

# Wave Height & Period Distribution Off the Southwest Coast of India

M BABA & C M HARISH

Centre for Earth Science Studies, Regional Centre, Cochin 682018

Received 28 March 1984; revised received 22 October 1984

Wave-by-wave analysis is conducted on typical wave records collected at 4 locations. Distributions of wave heights and periods are investigated against some theoretical distribution functions. The shallow water wave heights in general may be explained by Gluhovski distribution, which takes the form of Rayleigh for relative wave heights ( $\bar{H}/d$ ) equal to zero. For  $\bar{H}/d$  approaching zero, which is observed in the case of postmonsoon swell dominated wave climate, wave height distribution follows Rayleigh. Rayleigh distribution for the square of the wave periods fits the swell wave conditions than that of the sea. The present data do not conform to the joint distribution suggested by Gluhovski or Bretschneider as considerable correlation, with values of  $\gamma(H, T)$  ranging from 0.06 to 0.69, is observed between heights and periods. Some of these observations are found to be extendable also to other areas in the west coast of India.

Several locations in Kerala coast (600 km long) are undergoing severe erosion. In order to combat this problem, various coastal engineering measures are being contemplated and the need for wave data, required for design purposes, is felt. Hence a programme has been undertaken to study the wave climate along this coast by direct measurements and also by theoretical methods.

Studies on the distribution characteristics of wave parameters for the west coast of India covering both rough as well as fair weather season are limited<sup>1-3</sup>. Similar studies are of particular importance also in view of the recommendations of the International Commission for the study of waves<sup>4</sup>. In this paper distribution of wave heights and periods with special reference to Kerala coast is discussed.

## Methods

Wave records were collected at 3 h intervals along 4 locations (Fig. 1) of Kerala coast using pressure-type recorders<sup>5,6</sup>. Details regarding the recording sites, transducer installation and the records used for analysis from each location are given in Table 1.

Thirty seven records representing all months of 1981 were selected for detailed analysis. Samples were selected in such a way that they represent middle of the month and correspond to 1200 hrs, wherever possible. Wave-by-wave zero-upcrossing analysis for heights and periods were done on 15 min records. On an average, 100 waves were obtained from each record. For pressure attenuation the following formula was used:

$$H = nH' \frac{\cosh\left(\frac{2\pi d}{L}\right)}{\cosh\left[\frac{2\pi d}{L}\left(1 - \frac{Z}{d}\right)\right]} \quad \dots (1)$$

where  $H$  is corrected wave height;  $H'$ , uncorrected wave height as obtained from record;  $Z$ , height of water column above transducer;  $d$ , water depth at transducer;  $L$ , wave length; and  $n$ , instrumentation factor, taken as 1.25 following Dattatri<sup>1</sup>.

The height and period data obtained manually from the paper chart records were processed in computer for wave height attenuation and for corresponding frequency distributions. As abnormal values were obtained for wave height when periods are small, no attenuation correction was applied for waves with a period < 4 sec.

## Theoretical Considerations

Assuming a narrow band wave spectrum, Longuet-Higgins<sup>7</sup> recommended the use of Rayleigh distribution for deep water wave heights. According to

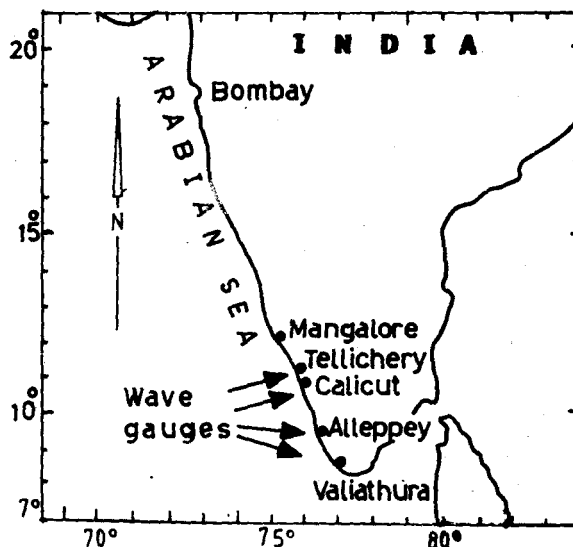


Fig. 1—Location map

Table 1—Details of Recording Sites

| Location    | Type of bottom sediments | Bottom slope | Depth of recording site (m) | Depth of transducer (m) | Serial Nos of records selected for analysis |
|-------------|--------------------------|--------------|-----------------------------|-------------------------|---------------------------------------------|
| Valiathura  | Sand                     | 0.07         | 5.5                         | 3.0                     | 1-11                                        |
| Alleppey    | Mud                      | 0.0025       | 5.5                         | 3.0                     | 12-23                                       |
| Calicut     | Mud                      | 0.0025       | 3.0                         | 1.5                     | 24-30                                       |
| Tellicherry | Mud                      | 0.0025       | 3.0                         | 1.5                     | 31-37                                       |

this, the probability of a wave height,  $H$  exceeding a given value may be written in terms of mean wave height  $\bar{H}$  as

$$P(H) = \exp \left[ -\frac{\pi}{4} \left( \frac{H}{\bar{H}} \right)^2 \right] \quad \dots (2)$$

where  $H$  is the individual wave height.

