

Computations of some acoustic ray parameters in the Bay of Bengal

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Methods to compute geometric path, travel time and intensity of sound rays were detailed following a range dependent numerical computer program to study the acoustic properties and their variability in the Bay of Bengal, utilising sound speed data derived from CTD profiles taken during winter and pre-monsoon along a section off Madras. Between the 2 seasons, intensity levels varied from -74 to -88 dB for distances of 200 km and the flight of the eigen rays presented significant differences. The results indicate the independent nature of the eigen rays for a given source-receiver pair.

Acoustic properties are utilised to monitor meso-scale features of oceans using remote sensing¹. The design of sources and receivers for such experiments demands a detailed knowledge on the acoustic intensity and estimation of losses over different frequencies and ranges. For this, the environmental setting and an understanding of the changes that occur over different time and space scales in the seas under the influence of various internal and external forcings are pre-requisite.

As a part of the investigation aimed at developing techniques for remote sensing of the ocean interior, acoustic propagation characteristics that depend on the refractive index of the waters of the Bay of Bengal are examined in this study. Expressions for acoustic propagation characteristics^{2,3} such as intensity and associated losses are utilised for computing the intensity within the sound channel of the Bay of Bengal. The calculations are based on the as-

sumption that (i) changes in the intensity occur only due to contraction or expansion of the ray tube and (ii) the power transmitted between 2 adjacent rays (separated by $1/100$ of a degree) is conserved.

Mathematical Formulation

The hypothetical source $S(x_0, y_0)$ and receiver(s) R_i (where $i=1,2,\dots,n$), located along a vertical section of the transmitting medium (Fig. 1), are separated by several kilometers.

Following Arthur *et al.*⁴, the ray geometry is governed by

$$dx/ds = \cos \theta \quad \dots (1)$$

$$dy/ds = \sin \theta \quad \dots (2)$$

$$d\theta/ds = 1/c [\sin \theta \partial c / \partial x - \cos \theta \partial c / \partial y] \quad \dots (3)$$

where, θ is the angle of emergence of sound ray (Fig. 1) and s is the arc length along the ray. The range-dependent sound speed profile is considered by in-

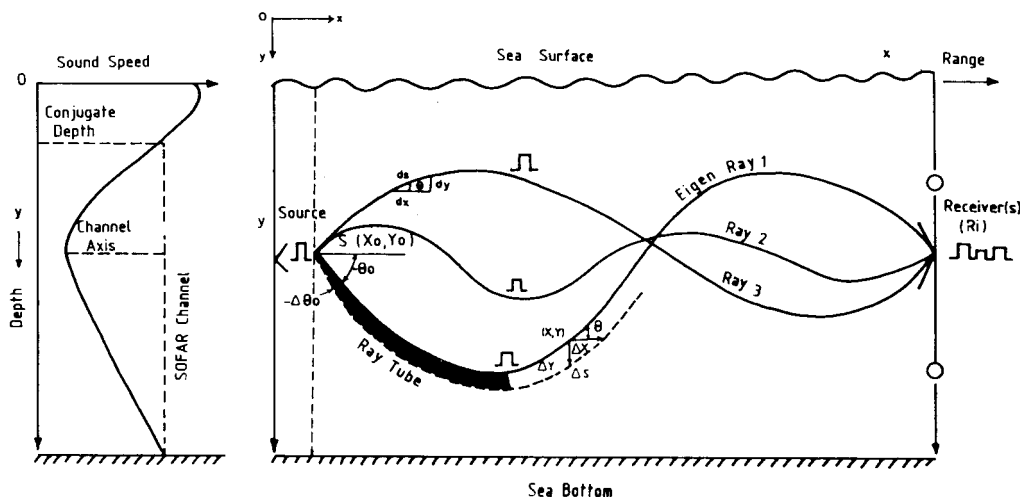


Fig. 1—Schematic presentation of acoustic rays from source to receiver

roducing $\partial c/\partial y$ in Eq. 3 neglecting horizontal sound speed gradient $\partial c/\partial x$.

Refractive index—Eqs 1-3 require specification of sound speed $c(i, j)$ at nodes or points of intersection. In order to produce a uniform grid, interpolation algorithms that consider random data points to produce data at required intervals (at the nodal points) have to be utilised. The path of the ray traversing the medium between the source and the receiver is controlled by the index of refraction which depends on the vertical profile of temperature and salinity, usually obtained either as a continuous profile as in STD or CSTD or at discrete points (conventional hydrocasts) at any oceanographic station.

For STD or CSTD data, cubic spline interpolation⁵ is useful as it takes into account the second order derivative of the required parameter. In the case of discrete data, weighted parabolic interpolation^{6,7} is appropriate. This technique operates with the arithmetic mean of two 3-point parabolic interpolations using overlapping sets of points and leads to an overshooting in case changes in the gradients occur rapidly.

Numerical ray tracing—The solution of ray tracing with initial conditions specifying any given point (source coordinates and direction) on the ray is usually known as the initial-value problem. The numerical computations in the form of algorithm for Eqs 1 to 3 are:

$$X^1_{ij} = f_i(X_1, X_2, X_3, X_4)$$

$$X_i = X_i + ds \sum_{j=1}^4 (Y_i - B_j Z_i) A_j \quad \dots (4)$$

$$Z_i = Z_i + 3 \sum_{j=1}^4 (Y_i - B_j Z_i) A_j - C_j Y_i$$

where $i = 1, 2, 3, 4$; $Y_1 = 1$; $Y_2 = \cos \theta$;

$Y_3 = \sin \theta$; $Y_4 = \cos \theta \ V_y/V$; and with initial conditions of

$$X_1(0) = 0$$

$$X_2(0) = X_0 \text{ (source } X\text{-coordinate)} \quad \dots (5)$$

$$X_3(0) = Y_0 \text{ (source } Y\text{-coordinate)}$$

$$X_4(0) = \theta_0 \text{ (horizontal angle)}$$

and the coefficients of A_j , B_j and C_j being:

$$A_1 = 1/2, B_1 = 2, C_1 = 1/2$$

$$A_2 = 1 - \sqrt{1/2}, B_2 = 1, C_2 = 1 - \sqrt{1/2}$$

$$A_3 = 1 + \sqrt{1/2}, B_3 = 1, C_3 = 1 + \sqrt{1/2}$$

$$A_4 = 1/6, B_4 = 1, C_4 = 1/2$$

In the present study Runga-Kutta-Gill's method⁸ has been chosen to obtain the numerical solution.

