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Radar polarimetry: The need aspect and the density matrix approach*

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Recent developments in imaging radars with polarimetric capabilities are briefly surveyed and the need to develop the technology indigenously in India is emphasized. A new method to analyse the data is developed using the concept of the density matrix which was introduced into quantum theory by von Neumann, Dirac, and Landau.

1 Introduction

Interest in the development and utilization of polarimetric radars is now widespread and radar polarimetry is currently a subject which is being intensively persued in several advanced countries of the world. Classically, the radar systems were concerned with variations in amplitude, frequency or phase of the electromagnetic signals. Polarization, which is an important characteristic of the electromagnetic wave, is now being exploited to sense the detailed structure of the imaged scene or object. Polarimetric radars provide additional data in the form of cross-polar and co-polar cross-sections, which play a very important role in target discrimination. Two specific branches of remote sensing where such studies have proved promising are (i) ground-based remote sensing of the earth's atmosphere, (ii) air-borne/space-borne remote sensing of the surface of the earth and other planets.

2 Remote sensing applications of radar polarimetry

2.1 Ground-based applications

In the case of ground-based atmospheric radars, the introduction of polarimetric capability leads to (i) discrimination between rain and hail formation and (ii) characterization of rainfall estimates more accurately in the path between geo-synchronous satellites and their earth stations, especially for those links for which rain attenuation could cause considerable outage (like in 11/14 GHz and 20/30 GHz links). Considering, for example, the destruction caused to apple orchards by hail storms, the polarimetric capability can help in saving annually in this one use only more than Rs 100 crore if a suitable cloud seeding technique is used to convert hail formation to rain. It can safely be recommended therefore as an essential feature for multiparameter weather radar systems. The existing precision coherent monopulse C-band tracking radar (PCMC), jointly developed by ISRO and BEL essentially for rocket tracking applications, can, with certain feasible modifications, be converted into a multi-parameter polarimetric Doppler weather radar.

2.2 Air-borne/space-borne applications

In air-borne/space-borne synthetic aperture radars (SARs) for remote sensing, the polarimetric capability enhances contrast and leads to better target discrimination. Experiments already conducted and being conducted at NASA/JPL using DC-8 SAR at C band have shown a capability to discriminate the returns from tree tops and tree trunks. Such a discrimination is obviously very useful in the IGBP context. The NASA/JPL DC-8 has, since spring of 1988, mapped different types of terrains such as the Californian deserts, mountains, Alaskan icefields and Scottish lochs. The observed polarization signatures reveal distinctive scattering mechanisms pre-

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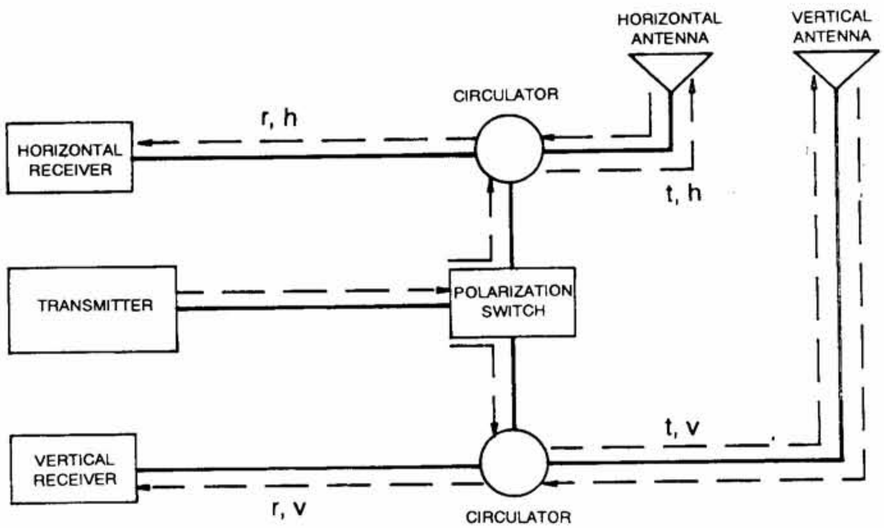