Further studies<sup>8,9</sup> have shown that the above distribution is applicable well beyond the narrow band assumption. Many workers<sup>1,10,11</sup> ascribe Rayleigh distribution for coastal wave heights. But it has been observed<sup>1,10,11</sup> that for the coastal wave data on many occasions, the heights of larger waves are over-estimated when Rayleigh distribution function is used. Gluhovski<sup>12</sup> after analysing large number of wave records from different coasts found that Rayleigh distribution may be modified by a depth factor  $h^* = \frac{\bar{H}}{d}$ ,

where  $d$  is the water depth. He suggested the following distribution function for wave heights at a given depth  $d$  for shallow water conditions

$$P(H) = \exp \left[ -\frac{\pi}{4 \left( 1 + \frac{h^*}{\sqrt{2\pi}} \right)} \left( \frac{H}{\bar{H}} \right)^{\frac{2}{1-h^*}} \right] \quad \dots (3)$$

This function assumes the form of Eq. 2 for  $h^* = 0$ , i.e. for deep water conditions. According to Eq. 3 for  $h^* = 0.5$ ,  $\frac{H_{\max}}{\bar{H}} = 1.63$  or  $H_{\max} = 1.63 \bar{H}$ . Then  $\frac{H_{\max}}{d} = 1.63$

$\bar{H}/d = 1.63 \times 0.5 \approx 0.8$ . For such a value of  $\frac{\bar{H}}{d}$  the wave breaking commences. Hence  $h^* = 0.5$  indicates the breaker zone, where Eq. 3 transforms to

$$P(H) = \exp \left[ -\frac{\pi}{4.8} \left( \frac{H}{\bar{H}} \right)^4 \right] \quad \dots (4)$$

The suitability of this distribution function (Eq. 3) has been established for bottom slopes of 0.1 to 0.001 comprising sand or gravel<sup>12</sup>.

For wave periods, on the basis of wave data from various deep and shallow water locations Krilov<sup>13</sup> and, later, based on both field and laboratory data Bretschneider<sup>14</sup> have recommended the use of

Rayleigh distribution for square of the wave periods, the cumulative form of which is expressed as:

$$P(T) = \exp \left[ -0.675 \left( \frac{T}{\bar{T}} \right)^4 \right] \quad \dots (5)$$

where  $T$  = individual wave period and  $\bar{T}$  = mean wave period. The distribution suggested by Longuet-Higgins<sup>15</sup> for wave periods is not examined here as no spectral analysis on the wave data could be done in the present study.

Bretschneider<sup>16</sup>, assuming Rayleigh distribution for wave heights and also for square of the wave periods and zero correlation between them suggested the following joint distribution function:

$$P(H, T) = P(H)P(T) = \exp \left\{ -\frac{\pi}{2} \left[ \left( \frac{H}{\bar{H}} \right)^2 + 0.417 \left( \frac{T}{\bar{T}} \right)^4 \right] \right\} \quad \dots (6)$$

Another function for the joint probability of  $H$  and  $T$  has been given by Gluhovski<sup>12</sup>:

$$P(H, T) = \exp \left\{ -\frac{\pi}{4} \left[ \frac{1}{1 + 0.4h^*} \left( \frac{H}{\bar{H}} \right)^{\frac{2}{1-h^*}} + 0.833 \left( \frac{T}{\bar{T}} \right)^4 \right] \right\} \quad \dots (7)$$

## Results and Discussion

**Wave height distribution**—Preliminary analysis of data shows that the mean wave heights range from a value as low as 0.09 m at Tellicherry to 2.17 m at Valiathura for the whole period (Table 2). Highest values at all stations are observed during southwest monsoon season (June to September). The ratio of maximum to mean wave height  $\left( \frac{H_{\max}}{\bar{H}} \right)$  ranges from 1.73 to 3.08 (Table 2). According to Rayleigh Distribution (Eq. 2) this ratio for a wave with percentage exceedance equal to 1 is 2.42. The observed values are lower than this value in majority of the cases (it may be remembered that about 100 waves were considered from each record and hence the maximum