For the present analysis, step-size (ds) is chosen depending on the medium at the end point of the ray segment. In the region of thermocline or halocline, ds is considered small compared to that within the layers further below and characterized by high gradients in sound speed. The changes in range (dx), depth (dy) and angle ($d\theta$) are computed. During integration, if the ray path does not pass through the nodal points which would define sound speed field for computing the gradients and the refractive index, they may be restricted to pass through arbitrary points in the plane. In such a case, suitable interpolation function has to be defined to evaluate both sound speed and its partial derivatives for a given ds . A 3-point Lagrangian interpolation is used here to construct such a function along the vertical and in the neighbourhood of the sound speed field taking 2 points above and one point below and vice versa, prior to averaging. Knowing the information along ds and at the arbitrary points, integration is carried out using the Runga-Kutta-Gill's technique. At the upper and lower boundaries, the ray gets reflected following equal angle law. Irregular bathymetric features and the curvature of the earth have not been considered here. The integration repeats until all rays within a given beam arrive at the receiver. If not, the algorithm⁹ takes on computation of the ray having the next assigned angle and follows the same procedure ('Shooting method'). The ray co-ordinates are kept in memory space allocated in a matrix form. The time of flight (T_i) is determined following the recurrence formula

$$T_i = T_i + \sum_{j=1}^4 \{(Y_2 - B_j Z_2) + (Y_3 - B_j Z_3)\} A_j ds \quad \dots (6)$$

where i covers full range.

Once the eigen rays are identified from the ray computation, the intensity calculations are made following the ray tube method².

Intensity computations—The acoustic intensity at any point along the ray path is given by the expression

$$\text{Intensity} = I(x)/I_0 = \sin \theta_0 / X \cos \theta \quad \dots (7)$$

where I_0 = reference intensity at the source; θ_0 = eigen ray angle at the source with reference to the horizontal; θ = ray angle at the point of measurement with reference to the horizontal; X = distance from the source to the point of measurement; ΔY = vertical distance at the point of measurement between eigen ray and ray in its immediate vicinity; and $\Delta \theta_0$ = angle between eigen ray and ray in its immediate vicinity at the source.

At the point where $\Delta Y \rightarrow 0$, the above expression will not hold good (tends to infinity mathematically).

cally) and the previous value only gets substituted for computational continuity.

Considering the phase shift for any given frequency the intensity has been computed following³

$$I/I_0 = |\sum P_k e^{j2\pi f t_k}|$$

$$= \sum_j \sum_k P_j P_k \cos [2\pi f(t_j - t_k)] \quad \dots (8)$$

where $P_k = [I_k(x)/I_0]^{1/2}$ and t_k are the relative pressures and arrival times of the eigen rays. These have wider applications in signal detection.

Data

Environmental data along a section (Leg A, Fig. 2) in the western Bay of Bengal, collected using MI-COM STD recorder during winter (January/February) and premonsoon (March/April) of 1988 were utilised in this study. Hydrographic data at standard depths up to 200 m and at 50 m depth intervals beyond 200 m were chosen for carrying out oceanographic computations¹⁰. For the computation of sound, speed formula of Chen and Millero¹¹ was employed. Sound speed profiles at all stations (along Leg A) for both the periods were plotted (Fig. 3).

In Fig. 4, a plot of the interpolated sound speed profiles at range interval of 20 km is depicted. The computer plots of the vertical sound speed structure along Leg A for winter and premonsoon are shown in Figs 5A and B. A ray diagram showing multiple paths emanated from a sound source located at 1500 m in the deep sound channel is constructed (Fig. 6). From these multiple rays, the rays connecting the given source and the receiver, usually called 'eigen rays', are obtained. Fig. 7 represents eigen ray plots for ranges 60-200 km.

Results and Discussion

Sound speed structure—This along a section off Madras ($13^\circ 5' - 14^\circ N$ to $81^\circ 5' - 87^\circ E$ in the western Bay of Bengal for winter and premonsoon seasons is shown in Figs 5A and B. To make small scale features prevailing in the medium conspicuous, the contours were drawn at 0.5 m.sec^{-1} intervals. During winter a thin (10 m) isovelocity layer exists within the upper 50 m column which is absent in premonsoon. During premonsoon, strong gradients in sound speed prevail up to 250 m depth whereas they are less intense in winter. Off Madras strong gradients are found indicating the presence of a frontal structure. This structure will influence the ray geometry. Sound speeds at the channel axis reveal that the magnitudes are higher during winter compared to premonsoon. During winter, numerous pockets with low sound speed occur in contrast

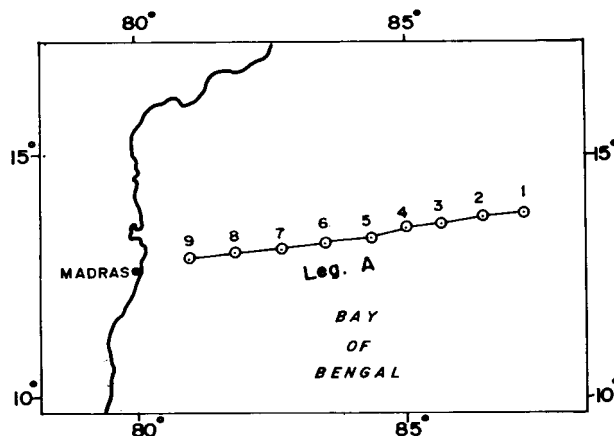


Fig. 2—Station locations

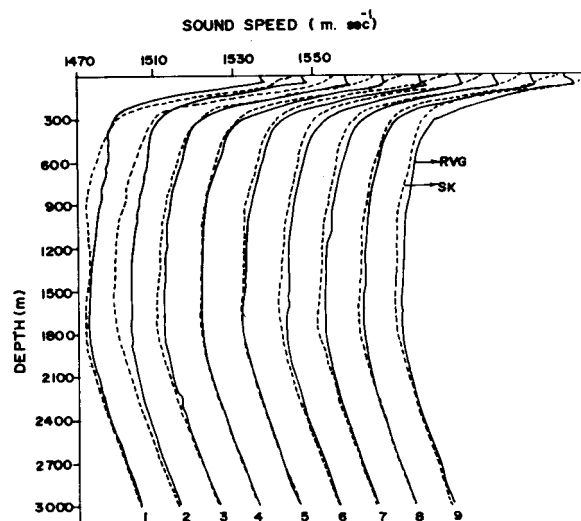


Fig. 3—Sound speed profiles for Leg A (Fig. 1)