Fig. 1—Schematic diagram of a polarimetric radar

sented by diverse types of terrains like urban areas, agricultural tracts, lava fields, high seas and ice-fields. In our country, therefore, one can and should explore possibilities of introducing polarimetric capabilities into the air-borne SAR system being developed at SAC. Space-based SAR systems were used successfully in Sea Sat (USA), KOSMOS 1500 (USSR), Shuttle Imaging Radar (SIR-NASA), ERS (ESA) for microwave remote sensing of the earth's surface for resources mapping and sea state studies. Future systems include SIRC (NASA) and RADAR SAT (Canada).

3 Radar polarimetry

3.1 The basic relation

A polarimetric radar (Fig. 1) is basically a microwave device which transmits a polarized pulse of electromagnetic radiation and accepts subsequently the echo in a preset well-defined state of polarization. The so-called* horizontal and vertical components (E_h, E_v) of the electric field $\mathbf{E}_s$ associated with the scattered signal are given by[1,2]

$$\begin{pmatrix} E_h \\ E_v \end{pmatrix}_s = \frac{1}{r}\exp(ikr)\begin{pmatrix} S_{hh'} & S_{hv'} \\ S_{vh'} & S_{vv'} \end{pmatrix}\begin{pmatrix} E'_h \\ E'_v \end{pmatrix}_i \qquad \ldots(1)$$

where r is the distance between the target and the receiving antenna, k the wave number, (E'_h, E'_v) the horizontal and vertical components of the field $\mathbf{E}_i$ illuminating the target. The complex valued 2×2 matrix S, characterizing the target may vary, in general, with k and depends also on the angles at which the pulse is incident and the echo is observed.

3.2 Dirac notation

A signal is said to be linearly polarized along $\hat{h}$, if $E_h = E$ and $E_v = 0$. Likewise, if $E_h = 0$ and $E_v = E$, the signal is linearly polarized along $\hat{v}$. These two orthogonal states of polarization may be denoted respectively by

$$\begin{pmatrix} E \\ 0 \end{pmatrix} = E\begin{pmatrix} 1 \\ 0 \end{pmatrix} = E\,|h\rangle;\ \begin{pmatrix} 0 \\ E \end{pmatrix} = E\begin{pmatrix} 0 \\ 1 \end{pmatrix} = E\,|v\rangle \qquad \ldots(2)$$

where $|\ \rangle$ is referred to as ket by Dirac[3]. Any arbitrary orthogonal pair of linearly polarized states [see Fig. 2(a)] are expressible as

$$-\sin\alpha\,|h\rangle + \cos\alpha\,|v\rangle \equiv |\varepsilon_x\rangle;\ \cos\alpha\,|h\rangle + \sin\alpha\,|v\rangle \equiv |\varepsilon_y\rangle \qquad \ldots(3)$$

so that $(\hat{\varepsilon}_x, \hat{\varepsilon}_y, \hat{k})$ form a right-handed Cartesian coordinate system (RHCCS). Any arbitrary state of polarization may then be written, in general, as

*The unit vectors $(\hat{h}, \hat{v})$ identifying the horizontal and vertical are usually defined as

$$\hat{h} = \hat{\imath}_3 \times \hat{k}/|\hat{\imath}_3 \times \hat{k}|;\ \hat{v} = \hat{h}\times\hat{k} \qquad \ldots(1\text{a})$$

where $\hat{k}$ denotes the unit vector along the direction of wave propagation and $(\hat{\imath}_1, \hat{\imath}_2, \hat{\imath}_3)$ denote the unit vectors defining a fixed right-handed cartesian coordinate system (RHCCS). Note that $(\hat{k}, \hat{v}, \hat{h})$ form a RHCCS whose orientation varies as $\mathbf{k}$ is changed. Therefore, $(\hat{k}_T, \hat{v}_T, \hat{h}_T)$ at the transmitter and $(\hat{k}_R, \hat{v}_R, \hat{h}_R)$ at the receiver antennas will in general be different. However, in the particular case when the same fixed antenna is used to study back-scattering, it is convenient to choose $\hat{k}_R = -\hat{k}$ so that $(\hat{k}_R, \hat{v}_R, \hat{h}_R) = (\hat{k}_T, \hat{v}_T, \hat{h}_T)$, which is referred to as back-scattering alignment (BSA) convention. If (1a) itself is used both at T and R, it is referred to as forward scattering alignment (FSA) convention.