Table 2—Details of Wave Records Analysed with Results

| Sl  | Date    |    | $\bar{H}$ | $\bar{T}$ | $\frac{H_{\max}}{\bar{H}}$ | $h^*$ | $\chi(H, T)$ | $\chi(H, T^2)$ |
|-----|---------|----|-----------|-----------|----------------------------|-------|--------------|----------------|
| No. |         |    | (m)       | (sec)     |                            |       |              |                |
| 1   | 15 I    | 81 | 0.46      | 8.54      | 1.77                       | 0.08  | 0.56         | 0.36           |
| 2   | 21 II   | 81 | 0.48      | 9.72      | 1.95                       | 0.09  | 0.59         | 0.47           |
| 3   | 16 III  | 81 | 0.56      | 9.45      | 2.51                       | 0.10  | 0.61         | 0.54           |
| 4   | 15 IV   | 81 | 0.96      | 8.77      | 2.82                       | 0.18  | 0.64         | 0.56           |
| 5   | 15 V    | 81 | 0.71      | 6.30      | 3.08                       | 0.13  | 0.52         | 0.46           |
| 6   | 12 VI   | 81 | 2.17      | 8.23      | 2.39                       | 0.40  | 0.06         | 0.06           |
| 7   | 28 VII  | 81 | 1.92      | 6.87      | 2.08                       | 0.35  | 0.47         | 0.37           |
| 8   | 14 IX   | 81 | 1.77      | 9.58      | 2.43                       | 0.32  | 0.26         | 0.10           |
| 9   | 15 X    | 81 | 0.88      | 11.49     | 2.34                       | 0.16  | 0.48         | 0.38           |
| 10  | 16 XI   | 81 | 0.67      | 9.66      | 2.36                       | 0.25  | 0.44         | 0.37           |
| 11  | 11 XII  | 81 | 0.76      | 10.56     | 2.31                       | 0.14  | 0.33         | 0.23           |
| 12  | 15 I    | 81 | 0.25      | 8.55      | 2.29                       | 0.05  | 0.09         | 0.03           |
| 13  | 16 II   | 81 | 0.35      | 8.24      | 2.08                       | 0.06  | 0.43         | 0.28           |
| 14  | 16 III  | 81 | 0.38      | 10.61     | 2.05                       | 0.07  | 0.46         | 0.37           |
| 15  | 15 IV   | 81 | 0.55      | 10.28     | 1.89                       | 0.10  | 0.69         | 0.65           |
| 16  | 15 V    | 81 | 0.42      | 5.85      | 2.93                       | 0.08  | 0.57         | 0.47           |
| 17  | 15 VI   | 81 | 1.66      | 8.78      | 1.80                       | 0.30  | 0.38         | 0.30           |
| 18  | 15 VII  | 81 | 0.60      | 8.81      | 2.49                       | 0.11  | 0.35         | 0.25           |
| 19  | 10 VIII | 81 | 1.50      | 8.84      | 2.33                       | 0.27  | 0.15         | 0.01           |
| 20  | 15 IX   | 81 | 0.37      | 7.46      | 2.25                       | 0.07  | 0.47         | 0.36           |
| 21  | 15 X    | 81 | 0.26      | 13.30     | 1.81                       | 0.05  | 0.36         | 0.27           |
| 22  | 16 XI   | 81 | 0.17      | 10.90     | 2.69                       | 0.03  | 0.56         | 0.61           |
| 23  | 15 XII  | 81 | 0.21      | 11.03     | 2.12                       | 0.04  | 0.55         | 0.47           |
| 24  | 11 V    | 81 | 0.52      | 13.64     | 1.97                       | 0.17  | 0.42         | 0.33           |
| 25  | 15 VI   | 81 | 1.36      | 10.13     | 1.98                       | 0.45  | 0.14         | 0.07           |
| 26  | 15 VII  | 81 | 0.68      | 9.14      | 2.43                       | 0.23  | 0.43         | 0.26           |
| 27  | 11 VIII | 81 | 0.48      | 9.41      | 2.73                       | 0.16  | 0.33         | 0.21           |
| 28  | 15 X    | 81 | 0.28      | 13.87     | 1.73                       | 0.09  | 0.14         | 0.16           |
| 29  | 16 XI   | 81 | 0.15      | 10.53     | 2.23                       | 0.05  | 0.42         | 0.28           |
| 30  | 15 XII  | 81 | 0.25      | 10.10     | 2.80                       | 0.08  | 0.46         | 0.46           |
| 31  | 16 VI   | 81 | 0.70      | 9.10      | 1.94                       | 0.23  | 0.28         | 0.19           |
| 32  | 15 VII  | 81 | 0.34      | 8.70      | 2.20                       | 0.11  | 0.54         | 0.35           |
| 33  | 13 VIII | 81 | 0.85      | 11.44     | 1.85                       | 0.28  | 0.66         | 0.55           |
| 34  | 15 IX   | 81 | 0.31      | 9.48      | 2.58                       | 0.10  | 0.52         | 0.41           |
| 35  | 05 X    | 81 | 0.09      | 14.73     | 2.26                       | 0.03  | 0.10         | 0.05           |
| 36  | 16 XI   | 81 | 0.25      | 10.88     | 2.46                       | 0.08  | 0.09         | 0.02           |
| 37  | 15 XII  | 81 | 0.19      | 10.56     | 2.16                       | 0.06  | 0.37         | 0.28           |

wave height may have a percentage exceedance of 1). This departure observed for highest waves are further examined for the entire distribution of wave heights of each set.