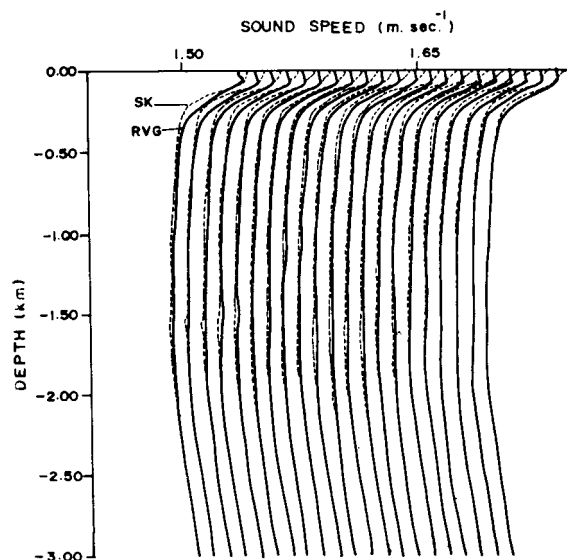


Fig. 4—Interpolated profiles of sound speed at 20 km intervals

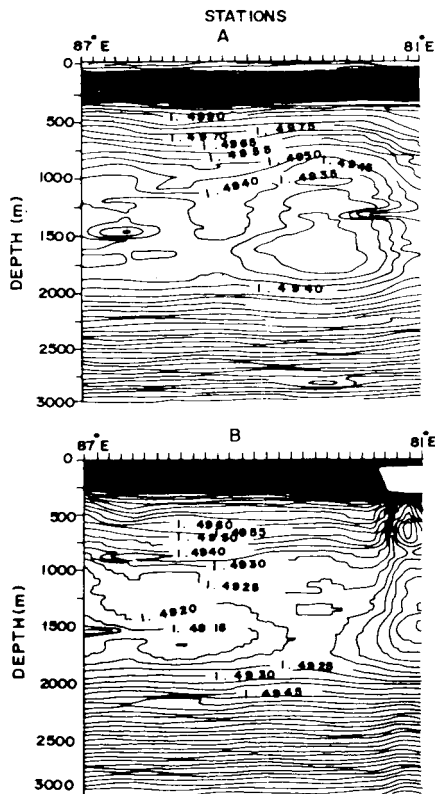


Fig. 5—Vertical cross section of sound speed for Leg A during (A) winter and (B) premonsoon

to premonsoon season along the sound channel axis. From sound speed contours near SOFAR (Sound Fixing And Ranging) axis, it is found that the near axial sound rays reach the receiver faster during winter than during premonsoon giving rise to changes in the arrival pattern (Table 1). Below 2000 m, the sound speed increases slowly with uniform gradient.

The sound speed (Figs 5A and B) is higher at (1536 to 1541 m.sec⁻¹) the surface than at the bot-

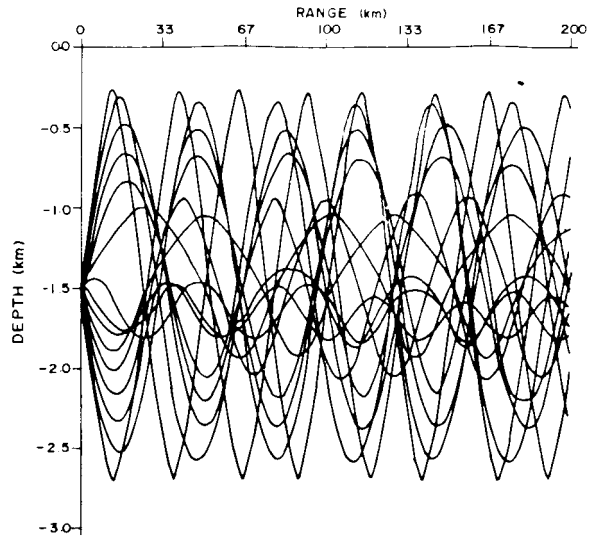


Fig. 6—A sample of acoustic ray paths

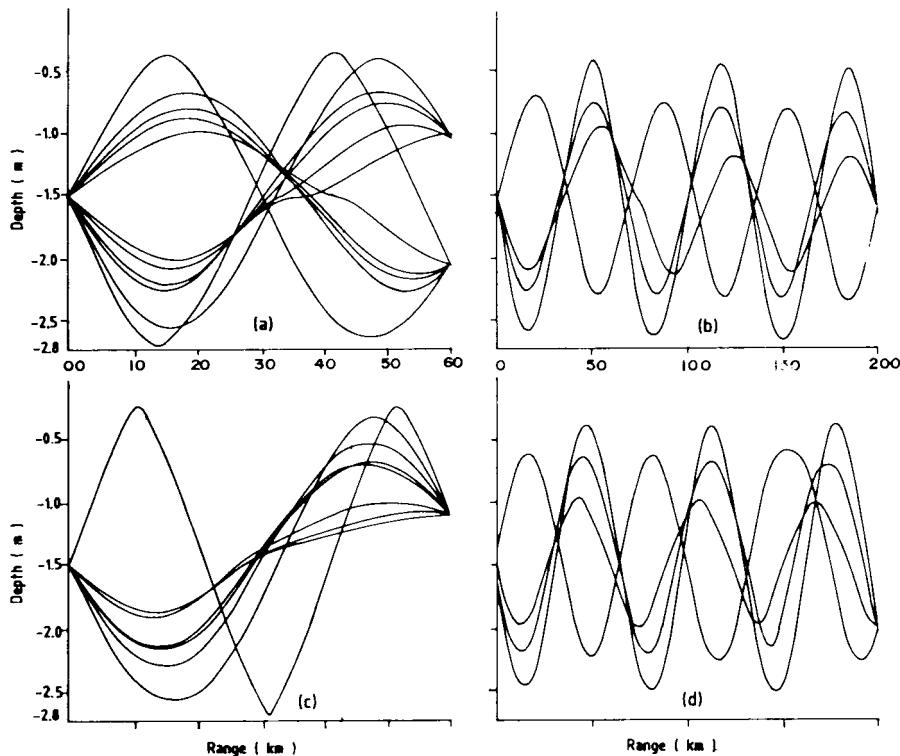


Fig. 7—Simulated sound rays over 00 and 200 km for winter (a) and (b) and for premonsoon (c) and (d) respectively

