind driven model. In this method, observations distributed in time and space are used to constrain the entire phase-space trajectory of the model and this is the major difference from the other sequential data assimilation methods. However the method is computationally expensive to use, but due to use of all past observations simultaneously for fitting the trajectory, it appears to be much more logical to use than other sequential assimilation methods. In the present study, it has been tried to utilise the technique for a linear reduced gravity model of the northwestern Indian Ocean with analysed wind stress data. (The

wind stress data was kindly provided by Prof. O'Brien of Florida State University). In view of the large number of upcoming spaceborne sensors like ERS-1/2, Topex, OCEANSAT-II with a radar altimeter to map the ocean surface, it will be important to study the assimilation of satellite data into numerical ocean models. The present study is a step in that direction.

Materials and Methods

This approach minimises the cost function representing misfit between the model and the data. An extensive review has been provided by many investigators⁹⁻¹¹. In this specific case, the observations will consist of measurements of oceanographic fields, e.g. the measurements of interface depths. Minimization has to be done with respect to a set of control variables. The control variables are the model initial conditions and the problem is of constrained minimization. Since the cost function is an implicit function of the initial variables, minimisation amounts to setting the partial derivatives of the cost function with respect to the initial variables to zero. The constrained problem is solved by redefining it so that it becomes a problem of unconstrained minimisation. The required partial derivatives constitute a very high dimensional vector and cannot be found by simple differentiation. An elegant way of tackling the problem is to use the adjoint equation technique. The optimisation is done by solving a

set of equations consisting of the model equations and the corresponding adjoint equations forced by the model data misfit. The adjoint equations are derived using Lagrange multiplier technique. A Lagrangian is constructed as the sum of the cost function and the constraints (model equations) multiplied by corresponding Lagrange multipliers. The minimum of the cost function coincides with a stationary point of the Lagrangian. To find this point one has to set the partial derivatives of the Lagrangian equal to zero. The partial derivatives with respect to the Lagrange multipliers recovers the model equations and the partial derivatives with respect to the model variables constitute a set of adjoint equations.

The solution of the adjoint equations at the initial time provides the partial derivatives of the cost function with respect to the initial variables. These partial derivatives are required for minimising the cost function using optimisation methods like conjugate-gradient or quasi-Newton. The algorithm thus proceeds in an iterative fashion. Each iteration consists of a forward run of the ocean model, a backward run of the adjoint model to provide the required partial derivatives for minimising the cost function. The minimum thus found provides the improved initial values and the integration of the model with these improved initial values results in a better forecast capability.

Model and the adjoint equations

The model developed for this study is a linear reduced gravity model of the northwestern Indian Ocean in spherical coordinates. The area involved is from 29°S to 23°N and from 40°E to 75°E. The reduced gravity simply describes the ocean consisting of two layers, one active upper layer and another passive lower layer. The density difference between the layers is $\Delta\rho$ and the mean density is ρ . The standard assumption of hydrostaticity has been invoked and reduced gravity is denoted by $g' (=g\Delta\rho/\rho)$. The motion of the upper layer represents the first baroclinic mode. Both the layers are homogeneous horizontally and vertically. The model equations in the transport form with spherical co-ordinates are written as⁹

$$\frac{\partial U}{\partial t} - f V = -\frac{c^2}{a \cos \theta} \frac{\partial h}{\partial \varphi} + A \nabla^2 U \quad \dots (1)$$

$$\frac{\partial V}{\partial t} + f U = -\frac{c^2}{a} \frac{\partial h}{\partial \theta} + A \nabla^2 V \quad \dots (2)$$

$$\frac{\partial h}{\partial t} + \frac{1}{a \cos \theta} \frac{\partial U}{\partial \varphi} + \frac{\partial}{\partial \theta} (V \cos \theta) = 0 \quad \dots (3)$$

Here $U=uh$ and $V=vh$ represent the eastward and northward components of upper layer transport, respectively, where (u,v) are the depth-independent (φ,θ) velocity components in the upper layer and h is the thickness of the upper layer, φ and θ denote longitude and latitude respectively, t is the time, c is the wave speed of the first baroclinic mode and the value chosen for c^2 is $6 \text{ m}^2 \text{ sec}^{-2}$ which is typical for the Arabian Sea, f is the Coriolis parameter and a is the radius of the earth. The coefficient for eddy viscosity A has been chosen to be $1000 \text{ m}^2 \text{ sec}^{-1}$. g' has been taken as 0.03 m sec^{-2} and $\rho = 1023 \text{ kg m}^{-3}$.