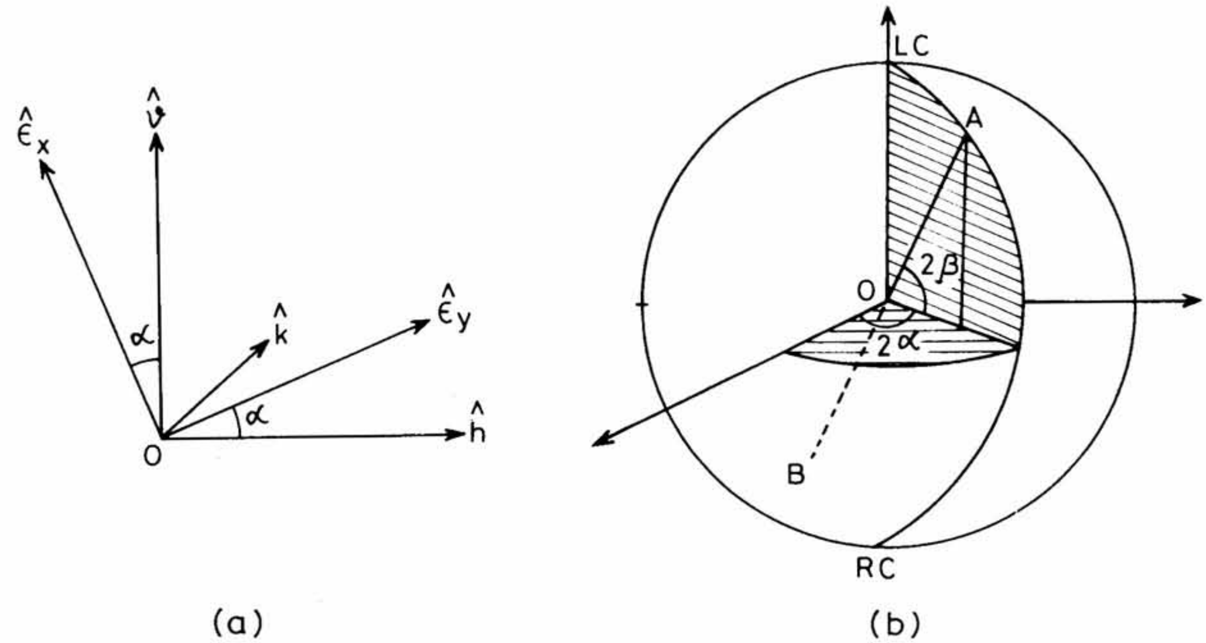

Fig. 2—Representation of state of polarization [(a) right-handed cartesian coordinate system, and (b) Pincaré sphere]

$$|\alpha,\beta\rangle \equiv \cos\beta|\varepsilon_x\rangle - j\sin\beta|\varepsilon_y\rangle = a|h\rangle + b|v\rangle = \begin{pmatrix} a \\ b \end{pmatrix} \quad \ldots(4)$$

where $j=\sqrt{-1}$ and a, b are in general complex. It is readily seen that all states of linear polarization are realized in Eq. (4) by setting $\beta=0$ and $-\pi/2 \leqslant \alpha \leqslant \pi/2$. Setting $\beta=\mp\pi/4$ leads respectively to right-circular and left-circular states ($|\mathrm{RC}\rangle$ and $|\mathrm{LC}\rangle$) of polarization[4]; it is being understood that the complex waveform $\exp[-j(\omega t-\mathbf{k}.\mathbf{r})]$ is used. For all other values of β, viz., $-\pi/4<\beta<\pi/4$, states of elliptical polarization are realized. It may be noted that $|\alpha,\beta\rangle$ is a vector in a linear two dimensional complex vector space (LTDCVS). Defining[3] the bra state $\langle\alpha,\beta|$ corresponding to $|\alpha,\beta\rangle$ as the Hermitian conjugate of Eq. (4), the inner product (or bracket) of two states $|\alpha_1,\beta_1\rangle$ and $|\alpha_2,\beta_2\rangle$ may be defined through (matrix multiplication)