Table 1—Acoustic parameters at different ranges

Range (km)	R _d (m)	n	θ _o /θ _r	ds (km)	T (sec)	I/I ₀	−dB	f ₄₀₀
January-February 1988 along Leg A								
60	1000	2	−2.10 2.61	60.023	40.19202	4.295 × 10 ^{−8}	73.66	64.77
		2	−2.60 −1.69	60.034	40.19399	4.901 × 10 ^{−8}	73.09	
		2	−3.60 −3.07	60.062	40.19437	2.085 × 10 ^{−8}	66.80	
		2	−3.90 −3.52	60.074	40.19432	1.543 × 10 ^{−8}	78.11	
		2	−5.60 −5.18	60.147	40.19898	1.049 × 10 ^{−8}	69.78	
60	2000	2	5.70 5.41	60.158	40.19999	5.163 × 10 ^{−8}	82.87	66.48
		2	3.30 2.37	60.049	40.19200	3.339 × 10 ^{−8}	74.76	
		2	2.90 1.80	60.039	40.19200	2.456 × 10 ^{−8}	76.09	
		2	2.20 −0.40	60.024	40.19100	1.601 × 10 ^{−8}	77.95	
		2	−6.50 −0.60	60.253	40.27400	2.189 × 10 ^{−8}	76.59	
200	2000	6	5.10 4.90	200.420	133.99081	3.485 × 10 ^{−8}	74.57	73.61
		6	3.00 2.12	200.147	133.98541	7.034 × 10 ^{−8}	91.52	
		6	2.70 −1.96	200.118	133.98190	1.763 × 10 ^{−8}	87.53	
		6	2.10 −1.74	200.052	133.98679	1.343 × 10 ^{−8}	88.71	
		6	−5.90 −5.44	200.585	134.05029	1.756 × 10 ^{−8}	87.55	
200	1500	6	3.90 −3.84	200.258	134.00851	1.949 × 10 ^{−8}	77.10	71.78
		6	−2.60 −2.05	200.109	133.98990	2.029 × 10 ^{−8}	86.92	
		6	−3.70 −3.58	200.242	134.00671	7.911 × 10 ^{−8}	81.01	
		6	−5.60 −5.54	200.514	134.02161	2.623 × 10 ^{−8}	75.81	
300	2100		−5.60 −4.86	300.829	201.07709	3.839 × 10 ^{−8}	94.15	70.48
			−2.80 −0.10	300.207	200.98331	1.204 × 10 ^{−8}	79.19	
			−4.60 3.68	300.525	200.98209	1.021 × 10 ^{−8}	79.90	
			−5.00 4.36	300.624	200.98499	1.030 × 10 ^{−8}	79.86	
		March-April 1988 along Leg A						
60	1100	2	−4.88 −4.54	60.104	40.23281	9.976 × 10 ^{−8}	100.01	60.66
		2	−3.93 −3.42	60.067	40.23323	7.415 × 10 ^{−8}	71.29	
		2	−1.73 −7.82	60.016	40.23514	5.108 × 10 ^{−8}	72.91	

contd

Table 1—Acoustic parameters at different ranges—*Contd.*

Range (km)	R_d (m)	n	θ_o/θ_r	ds (km)	T (sec)	I/I_0	- dB	f_{400}
200	2000	2	-1.87 -6.28	60.017	40.23488	1.128×10^{-8}	69.47	71.45
		2	-2.20 -1.34	60.023	40.23548	1.271×10^{-8}	78.95	
		2	-4.00 -3.47	60.069	40.23244	4.341×10^{-8}	73.62	
		3	8.00 -8.00	60.438	40.42968	1.674×10^{-8}	87.76	
		2	-4.84 -4.50	60.103	40.23363	8.472×10^{-8}	70.71	
		2	-3.90 -3.42	60.067	40.23447	1.115×10^{-8}	69.52	
		2	-6.20 -5.97	60.173	40.23635	5.990×10^{-8}	82.22	
		6	-2.70 4.56	200.124	134.12762	3.568×10^{-8}	74.47	
		6	-5.60 -5.04	200.524	134.14996	2.688×10^{-8}	85.70	
		6	4.21 3.44	200.275	134.08960	2.179×10^{-8}	76.61	
		6	-4.20 -3.00	200.278	134.13455	8.845×10^{-8}	80.53	

Range—distance between source to receiver
 R_d —depth of receiver
 n—number of loops
 θ_o —ray launching angle at source
 θ_r —ray angle at receiver

ds—ray path length
 T—travel-time
 I_x/I_0 —acoustic intensity ratio
 - dB—decibar (unit of intensity)
 f_{400} —frequency at 400 Hz

tom (1503 to 1505 m.sec⁻¹). The upper boundary of the wave guide occurs at depth of 190 to 260 m. Sound signal entering this channel would get trapped and propagates over long ranges with little or no attenuation. The variability, if any, produces travel time perturbations that form the basis for monitoring the oceanic water body. The acoustic parameters (Table 2) for different ranges (x_i) exhibit variations in space and in time. The computation of seasonally range dependent sound rays enables a realistic estimation of acoustic parameters prior to conduct of any field experiment.

Eigen rays—The sound rays (Fig. 7) from the source reach the receivers situated at 60 and 200 km. The upper and lower graphs represent winter and premonsoon conditions. Converging regions could be seen around 30 km range in all these diagrams. These rays refract at the upper boundary and reflect at the lower boundary of the channel. From Fig. 7 one can see that source-receiver positions are not independent of seasons because these rays—eigen rays—have the freedom to select the paths that offer the combination of sound speed and current velocity that best satisfy well known Fermat's princi-

ple (path of minimum travel time). So one can trust that these eigen rays are seasonally range dependent.