The grid employed is Arakawa C-grid and the spacing is 0.5° , uniform in both meridional and zonal directions. The time step is 30 minutes. The finite difference scheme is leap frog with a forward time step after every 99 steps to suppress the computational mode. The coastlines are treated as no-slip boundaries.

The primary forcing of the model is windstress and τ^φ and τ^θ are the eastward and northward components of windstress.

The cost function J is defined as

$$J = \int \frac{1}{2} (h - h_{\text{obs}})^2 d\sigma \quad \dots (4)$$

where h_{obs} represents an observation of the upper layer thickness. The integration is taken over the entire spatiotemporal domain over which the model is integrated. The Lagrangian is formed as

$$L = \int \left[\left(\beta_u E_u + \beta_v E_v + \beta_h E_h \right)^2 + \frac{1}{2} (h - h_{\text{obs}})^2 \right] d\sigma \quad \dots (5)$$

where, β_u, β_v and β_h are the Lagrange multipliers for U, V , and h respectively and E_u, E_v and E_h are the left hand sides of U, V and h equations when all the terms have been brought to the left hand side and the right hand side represents zero. The stationary points of the Lagrange function (which correspond to the minimum of the cost function) can be found by letting the first variation of L with respect to the

variables $U, V, h, \beta_u, \beta_v$ and β_h vanish. The first variation of L with respect to β_u, β_v and β_h gives the original model equations (1-3). Constraining the first variation with respect to U, V and h to zero leads to the adjoint equations :

$$-\frac{\partial \beta_u}{\partial t} + f\beta_v = \frac{c^2}{a \cos \theta} \frac{\partial \beta_h}{\partial \varphi} + A \nabla^2 \beta_u \quad \dots (6)$$

$$-\frac{\partial \beta_v}{\partial t} - f\beta_u = \frac{c^2}{a} \frac{\partial \beta_h}{\partial \theta} + A \nabla^2 \beta_v \quad \dots (7)$$

$$-\frac{\partial \beta_h}{\partial t} - \frac{1}{a \cos \theta} \left[\frac{\partial \beta_u}{\partial \varphi} + \frac{\partial}{\partial \theta} (\beta_v \cos \theta) \right] + (h - h_{\text{obs}}) = 0 \quad \dots (8)$$

It can be easily seen that the adjoint equations are forced by data misfits represented by the last term in equation (8). Comparing equations (6-8) with model equations, one can see that they have similar form, except that adjoint equations corresponds to evolution backward in time. The Lagrange variables therefore carry information about the misfit backwards to the initial time of the data assimilation period. The gradient of the cost function is contained in the values of the adjoint

variables at the model initial time¹¹. The temporal domain in which the cost function is minimised has been chosen as 10 days.

Results and Discussion

In this study, the model has been spun up from rest with the initial upper layer thickness of 200m at each grid. A steady annual cycle was achieved by running the model for six years with monthly averaged wind stress data of 1993. The current pattern simulated by the model resemble the generally observed patterns in the Arabian Sea. The data for January 15 has then been stored as initial state of the model.

The model has been run for 10 days starting from initial conditions. The upper layer thickness (ULT) values at each space and time levels have been stored to be used as synthetic observations (h_{obs}). Figure 1 shows the initial sea level and current values. The ULT values have been converted to sea surface elevation (SSE) by multiplying upper layer thickness by density difference of two layers i.e. $\text{SSE} = \text{ULT} * (\Delta \rho / \rho)$. This data was further used in assimilation