$$\langle\alpha_1,\beta_1|\alpha_2,\beta_2\rangle = \langle\alpha_2,\beta_2|\alpha_1,\beta_1\rangle^* = a_1{}^*a_2 + b_1{}^*b_2 \quad \ldots(5)$$

where z^* denotes the complex conjugate of a given complex number z. We note further that

$$a=\langle h|\alpha,\beta\rangle = -(\sin\alpha\cos\beta + j\cos\alpha\sin\beta)$$

and

$$b=\langle v|\alpha,\beta\rangle = (\cos\alpha\cos\beta - j\sin\alpha\sin\beta) \quad \ldots(6)$$

in Eq. (4). Two states of polarization are said to be orthogonal in the LTDCVS if their inner product vanishes. It is in this sense that $|\mathrm{RC}\rangle$ and $|\mathrm{LC}\rangle$ are said to be orthogonal. In general, it may be noted that Eq. (5) vanishes if $\alpha_2=\alpha_1+\pi/2$; $\beta_2=-\beta_1$. We note, moreover, that

$$\sum_{i=1}^{2} |\alpha_i,\beta_i\rangle\langle\alpha_i,\beta_i| = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \ldots(7)$$

for any pair of orthogonal states

3.3 Poincaré sphere

Every state of polarization, as given by Eq. (4), is uniquely represented by a point **A** [as in Fig. 2(b)] with polar coordinates $\theta=\pi/2-2\beta$, $\phi=2\alpha$ on the Poincaré sphere[5] and vice versa, so that the sense of rotation is left handed in the upper half, right handed in the lower half, while the equatorial line represents all states of linear polarization. It may be noted that the state $|\alpha+\pi/2, -\beta\rangle$ which is orthogonal to $|\alpha,\beta\rangle$ is represented by a point B which is diametrically opposite to A. This is so even for points on the equator.

3.4 Co-polarization and cross-polarization response

It is clear from Eq. (1) that the scattered signal $\mathbf{E}_s$, in general, is of the form

$$E_h|h\rangle + E_v|v\rangle = E(a|h\rangle + b|v\rangle) = E|\alpha,\beta\rangle \quad \ldots(8)$$

where

$$E=(|E_h|^2+|E_v|^2)^{1/2};\ a=E_h/E;\ b=E_v/E \qquad \dots(9)$$

even when the illuminating pulse $\mathbf{E}_i$ is linearly polarized, say, along $\hat{h}_T$ or $\hat{v}_T$. In general, however, let P_T denote the power and $|\alpha_T, \beta_T\rangle$ the state of polarization of the transmitted signal. If the receiver antenna is preset to admit $|\alpha_R, \beta_R\rangle$ and exclude $|\alpha_R+\pi/2, -\beta_R\rangle$, the scattered signal gets resolved into these two orthogonal states and the power, P_{RT} measured by the receiver is determined by the amplitude of $|\alpha_R, \beta_R\rangle$. The bistatic cross-section takes the form

$$\sigma_{RT}=\lim_{r\to\infty} 4\pi r^2 P_{RT}/P_T = K\,|\langle\alpha_R\,\beta_R|S|\alpha_T\,\beta_T\rangle|^2 \qquad \dots(10)$$

where K is independent of the target but depends on k and the angles determining the effective area of the receiver antenna. Eq. (10) gives co-polarization response, σ^{Co}, when one sets $|\alpha_R, \beta_R\rangle=|\alpha_T, \beta_T\rangle$ and cross-polarization response, σ^{Cr}, if $|\alpha_R, \beta_R\rangle=|\alpha_T+\pi/2, -\beta_T\rangle$. These are usually presented as a pair of three-dimensional plots of σ versus α_T, β_T as variables. The procedure adopted for arriving at these plots is usually referred to in the literature as polarization synthesis[2] and several techniques have been proposed, all of them involving different types of 4×4 matrices. We shall, however, show in sec. 4 that this can effectively and elegantly be carried out using only 2×2 matrices.