Angle-depth diagrams—The eigen rays for different source-receiver configurations are presented in terms of angle versus depth (Fig. 8). They are useful in the diagnosis of ray history such as the depth of scanning, reception angle (θ) at (arbitrary) the receiver, etc.; 'O' is the source-point (Fig. 8) and is the intersection of vertical and horizontal source lines. The rays usually form concentric curves with respect to this point. Since the sound speed profiles are range dependent, the rays form open curves. This feature also indicates that any given ray does not repeat the same angle and depth after some range. As the ray travels over a range, it traverses the curve in a counterclock fashion. Thus range is indicated by distance along the curve. The starting points are taken at the sound axis (1500 m) where the ray points are upward, that is, at the depth where the ray has the largest angle with the horizontal. The angles are presented in Table 1. During premonsoon season (Fig. 8c), kinks are seen at 1500 and 900 m depth. The gradients are much stronger than

Lat/Long. [Depth (m)]	January-February 1988				March-April 1988			
	Sound channel		Conjugate		Sound channel		Conjugate	
	depth (m)	speed (m. sec ⁻¹)	depth (m)	speed (m. sec ⁻¹)	depth (m)	speed (m. sec ⁻¹)	depth (m)	speed (m. sec ⁻¹)
1 14° 00'/87° 00' (3030)	1700	1492.8	250	1507.9	1550	1490.9	190	1505.5
2 13° 53'/85° 16' (3100)	1700	1493.0	240	1506.4	1600	1491.4	230	1503.0
3 13° 46'/85° 35' (3140)	1650	1493.0	245	1506.3	1700	1491.0	220	1503.6
4 13° 44'/84° 49' (3140)	1650	1493.1	255	1507.4	1600	1491.1	215	1503.8
5 13° 32'/84° 02' (3280)	1650	1492.1	235	1506.4	1650	1491.9	220	1503.3
6 13° 25'/83° 17' (3090)	1550	1492.0	250	1507.0	1750	1492.0	230	1504.4
7 13° 18'/82° 34' (3200)	1500	1493.0	235	1506.1	1650	1490.9	230	1504.1
8 13° 12'/81° 50' (3490)	1700	1494.1	195	1507.8	1525	1489.5	240	1507.8
9 13° 05'/81° 05' (3430)	1700	1493.3	180	1505.3	1650	1492.0	230	1503.4

Columns 2 and 6 stand for the axis of sound channel.

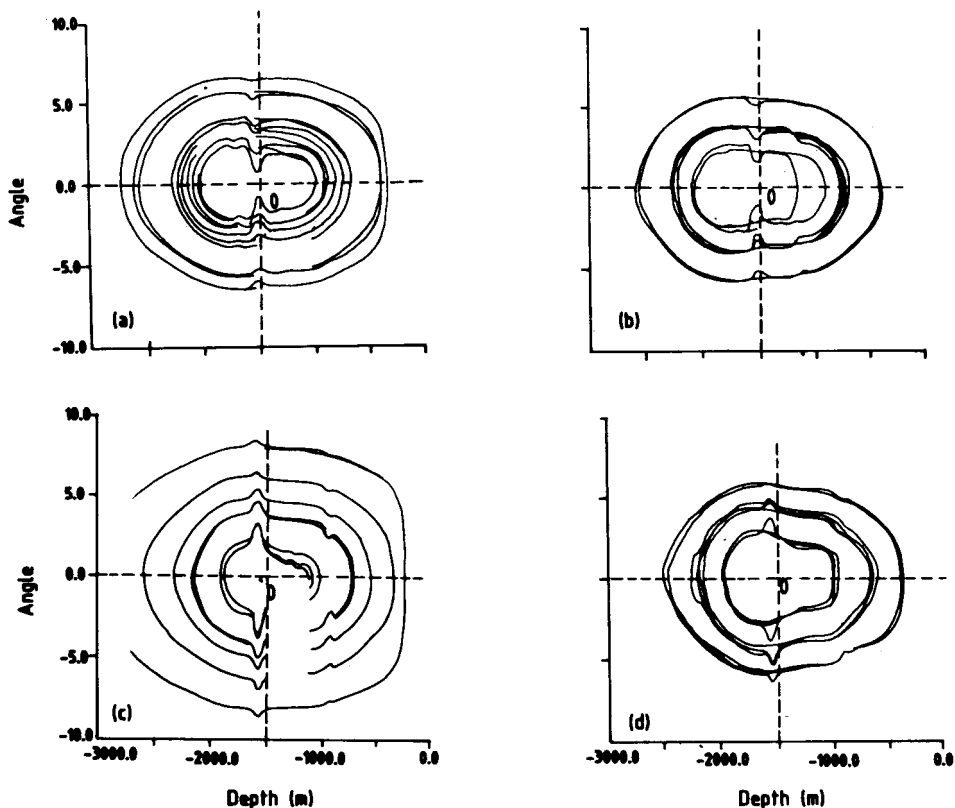


Fig. 8—Angle versus depth diagram of sound rays for conditions as described for Fig. 7

those at 900 m. Obviously, they are the artifacts of sound speed gradient to ray propagation or due to accidental observation. When the ray propagates from sound channel axis towards the surface (upper propagating ray) it undergoes less refraction (increasing the depth of penetration). For an upward/downward propagating ray from the channel axis the upward/downward kink is an indication of sound speed gradient due to warm/cold water. An interesting observation is the fact that the rays with flat angle undergo greater upward/downward penetration (indicated by upward/downward peaks) compared to those with steep angle. This is due to the existence of gradients in sound speed near the axis which is otherwise a zone of near homogeneous water. For ranges longer than 200 km, each ray forms more than one open cycle and repeats with some deviations indicating changes in the width of the convergence zone from central to western part of the Bay of Bengal due to the effect of refractivity and the nonhomogenous nature of the waters.

Acoustic intensity—For a few selected rays intensity computations are made for different ranges (Figs 9-12). During winter for a source depth of 1500 m and receiver depth of 1000 m, the eigen ray (-2.1°) has an intensity of -35 dB at the source and decreases gradually to -60 dB at about 15 km and then increases at 25 km. This indicates a reduction in the radius of the ray caused due to convergence near turning points (Fig. 7a). The acoustic energy of a ray emerging at -2.6° has 2 peaks with intensity

around -40 dB. For -3.6° ray, the fall in intensity is gradual and the magnitude lies between -40 dB to -60 dB. Similar pattern can be observed for -3.9° ray, but with a peak of -38 dB at 19 km. This only shows that different rays suffer losses differently and do not follow any pattern making their prediction complex.

The presence of multiple peaks in intensity versus range graph (Fig. 10) provides information on small scale features. The convergence zones seen in Fig. 7 can be identified by the larger peaks in this distribution. The intensity of ray (-3.7°) has a magnitude of -35 dB near the source and decreases to -50 dB at 17 km. The energy peak for upward propagating paths once again falls to -65 dB at the convergence zone (Fig. 7b). This process repeats over the desired range. From Fig. 10 one observes, for -2.6° ray at 120 km range, a reduction in intensity associated with a fall in the depth of scanning from 900 to 1200 m as shown in Fig. 7B. The sound speed distribution (Fig. 5A) reveals the above changes in scanning depth which were attributed to strong gradients in sound speed at depths of 1200 m.