The wave height distribution for the entire data set is presented in Fig. 2. Rayleigh distribution is also shown against the data. It becomes now evident that majority of wave heights with probabilities higher than 40% accumulate on the right hand side while those with lower probabilities scatter mainly on the left hand side of Rayleigh distribution curve.

Various authors<sup>1,2,10,11</sup> attributed different reasons for similar departures observed in data from other coasts. While Dattatri<sup>1</sup>, Thompson<sup>10</sup> and John<sup>2</sup> described this to be due to the type of recorder, Deo and Narasimhan<sup>11</sup> and John<sup>2</sup> doubt it to be due to the presence of both swell and sea waves in the record. But Harris<sup>17</sup> observed that there is not much of variation

between the distribution obtained from subsurface and surface wave gauges and in Thompson<sup>10</sup> the departure from Rayleigh was seen in the results from different types of gauges. Further, Harris<sup>17</sup> reported that pressure records, as properly corrected, give a better measure of surface waves, than some surface gauges. The second view that the swell and sea waves react differently to Rayleigh distribution appears to be true with the present data.

In Fig. 2 the data points are grouped depending on the value of  $h^*$ . The Gluhovski distribution (Eq. 3) for different  $h^*$  values are also superimposed. In the first range of  $h^*$  (i.e. 0 to 0.049) there are only 5 records, that too with very small wave heights, whose measurements may have some inaccuracies leading to wide scatter of the points around the corresponding curve. However, it may be stated that they are satisfactorily approximated by Rayleigh distribution curve. It is to

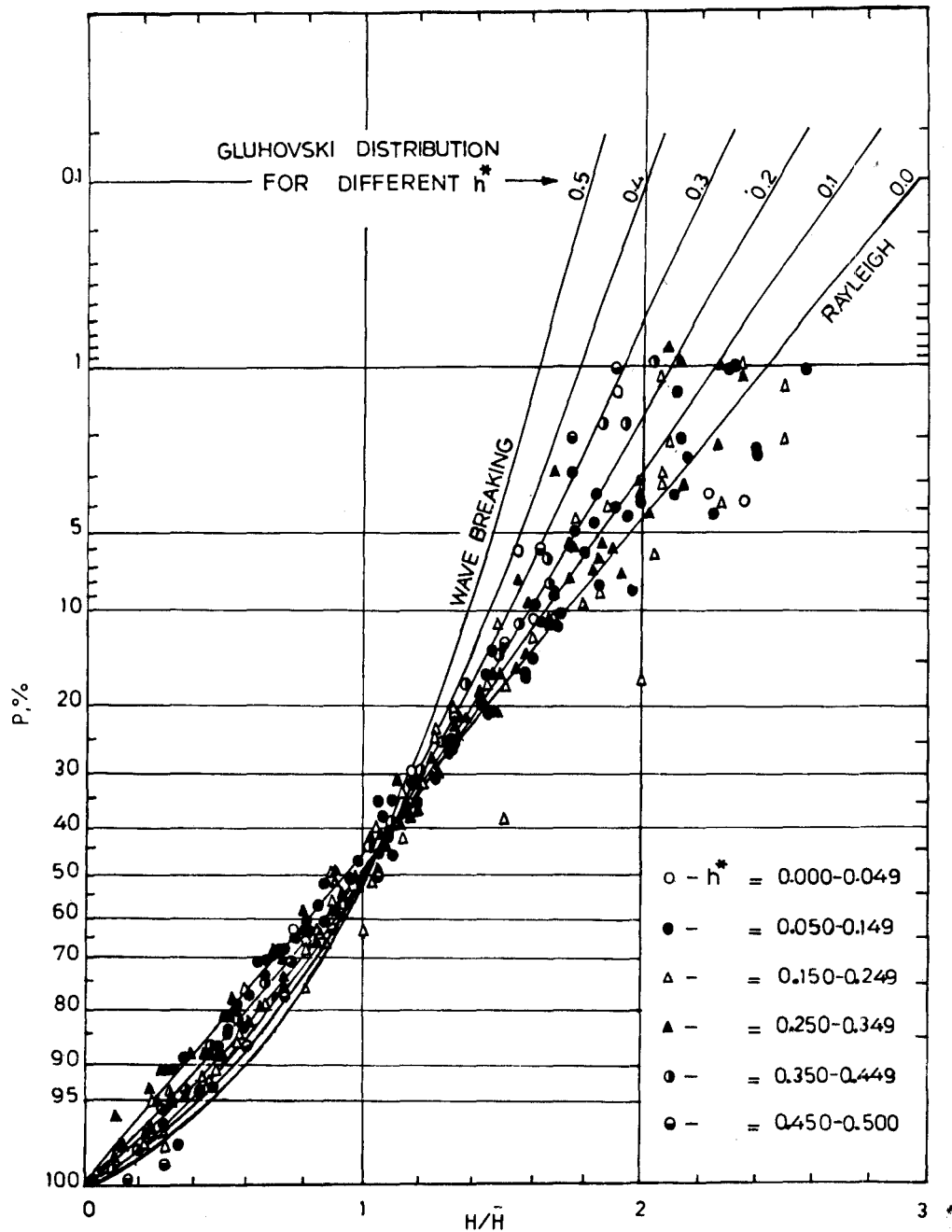


Fig. 2—Distribution of nondimensional wave heights ( $H/\bar{H}$ ) as a function of  $h^* = \bar{H}/d$