It may also be pointed out that the basic Eq. (1) is ideally valid for a point target. When the scenario under observation consists of a distributed set of targets or when the radar target is sufficiently large that it has to be considered as a distributed source of scattered waves, the various contributions may superpose coherently or add incoherently.

3.5 Coherent and incoherent signals

If several scattered waves $E_n|\alpha_n, \beta_n\rangle$, $n=1,2,\dots$ as given by Eq. (8) superpose coherently, the resultant may be expressed as

$$\sum_n E_n|\alpha_n, \beta_n\rangle = E_h|h\rangle + E_v|v\rangle = E|\alpha,\beta\rangle$$

$$= SE_i|\alpha_i, \beta_i\rangle \qquad \dots(11)$$

where use is made of Eq. (1) and

$$E_h=\sum_n E_{h,n};\ E_v=\sum_n E_{v,n};\ E_i=(|E_h^i|^2+|E_v^i|^2)^{1/2};$$

$$S=\sum_n S_n \qquad \dots(12)$$

i.e., the situation is adequately taken care of by simply adding the individual matrices S_n characterizing each of the scatterers to make up S appearing in Eq. (10).

On the other hand, if the signals add incoherently the expression (10) has to be modified to read

$$\sigma_{RT}=K\sum_n |\langle\alpha_R, \beta_R|S_n|\alpha_T, \beta_T\rangle|^2 \qquad \dots(13)$$

since each scattered wave contributes only to the power, there being no well-defined phase relationship between the waves.

4 Density matrix approach

4.1 Formalism

If the state of polarization of a wave is $|\alpha,\beta\rangle$ and its amplitude E, the entity $E^2|\alpha,\beta\rangle\langle\alpha,\beta|$, which behaves like an operator in LTDCVS, maybe referred to as the density operator. If $|\alpha_1, \beta_1\rangle$ and $|\alpha_2, \beta_2\rangle$ denote any pair of orthogonal states, they may be chosen as the basis for matrix representation of the operator. The density matrix[6-8] ρ is then defined in terms of its elements,

$$\rho_{ij}=E^2\langle\alpha_i, \beta_i|\alpha,\beta\rangle\langle\alpha,\beta|\alpha_j, \beta_j\rangle=\rho^*_{ji} \qquad \dots(14)$$

which shows that ρ is Hermitian, i.e., $\rho=\rho^+$ where ρ^+ denotes the Hermitian conjugate of ρ. Choosing $|\alpha_1, \beta_1\rangle=|h\rangle$ and $|\alpha_2, \beta_2\rangle=|v\rangle$, we have ρ explicitly as

$$\rho=\begin{pmatrix}|E_h|^2 & E_hE_v^* \\ E_vE_h^* & |E_v|^2\end{pmatrix}=\begin{pmatrix}E_h \\ E_v\end{pmatrix} E_h^*\,E_v^* \qquad \dots(15)$$

If ρ' denotes the density matrix, when a different pair $|\alpha_1', \beta_1'\rangle$ and $|\alpha_2', \beta_2'\rangle$ are chosen as the basis, it is clear, on using Eq. (7) that

$$\rho'=U\rho U^+ \qquad \dots(16)$$

where U denotes the unitary matrix with elements

$$U_{ij}=\langle\alpha_i', \beta_i'|\alpha_j, \beta_j\rangle \qquad \dots(17)$$

Considering the scattered wave given by Eq. (8) and making use of Eq. (1) and Eq. (15), we have

$$\rho=S\rho^i S^+ \qquad \dots(18)$$

where the density matrix, ρ^i refers to the signal illuminating the target. Using the identiy $\langle\alpha_R, \beta_R|S|\alpha_T, \beta_T\rangle^*=\langle\alpha_T, \beta_T|S^+|\alpha_R, \beta_R\rangle$, we observe that σ_{RT} given by Eq. (10) may now be expressed simply as

$$\sigma_{RT}=K\langle\alpha_R, \beta_R|\rho|\alpha_R, \beta_R\rangle \qquad \dots(19)$$