Premonsoon—Intensity versus range for launch angles 8° , -1.87° , -2.2° , -3.9° , -4° , -6.2° of 60 km for premonsoon season (Fig. 12) shows maximum energy (-10 dB) near the source for -1.87° ray while the remaining are confined to -20 dB and -30 dB. Two peaks are observed for the negative rays, one at 15 km range and the other at 50 km. This is associated with the depths of turn-over and

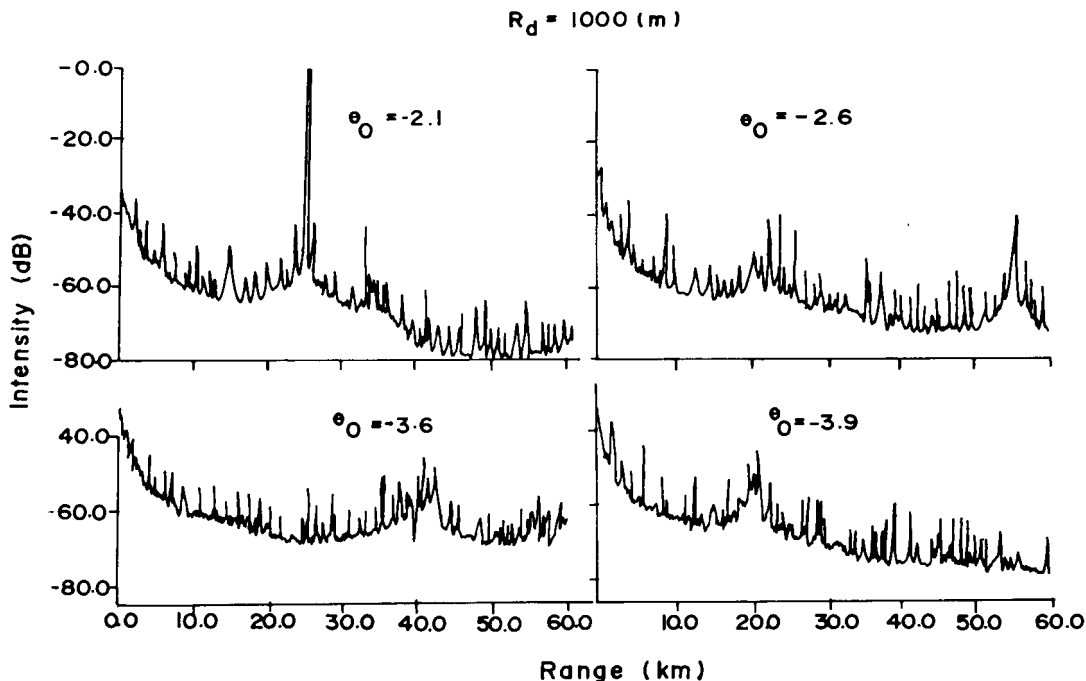


Fig. 9—Intensity changes for 60 km range during winter

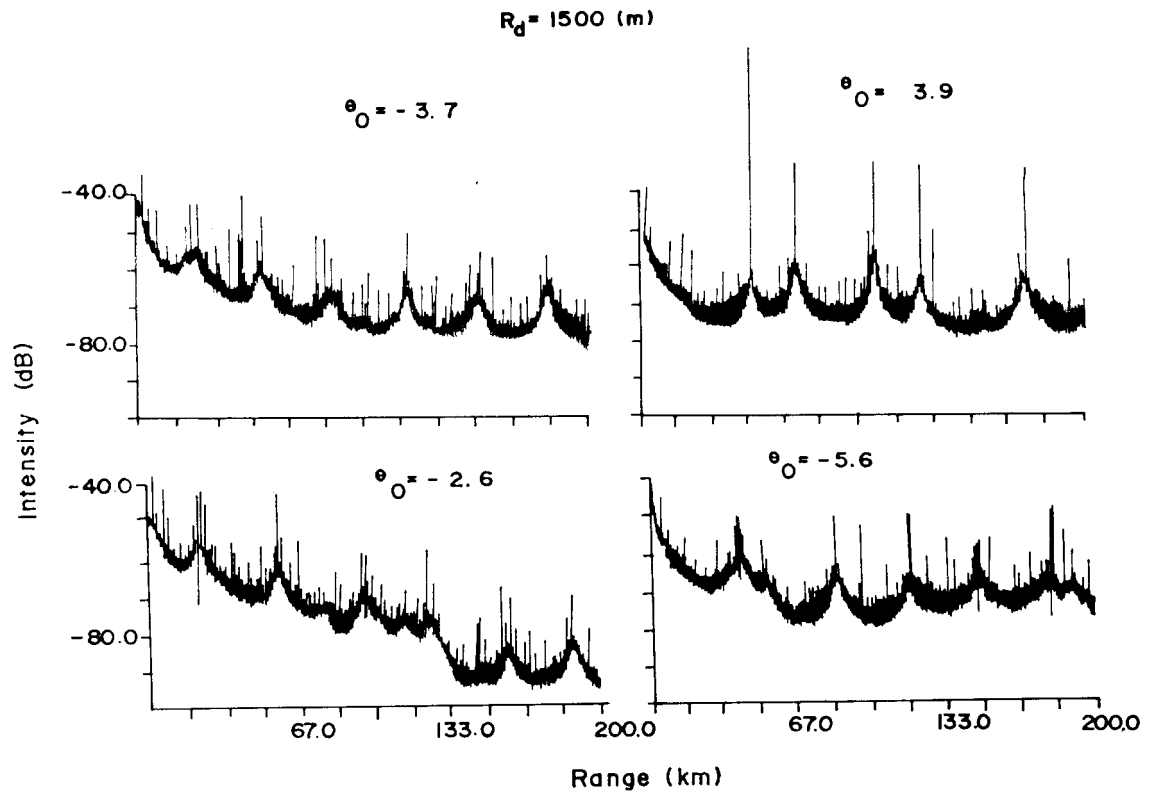


Fig. 10—Intensity changes for 200 km range during winter

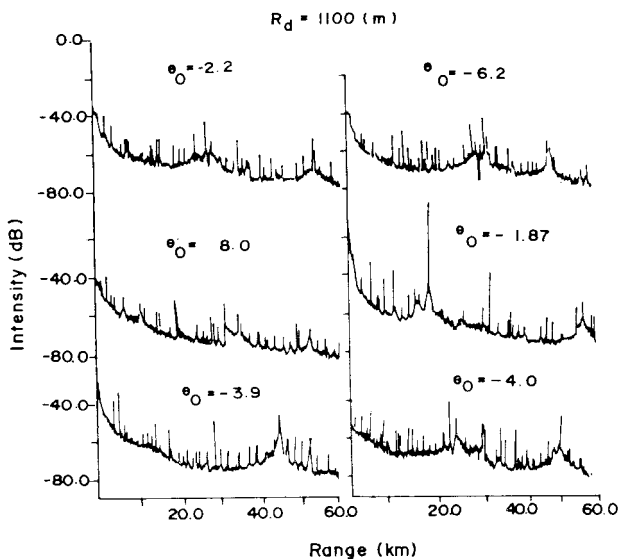


Fig. 11—Intensity changes for 60 km range during premonsoon

turn-under. Three maximum peaks are observed for 8° ray representing 3 turning points but with a shift in their spacings apart from the less sharp feature at the peaks. This is due to higher gradients of sound speed at the upper layer as compared to the lower.