be remembered that the Gluhovski distribution takes the shape of Rayleigh distribution for  $h^* = 0$ . From the next range onwards a general shift of points, corresponding to probabilities  $< 40\%$ , to  $\frac{H}{\bar{H}}$  values lower than those predicted by Rayleigh distribution is seen. This shift is quite evident, especially for ranges of  $h^*$  exceeding 0.25. The wide scatter of points near the lowest probability area may be attributed to the fact that, since they are the largest waves, they may be prone to breaking or they are in a near-breaking condition, especially during peak monsoon, or they are

statistically ill-represented, as each data set contains only 1 or 2 of them. A departure of points to  $\frac{H}{\bar{H}}$  values higher than those predicted by Rayleigh is observed for probabilities  $> 40\%$ . Even though the Gluhovski distribution curves for different  $h^*$  values (Fig. 2) do not exactly fit the points in the corresponding ranges of  $h^*$ , a definite trend for the distribution, especially for higher values of  $h^*$ , to follow Gluhovski may be seen.

Note the evident systematic decrease of  $\frac{H}{\bar{H}}$

increasing  $h^*$  in the case of topmost points lying near  $P=1$  which correspond to maximum heights.

It may have to be remembered that the water depth in the present studies varied only between 3 and 5.5 m. Hence the high values of  $h^*$  are mainly attributed to the increase in mean wave height. The maximum departure is observed for higher values of  $h^*$  and is coupled with the high waves which are more prone to the shallow water effects like shoaling, breaking, etc. at the study site. Gluhovski<sup>12</sup> had suggested the distribution (Eq. 3) specifically for such shallow water situations. Another observation made in the present study is that the nature of sea bottom (Table 1) does not have any influence on the wave height distributions, as they are more or less similar at all locations other conditions remaining the same. Hence for the shallow water wave heights even though Rayleigh distribution holds good for lower values of  $h^*$ , Gluhovski distribution function appears to be more suitable especially for higher  $h^*$  values.

A subsequent examination of 2 earlier wave data sets obtained from Mangalore<sup>1</sup> at a depth of 7.5 m and from Bombay<sup>11</sup> at a depth of 30 m along the west coast of India revealed that the wave height distributions are closer to that of Gluhovski (Fig. 3). It is also to be mentioned that in the case of Bombay data, which are obtained from a wave rider buoy (surface gauge), the departure from Rayleigh has been quite evident. Both the data sets reproduced here correspond to the peak monsoon wave climate and the corresponding  $h^*$  values exceed zero. Hence the wave heights follow the Gluhovski distribution.

Careful examination of Table 2 shows that the long period swells ( $\bar{T} > 10$  sec), occurring mainly during post-monsoon fair weather season, the value of  $h^*$  is generally  $< 0.1$ . Earlier it is found that the wave height distributions for  $h^*$  values nearer to zero are fairly explained by Rayleigh. This coincidence may be due to the fact that the Rayleigh distribution bases on the narrow band assumption, which is valid for swell waves. However, the finding that the Rayleigh distribution explains the heights of swell waves, may be considered only preliminary, taking into account the limitations in this part of the data as mentioned earlier. In view of this, the doubt that Rayleigh distribution is influenced by sea and swell<sup>2,11</sup> may be relevant.

*Wave period distributions*—Cumulative distribution of the wave periods are presented in Fig. 4. Rayleigh distribution function (Eq. 5) is also presented in the same diagram. Eventhough for probabilities  $> 30\%$  the above function seems to satisfactorily explain the distribution of the periods, for lesser probabilities the data seem to deviate from the theoretical curve. Dattatri *et al.*<sup>9</sup> and Deo and Narasimhan<sup>11</sup> also based on the west coast data reached the conclusion that this distribution function is not satisfactory.