We note that Eqs (18) and (19) hold good in a situation described by Eq. (11) as well, provided

$S=\sum_n S_n$, as was seen in Eq. (12). It is equally true that Eq. (19) holds good in a situation represented by Eq. (13), when incoherent contributions appear, provided.

$$\rho=\sum_n \rho_n=\sum_n S_n \rho^i S_n^+ \qquad \ldots (20)$$

We note, moreover, that ρ, whether given by Eq. (18) or Eq. (20), is Hermitian and may therefore be brought to its diagonal form, ρ°, by making an appropriate unitary transformation, U, which in the case of Eq. (14) must be of the form

$$U=\begin{pmatrix} a^* & b^* \\ -b & a \end{pmatrix} \qquad \ldots (21)$$

since $\rho^\circ_{11}=E^2$ and all other elements must be zero, when we choose the basis $|\alpha_1, \beta_1\rangle=|\alpha, \beta\rangle$ itself and $|\alpha_2, \beta_2\rangle=|\alpha+\pi/2, -\beta\rangle$.

It is a characteristic feature of 'pure' states of polarization that only one of the eigen values of ρ is non-zero. Denoting the eigen values by ρ_1, ρ_2 such that $\rho_1 \geqslant \rho_2$, we may define the ratio

$$r=(\rho_1-\rho_2)/(\rho_1+\rho_2) \qquad \ldots (22)$$

which takes a value 1 for pure polarized states. For $0<r<1$, the signal is said to be partially polarized, which results only when incoherent contributions are present. If $r=0$, the signal is unpolarized.

4.2 Standard form

Any 2×2 matrix may be expanded in terms of the four matrices

$$I,\ \sigma_1=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_2=\begin{pmatrix} 0 & -j \\ j & 0 \end{pmatrix},\ \sigma_3=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \ldots (23)$$

where I denotes the unit matrix and σ_i are the Pauli matrices. Thus

$$\rho=\frac{\mathrm{Tr}\,\rho}{2}[I+\boldsymbol{\sigma}\cdot\mathbf{P}] \qquad \ldots (24)$$

where Tr denotes the trace or spur, $\boldsymbol{\sigma}\cdot\mathbf{P}=\sum_{i=1}^{3}\sigma_i P_i$,

and

$$P_i=\mathrm{Tr}(\rho\sigma_i)/\mathrm{Tr}\,\rho \qquad \ldots (25)$$

which are real, since the matrices (23) as well as ρ are Hermitian. The form (24) is valid and $\mathrm{Tr}\,\rho=E^2$, irrespective of the basis chosen, although the numerical values and the physical interpretation of the P_i change if the basis is changed. For ρ given by Eq. (15) in the $|h\rangle, |v\rangle$ basis, the parameters are

$$P_1=2R_e(E_h E_v^*)/E^2,\ P_2=2I_m(E_h^* E_v)/E^2,$$
$$P_3=(|E_h|^2-|E_v|^2)/E^2 \qquad \ldots (26)$$

so that P_3, P_1, P_2 correspond to the well-known Stokes[9] parameters; where R_e is the real part of $(E_h E_v^*)$ and I_m is the imaginary part of $(E_h^* E_v)$. The same state is characterized by $P_1=0$, $P_2=0$ and $P_3=1$, if the basis chosen is $|\alpha,\beta\rangle$ and $|\alpha+\pi/2, -\beta\rangle$. It may also be noted that $\mathbf{P}\cdot\mathbf{P}\leqslant 1$, the equality holding for pure states. In fact $|\mathbf{P}|=r$, defined in Eq. (22).