Since eigen rays for different source-receiver configurations vary irrespective of seasons, to get a physical insight into the problem numerical calculations were carried out for angles 3° and 6° for both the seasons. Plots of ray tracing and intensity vs range (Figs 13 and 14) show that the depth of scanning for winter is more compared to premonsoon season. Considerable rise in intensity, viz. -88.22 to -68.99 dB is noticed along 3° ray from winter to premonsoon. This is equivalent to -19.23 dB ($1.1454 \times 10^{-7} \text{ W M}^{-2}$). For the ray with 6° emergence angle, the rise is about -10 dB (-84.66 to -74.96 dB) which is equivalent to $3.0627 \times 10^{-8} \text{ W M}^{-2}$.

Fig. 15 shows the time sequence of simulated arrival times of eigen rays at the receivers over different ranges and their intensity. The left side of the figure presents intensities cumulated with phase shift for different frequencies. Travel times at the receivers are obtained through total average speed along the paths whereas intensities are obtained based on the localized ray-tube radius. The flight of arrival time and intensity are independent parameters.

In principle, unlike medical tomography which is based on measurements of the absorption of X-rays or ultrasonic waves, acoustic tomography of the

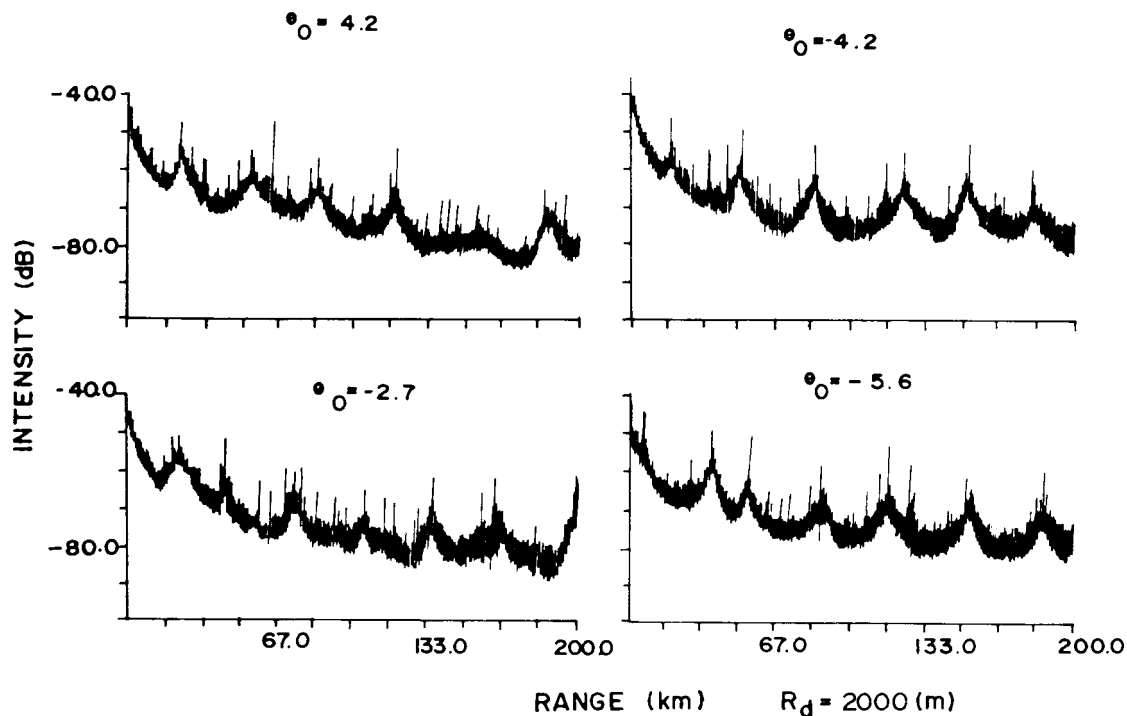


Fig. 12—Intensity changes for 200 km range during premonsoon

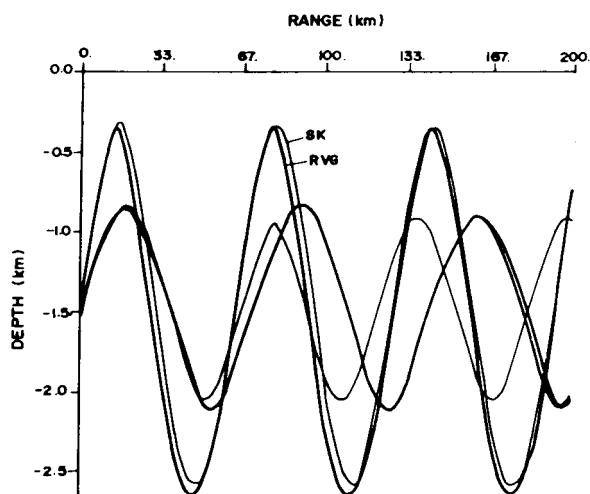


Fig. 13—Ray tube for 6° and 3° emergence angle

ocean presumes measurement of temporal delays of acoustic signals. These delays can be obtained through computer simulation of media characteristics and the field measurements. This will be the input to obtain anomalies of sound speed through the use of generalized inverse operators.