In order to test whether the function can explain the sea or swell waves independently, the data were divided into 2 groups depending on whether the mean period is  $<$  or  $> 10$  sec, which is assumed to arbitrarily represent the sea or swell waves respectively. It is found that those periods, which are dominated by swell waves ( $\bar{T} > 10$  sec) are satisfactorily explained by Rayleigh function. At the same time the other set of

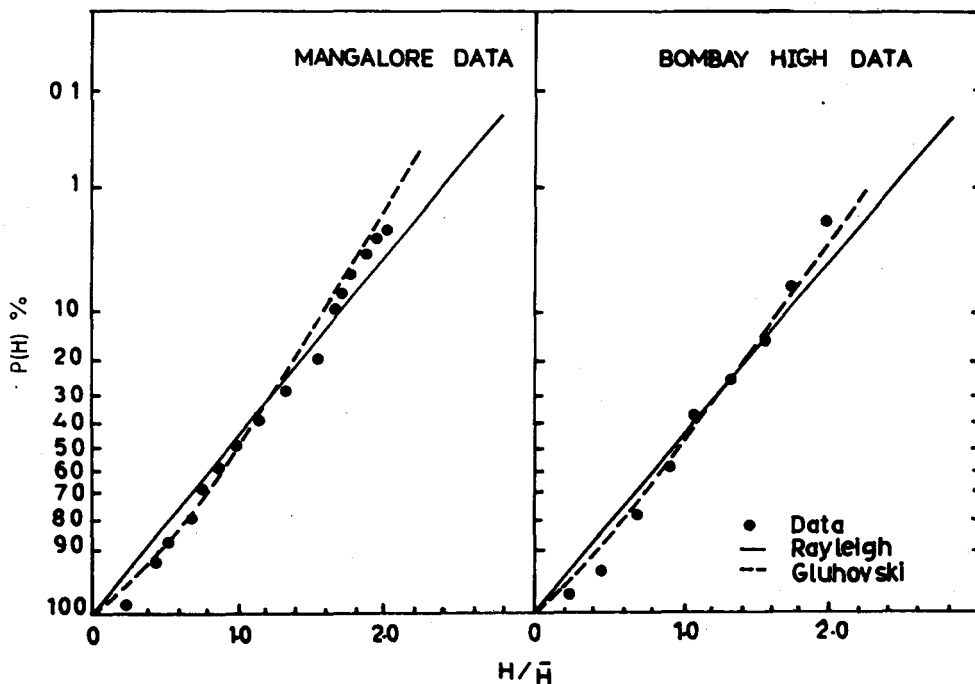
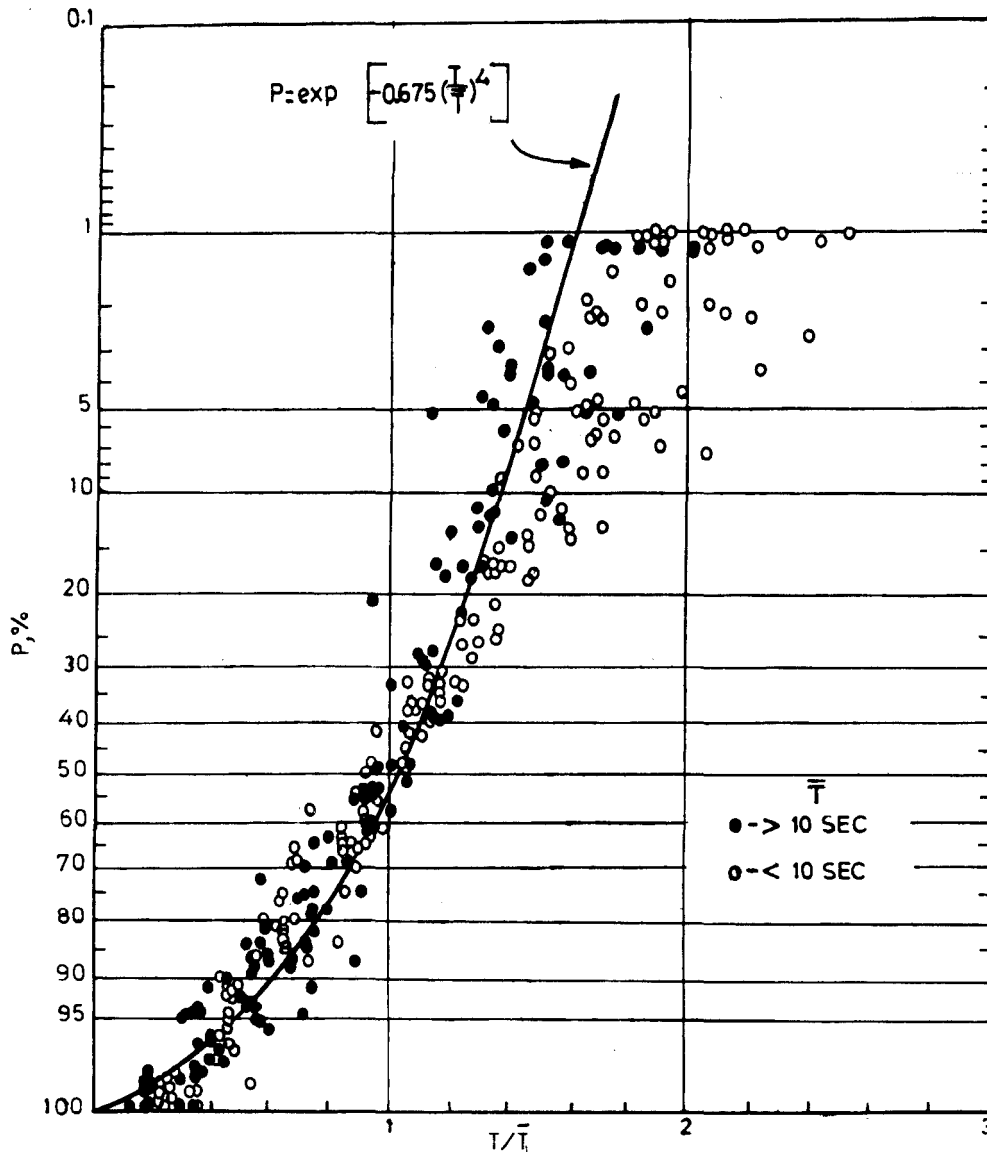