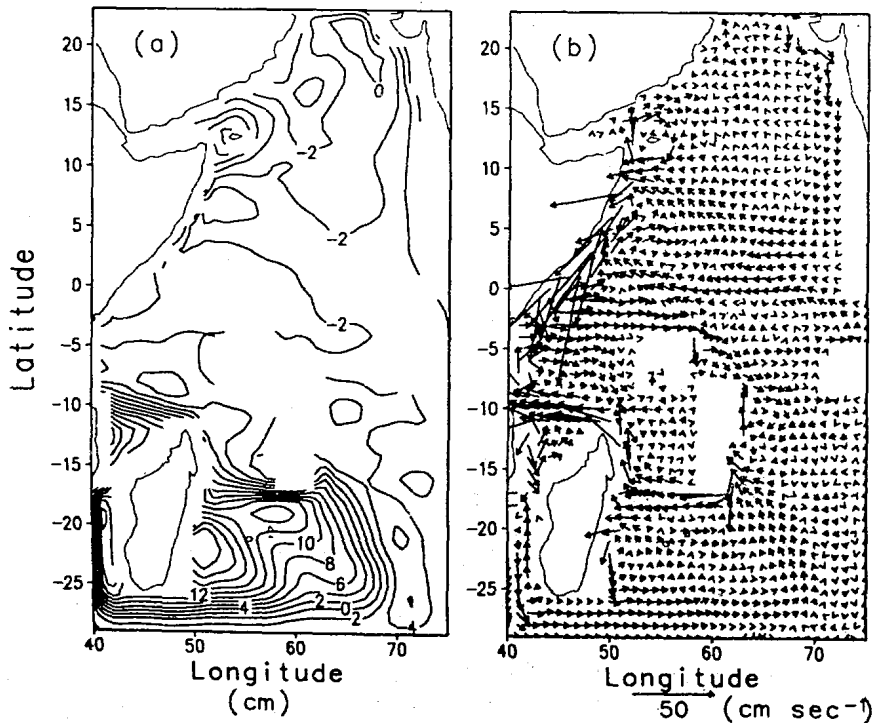


Fig. 1—(a) Initial sea surface elevation without noise., (b) Initial current field.

procedure. Thereafter, assimilation has been done with the assumption of initial thickness, U and V component of current as 200, 0 and 0 respectively. The assimilation was done for 10 days and data (h_{obs}) has been provided at all time step for retrieval of initial conditions. Figure 2 shows the retrieved initial values of SSE and current. It can be clearly seen that assimilation is able to retrieve the initial values with a very good accuracy. It has to be noted that, however only h values have been provided (no current data) in the assimilation procedure, even though current values have been retrieved very well.

Any observational data set may not be perfect as synthetic data, it will always have noise embedded in it. To simulate the actual data, Gaussian noise has been added in the synthetic perfect data set with the standard deviation of noise as one fourth of standard deviation of actual data. Figure 3 shows the initial SSE and current data with noise. After assimilation of 10 days, we recover retrieved initial values as shown in Figure 4. Even after putting noisy data of ULT, one was able to recover SSE values perfectly well. The surface current values have also been retrieved with very good accuracy, the rms error between initial data and retrieved being 5.

Since the satellite passes over a certain location after a few days, so the observation for a particular location may not be available all the time. At each locations, data will be available only after averaging over a few days. To imitate this situation, it has been tried to see the effect of observational data only at few time steps. A number of experiments have been done with observations of h provided 12 hourly, every 2 days, every 5th day and only at 1st and last time steps. Figure 5 shows the retrieved initial values after the assimilation, when the h data was provided at only 2 days interval. The data at each step was then linearly interpolated. The SSE values again perfectly march with the initial values. Even in the case of retrieved currents, which were not provided at all for assimilation run, one is able to retrieve it very well. The major current systems are clearly reproduced in retrieved values.

In the next experiment, observations were provided with noise at 2 days interval. Figure 6 shows the retrieve values. In this case, the

retrieved values of SSE have rms deviation of 3.4 which is comparable to standard deviation of initial data itself. It means that assimilation is able to reproduce the actual observations (i.e. with noise) not the noise free initial values, which may be understandable because assimilation procedure tries to minimise the misfit between observation and model. In the present experiment, observation itself is noisy, so retrieved values also have noise of the similar magnitude.

Figure 7 shows the retrieved values of SSE and surface currents in the case of observation data provided only at initial and final time steps. Even in this extreme case, retrieved h values are perfect. The assimilation procedure is also able to reproduce current values, which were not provided initially.