This article outlines information about resolvable travel times and the signal strengths. The flight time during winter is longer and true for all source-receiver configurations. The seasonal mesoscale variability could be examined through arrival sequence

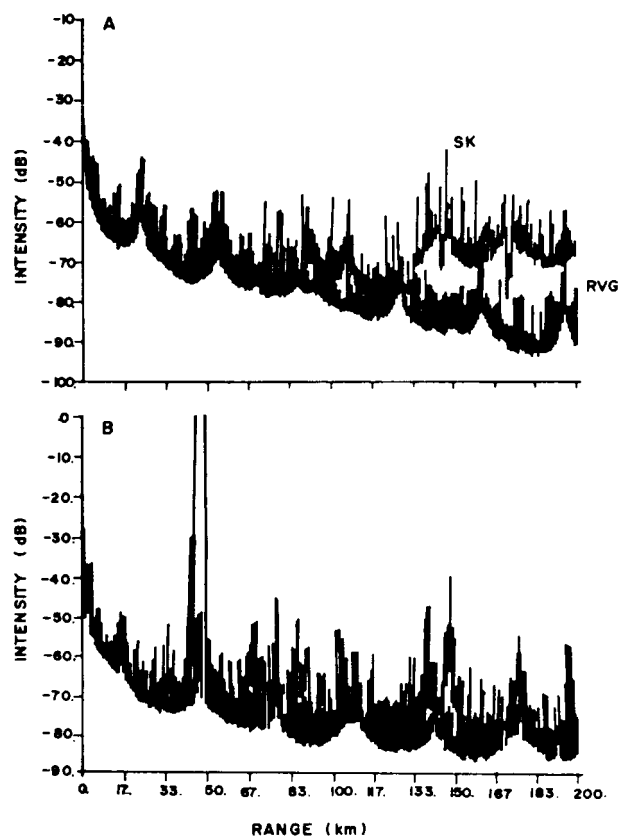


Fig. 14—Intensity changes between seasons for 200 km range with (A) 3° and (B) 6° angles

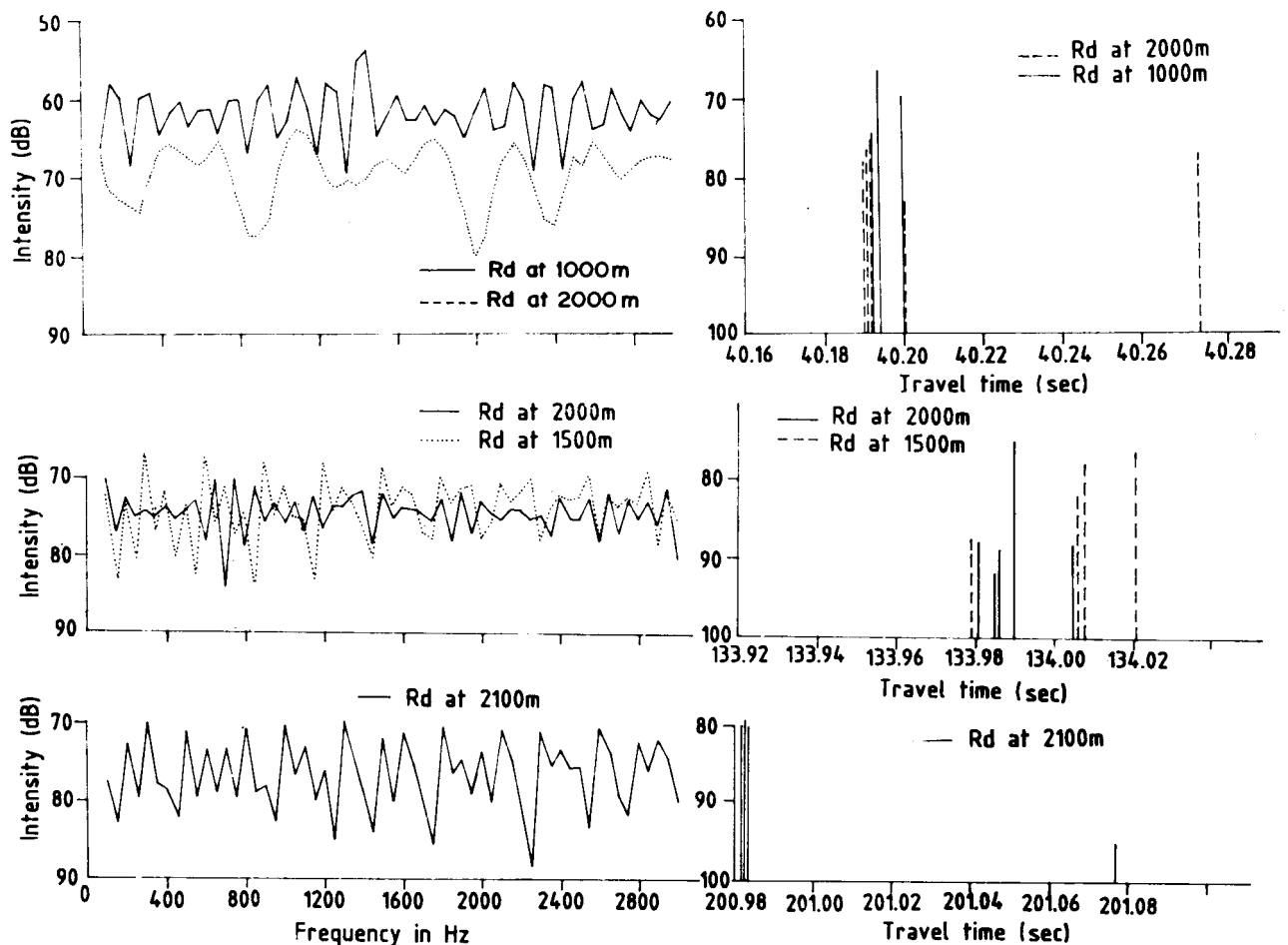


Fig. 15—Arrival times and intensity of the eigen rays at the receiver for different source-receiver configurations and for different ranges

of the eigen rays at the receivers which are resolvable in both seasons (approximately 197 millisecond). Travel time variance calculated varies from 0.01269×10^{-2} to 0.3338×10^{-2} except for one S-R combination Fig. 7(a) where the variance of order 10^{-3} is poorly resolvable even if the intensity levels are more for different frequencies and thus are not favourable for keeping the source at 1500 m and receiver at 1000 m in the actual field experiment.

The eigen rays and source-receiver configurations are not independent of seasons. The travel time along the path with typical launch angle for both seasons was examined considering 6° ray for premonsoon. This study shows that the ray reached the receiver at 200 km range in 143.17 sec with an average speed of $1495.18 \text{ m} \cdot \text{sec}^{-1}$ while for the winter, the ray has taken 134.06755 sec with an average speed of $1496.462 \text{ m} \cdot \text{sec}^{-1}$.

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