Fig. 3—Distribution of nondimensional wave heights at Mangalore (Dattatri<sup>1</sup>) and Bombay (Deo & Narasimhan<sup>11</sup>)

Fig. 4—Distribution of nondimensional wave periods ( $T/\bar{T}$ )

data, with mean periods  $< 10$  sec is clearly separated from the swell waves. Similar observations with this function are not seen in the literature. One of the possible reasons for such a behaviour of Rayleigh function for the square of the wave periods may be that this theoretical distribution has a strong base on the narrow bandedness of the wave spectra. As pointed out in the earlier section narrow band spectra may be expected during postmonsoon season due to the approach of long period swells. And hence the function is more closer to swell than sea waves.

**Joint distribution of wave heights and periods**—Joint distribution of wave heights and periods exhibit quite large variations. In general it is found that smaller waves are spread to a wide range of periods and the higher waves to a narrow band (Fig. 5). There is a general tendency for an increase in wave heights with

periods and it continues only up to intermediate periods. This may be due to the fact that higher waves become unstable (and consequently break) with decreasing periods. The fetch, duration and depth conditions are insufficient for the formation and propagation of higher waves with longer periods.

Thus a certain dependence of wave heights on wave periods could be observed in the data. But for the theoretical distributions of Bretschneider (Eq. 6) and Gluhovski (Eq. 7) the primary condition is a zero correlation between the 2 variables. The present data show correlation co-efficients between  $H$  and  $T$  ranging from 0.06 to 0.69. The correlation between  $H$  and  $T^2$  was consistently weaker. The wider range observed in the correlation coefficients may be due to the fact that the data represent different seasons and also different locations. Dattatri *et al.*<sup>9</sup> found a slightly