In this study, variational data assimilation technique has been developed for a reduced gravity model of northwestern Indian Ocean. Several experiments have been performed to see the effect on retrieval of initial values by assimilation procedure.

The model was initialised by analysed wind stress data. The perfect and noisy observations were generated and initial values have been retrieved using 10 days assimilation of data. The r.m.s. deviation between initial and retrieved values have also been computed. It has been seen that in the case of noise free data, r.m.s deviation between initial and retrieved values is very less, implying the success of assimilation procedure in recovering initial fields. It is true for all experiments, i.e. even if data is provided at all time steps or only at very few time steps. The r.m.s. error between actual initial values of current and recovered values, however, increases with less number of h observation data provided in assimilation.

For the case of noisy observational data, if the observation has been provided at all the time steps, the recovery of initial h and current field is very good, however if the observations are not available at all time steps, such as in the case of data given at 2 days interval, the recovery of initial SSE is not perfect. The error in recovery of SSE is same as error given in observation data field. For the case of currents, however the recovery is not at all good. It means that, without providing any current data, recovery of current with noisy observations

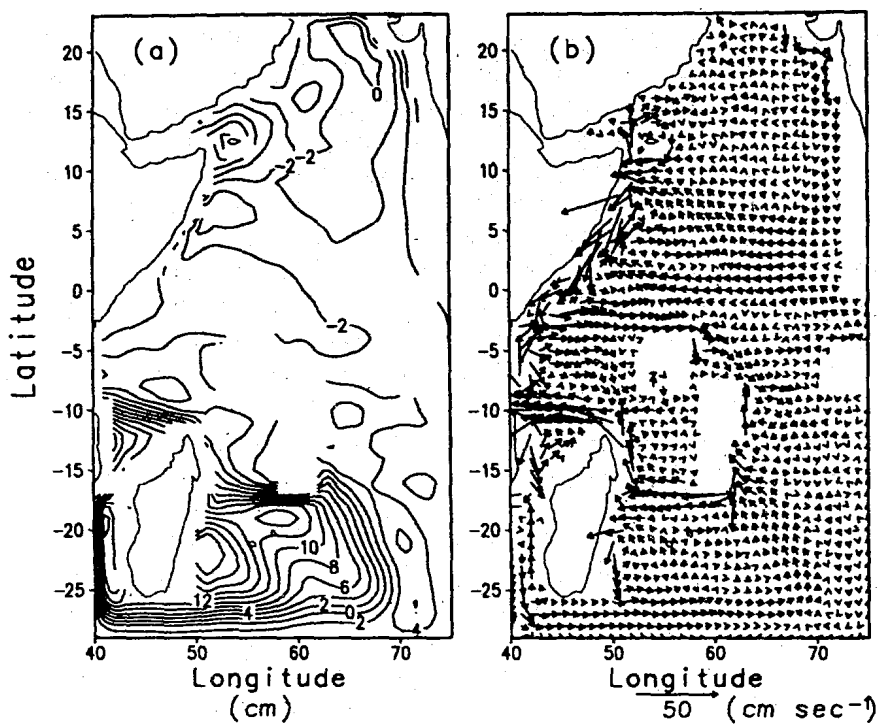


Fig. 2—(a) Retrieved initial sea surface elevation (data at all time steps), (b) Retrieved current field (data at all time steps)