4.3 Polarization synthesis

The 2×2 matrix, S, occurring in Eq. (1) may be expressed in a form, S_{RT} connecting illuminating states in the basis $|\alpha_{\mathrm{T}}, \beta_{\mathrm{T}}\rangle$, $|\alpha_{\mathrm{T}}+\pi/2, -\beta_{\mathrm{T}}\rangle$ to states in the $|\alpha_{\mathrm{R}}, \beta_{\mathrm{R}}\rangle$, $|\alpha_{\mathrm{R}}+\pi/2, -\beta_{\mathrm{R}}\rangle$ basis by using the unitary transformation (21) and S_{RT} may also be expanded in the standard form as

$$S_{\mathrm{RT}}=U_{\mathrm{R}} S U_{\mathrm{T}}^+=\frac{1}{2}(s_0+\boldsymbol{\sigma}\cdot\mathbf{s}) \qquad \ldots (27)$$

where s_0, s_1, s_2, s_3 are, in general, complex. Using Eqs (24) and (27) in Eq. (18), we have

$$\rho=\frac{E^2}{2}(I+\boldsymbol{\sigma}\cdot\mathbf{P})=\frac{1}{8}E_i^2(s_0+\boldsymbol{\sigma}\cdot\mathbf{s})(I+\boldsymbol{\sigma}\cdot\mathbf{P}_i)(s_0^* + \boldsymbol{\sigma}\cdot\mathbf{s}^*) \qquad \ldots (28)$$

where the right hand side may be simplified by the repeated use of the identity

$$(\boldsymbol{\sigma}\cdot\mathbf{A})(\boldsymbol{\sigma}\cdot\mathbf{B})=(\mathbf{A}\cdot\mathbf{B})+i\boldsymbol{\sigma}\cdot(\mathbf{A}\times\mathbf{B}) \qquad \ldots (29)$$

to give explicitly,

$$\mathbf{P}=E_i^2[(s_0\mathbf{s}^*+s_0^*\mathbf{s}+j\mathbf{s}\times\mathbf{s}^*)+(|s_0|^2+\mathbf{s}\cdot\mathbf{s}^*)\mathbf{P}_i$$
$$+j(s_0^*\mathbf{s}-s_0\mathbf{s}^*)\times\mathbf{P}_i-\mathbf{s}\times(\mathbf{P}_i\times\mathbf{s}^*)]/4E^2 \qquad \ldots (30)$$

and

$$E^2=\frac{1}{4}E_i^2[|s_0|^2+\mathbf{s}\cdot\mathbf{s}^*+\{s_0\mathbf{s}^*+s_0^*\mathbf{s}-j(\mathbf{s}\times\mathbf{s}^*)\}\cdot\mathbf{P}_i] \qquad \ldots (31)$$

so that σ_{RT} given by Eq. (19) is simply

$$\sigma_{\mathrm{RT}}=KE^2(1+P_3)/2 \qquad \ldots (32)$$

which can readily be used to plot σ^{Co} and σ^{Cr}, for any given S, in the same way as explained following Eq. (10). One should, of course, pay due attention to the conventions employed (BSA, etc.) in making comparisons.

4.4 Outlook

It is readily seen from Eq. (24) that ρ is completely determined if E^2 and $\mathbf{P}$ are measured. The presence of inelastic contributions is immediately detected if $\mathbf{P}\cdot\mathbf{P} < 1$. The more interesting question of reconstructing S for any target, using the density matrix formalism for radar polarimetry which has been developed here, will be taken up elsewhere.

5 Conclusions

The paper introduces the topic of radar polarimetry and discusses its important advantages in the area of remote sensing. It adopts the density matrix approach, well-known in Physics, to the problem of radar polarimetry, and this shows that scope exists for considerable reduction in the computational requirements in calculating the radar cross-sections. A comprehensive expression is derived for co-polar and cross-polar returns, using the density matrix formalism. The paper is also intended to highlight the need aspect of generating a study project at the national level to exploit the polarimetric capabilities in microwave remote sensing[10,11].

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