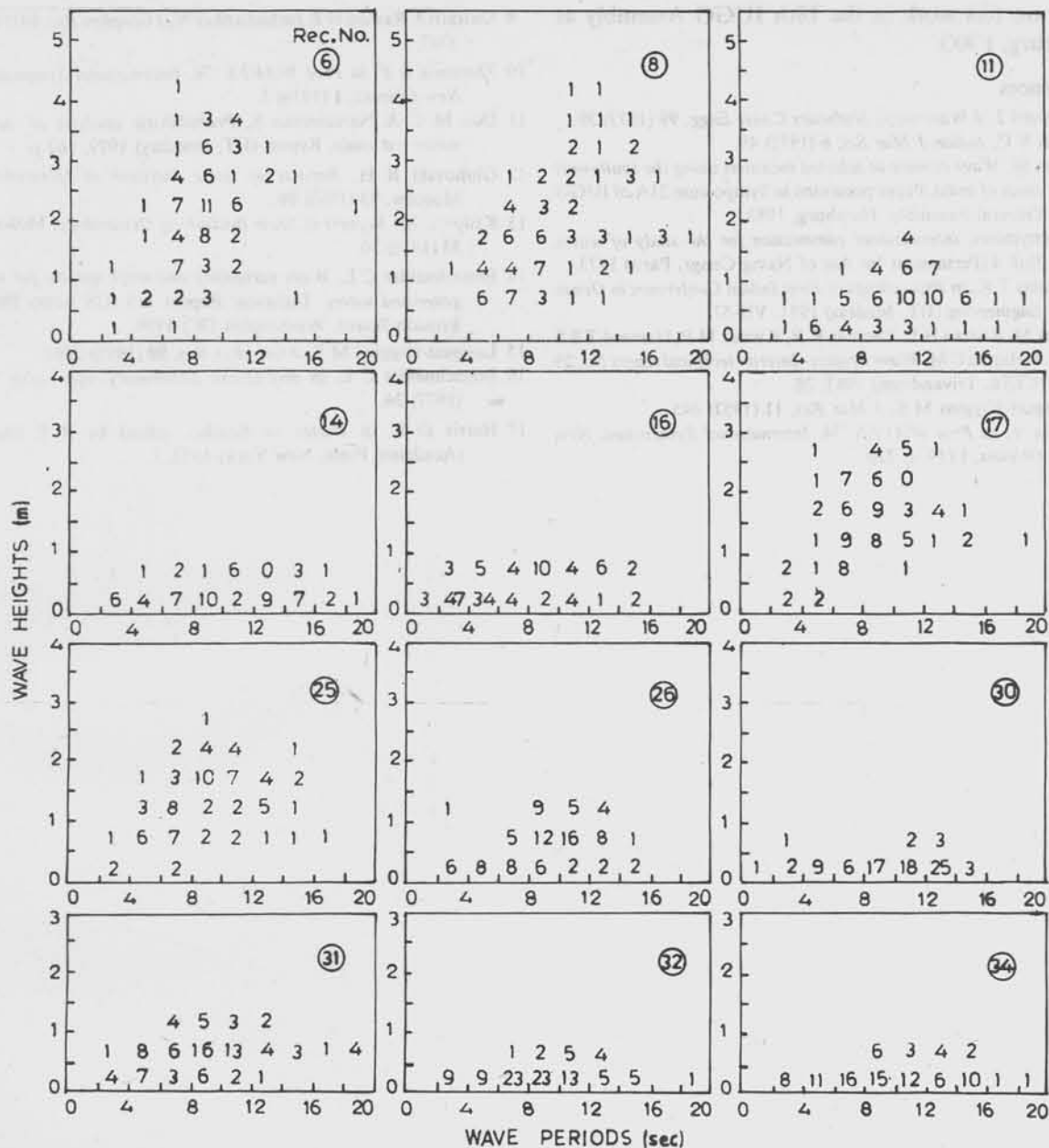


Fig. 5—Joint distribution of wave heights and periods for typical records

higher correlation for the monsoon data from Mangalore.

No dependence on the mean wave height or mean wave period could be observed for the correlation coefficients. However, it is found that the most frequent period, whatever may be the wave height, is practically equal to  $\bar{T}$ , which has also been reported by Gluhovski<sup>12</sup> in the case of both deep and shallow water data. As reported by many investigators<sup>9,11,12</sup> the highest waves do not correspond to the largest, but to the intermediate periods.

Hence it is concluded that, even though Gluhovski distribution is satisfactory for shallow water wave

heights in general and Rayleigh distribution for the swell wave periods and heights, the theoretical joint distributions of Bretschneider and Gluhovski do not find to fit the data. The joint distribution functions may have to be modified incorporating the high correlation observed between  $H$  and  $T$  to explain the actual field conditions.

#### Acknowledgement

The authors are grateful to Dr H K Gupta, Director and Dr T K Mallik for facilities and encouragement. International Association for Physical Sciences of the Oceans sponsored the presentation of a preliminary

paper on this work in the 18th IUGG Assembly at Hamburg, FRG.

## References

- 1 Dattatri J, *J Waterways Harbours Coast Engg*, **99** (1973) 39.
- 2 John V C, *Indian J Mar Sci*, **6** (1977) 49.
- 3 Baba M, *Wave climate at selected locations along the south-west coast of India*, Paper presented in Symposium 21A of IUGG General Assembly, Hamburg, 1983.
- 4 Anonymous, *International commission for the study of waves*, Bull 4 (Permanent Int Ass of Navig Congr, Paris) 1973.
- 5 Sivadas T K, in *Proceedings of First Indian Conference in Ocean Engineering* (IIT, Madras) 1981, VII-57.
- 6 Baba M, Kurian N P, Thomas K V, Kumar M P, Hameed T S S & Harish C M, *Wave Project - Interim technical report No. 29* (CESS, Trivandrum) 1983, 28.
- 7 Longuet-Higgins M S, *J Mar Res*, **11** (1952) 245.
- 8 Goda Y, in *Proc WAVES '74, International Symposium, New Orleans*, **1** (1974) 320.
- 9 Dattatri J, Raman H & Jothishankar N, *J Geophys Res*, **84** (1979) 3767.
- 10 Thomson E F, in *Proc WAVES '74, International Symposium, New Orleans*, **1** (1974) 1.
- 11 Deo M C & Narasimhan S, *Probabilistic analysis of ocean waves—a study*, Report (IIT, Bombay) 1979, 162 p.
- 12 Gluhovski B H, *Reports of State Institute of Oceanology, Moscow*, **93** (1963) 98.
- 13 Krilov U M, *Reports of State Institute of Oceanology, Moscow*, **33** (1956) 30.
- 14 Bretschneider C L, *Wave variability and wave spectra for wind generated waves*. Technical Report 118 (US Army Beach Erosion Board, Washington DC) 1959.
- 15 Longuet-Higgins M S, *J Geophys Res*, **80** (1975) 2688.
- 16 Bretschneider C L, in *Bull (Look Laboratory, Honolulu)*, **7**(1) (1977) 24.
- 17 Harris D L, in *Waves on Beaches*, edited by R E Meyer (Academic Press, New York) 1972, 1.