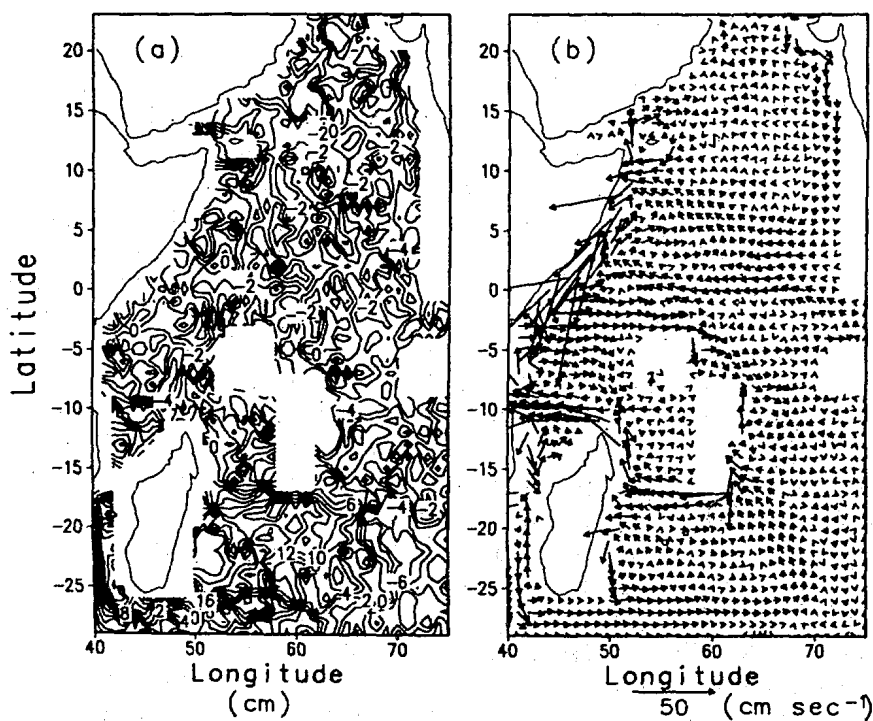


Fig. 3—(a) Initial sea surface elevation with noise., (b) Initial current field.

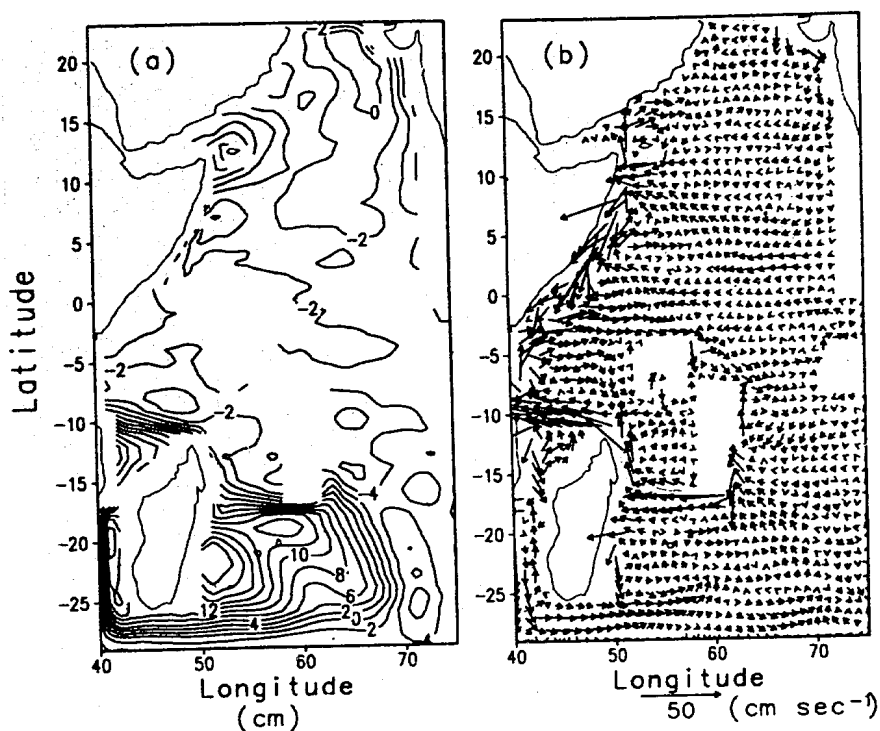


Fig. 4—(a) Retrieved initial sea surface elevation (data at all time steps with noise),
(b) Retrieved current field (data at all time)

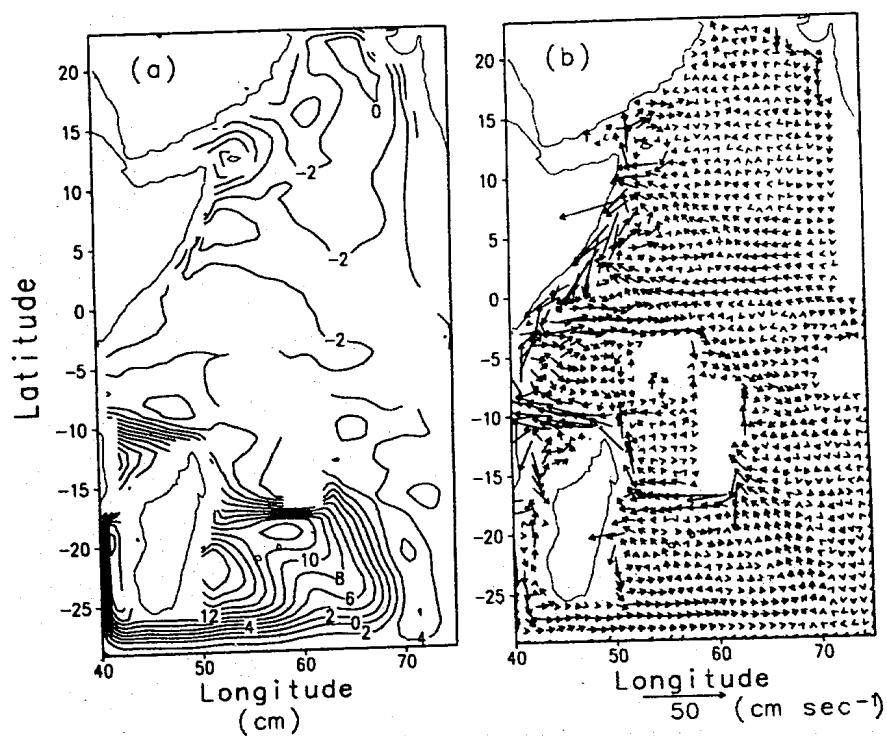


Fig. 5—(a) Retrieved initial sea surface elevation (data at 2 days interval), (b) Retrieved current field (data at 2 days interval)

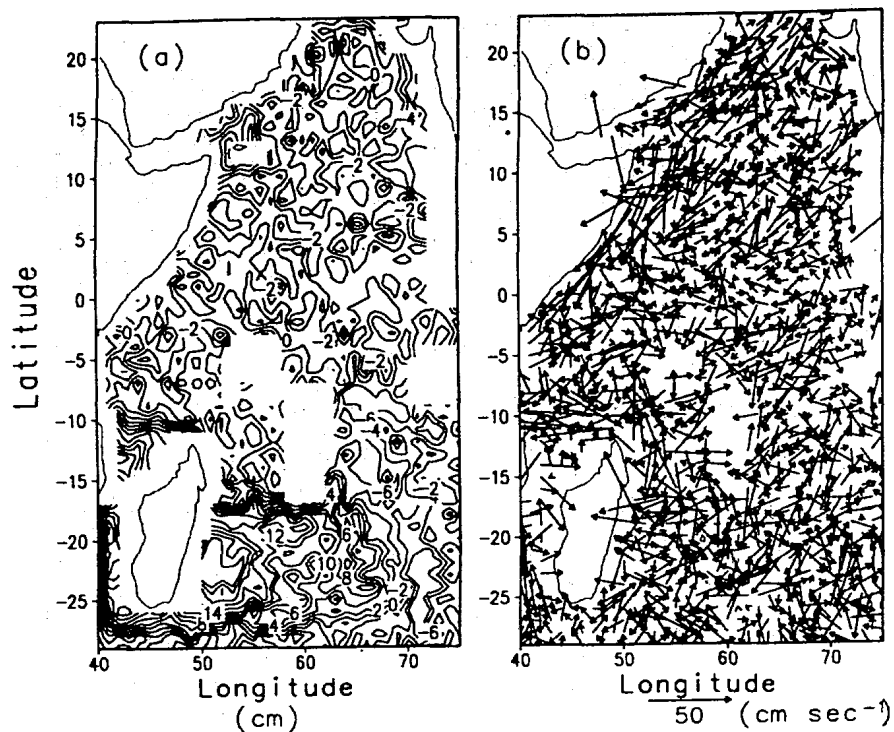


Fig. 6—(a) Retrieved initial sea surface elevation (data at 2 days interval with noise), (b) Retrieved current field (data at 2 days interval)

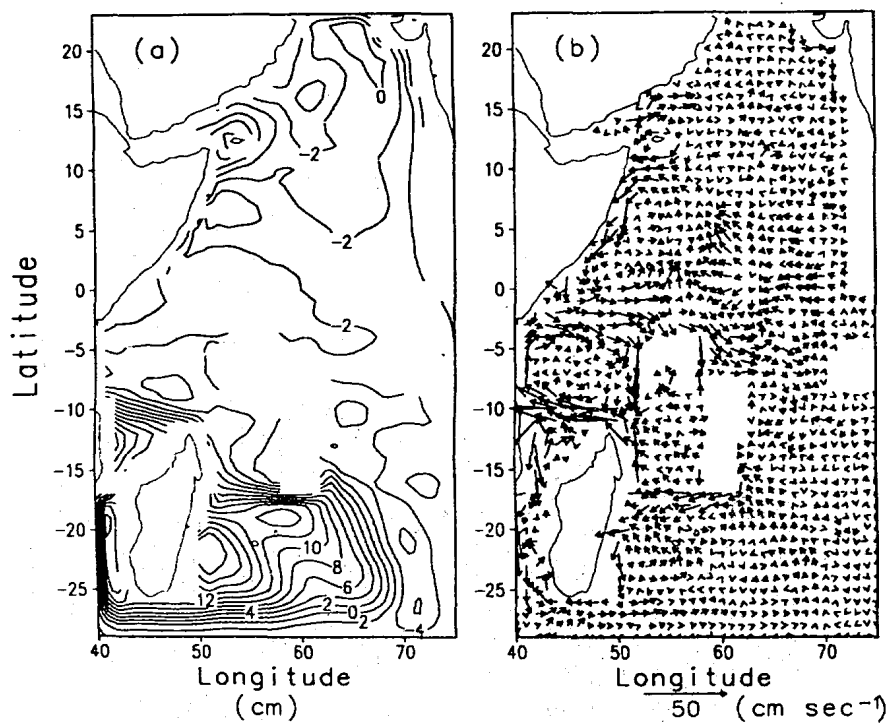


Fig. 7—(a) Retrieved initial sea surface elevation (data at first and last steps only), (b) Retrieved current field (data at first and last steps only)

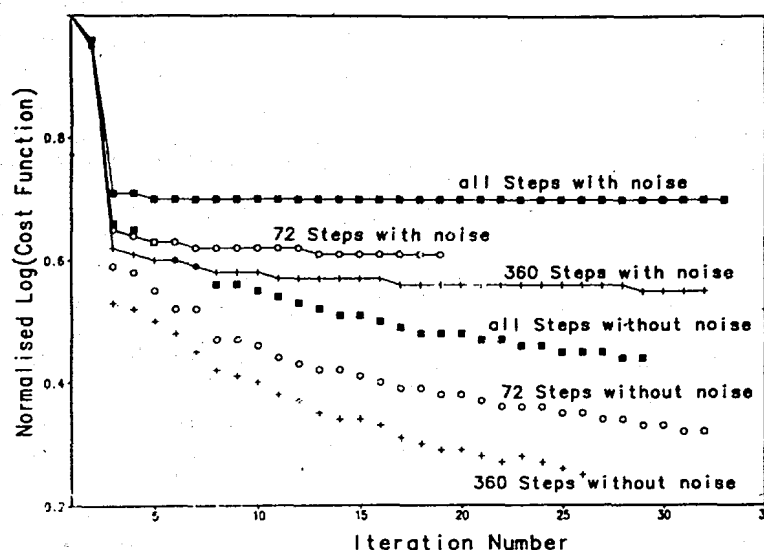


Fig. 8—Reduction in cost function with iteration numbers

only at a few time steps will not be possible, whereas sea level values can still be recovered with sufficient accuracy.

The results have shown that algorithm is computational efficient. As can be seen from Fig. 8, that convergence is achieved very fast in the case of noise free data and cost function reduced to approximately 10^{-6} times of original values, however in the case of noisy data, generally cost function reduces up to 10^{-3} times only. Minimisation routine used for the convergence is variable storage (or limited memory) quasi Newton method developed by Gilbert and Lemarechal¹².

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