

A new research approach of electromagnetic theory and its applications

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A new research approach of electromagnetic theory is presented. The relations of electromagnetic parameters in the scale-transformation coordinate system and in the original coordinate system are presented. The applied fields of the scale-transformation theory are introduced. The result obtained with the scale-transformation theory is in agreement with that in the literature. The method presented in this paper has the characteristics of briefness and is of convenience in engineering.

Keywords: Scale-transformation, Electromagnetic parameter, Electromagnetic scattering.

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1 Introduction

There exist the deterministic transformation relations among the rectangular coordinate system, spherical coordinate system and cylindrical coordinate system, thereby the Maxwell equations are expressed completely and compactly. The electromagnetic topics related to the targets such as sphere and cylinder are discussed in detail by many works¹⁻⁸ and their analytical solutions are presented, which bring much convenience to the theoretical study and engineering. The relativity developed by Albert Einstein in 1909 is of epoch-making significance, which provides a limitless vital force for acquiring the solutions of electromagnetic systems moving in high velocity. It is doubtless that the brief analytical solutions of scattering field, etc. for the targets of ellipsoids and elliptical cylinders are of great importance in practical applications. Scale-analyses both for coordinate and for electromagnetic fields offer a novel approach to this kind of problem. In this paper the idea of coordinate scales analyses is first carried out, and based on that idea the transformation relations for the parameters ϵ and μ and electromagnetic field are briefly introduced. It is proved that form of Maxwell equations is kept unchanged. The applications of this new ways are also introduced briefly. Finally, the validity of the new approach is

demonstrated by the agreement of the results obtained by us and that in the literatures.

2 The idea of scales transformation

2.1 Backdrop for scales transformation

It is well known that the electromagnetic topics related to the targets such as sphere and cylinder are discussed in detail by Mie theory and the available literatures, whereas the electromagnetic problems related to ellipsoids and elliptical cylinders are not investigated in depth. These problems bring much inconvenience for application. If an ellipsoid and a elliptical cylinder are changed into a sphere and a cylinder, respectively, we can use their results to investigate the scattering characteristics of the ellipsoidal targets. We, therefore, make the scale transformation.

It is assumed that there are two coordinate systems Σ and Σ' . The symbols x, y, z and r', θ, φ are, respectively, used to denote the rectangular coordinates and spherical coordinates in the system Σ . Similarly, x', y', z' and r', θ', φ' are the coordinates in the system Σ' . The corresponding coordinate axes in the two systems are parallel with each other. These symbols a, b, c are dimensionless and called scale factors.

Let

$$x' = \frac{x}{a}, \quad y' = \frac{y}{b}, \quad z' = \frac{z}{c}, \quad \dots (1)$$

namely

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = T \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \dots (2)$$

where

$$T = \begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{bmatrix} \quad \dots (3)$$

Equation (2) indicates the relation between the two systems. The summation of the three terms in the expression (1) is an equation of a sphere with radius of 1m. It is concluded from above that for an arbitrary ellipsoidal particle or an ellipsoidal cylinder, any one of the factors a , b and c equal to 1 is changed into a sphere or a cylinder with a radius of 1m, which, thus, makes it possible to investigate the scattering of electromagnetic wave by an ellipsoidal particle in the way of studying the scattering of electromagnetic wave by a spherical particle. Any physical quantity is measured with the three units, metre, kilogram and second. Now the fathom is changed, so the other physical quantities must be measured renewedly.

2.2 Scales transformation of electromagnetic field

The trigonometric functions are necessary in the expression of scattering field. It is obtained by using Eq. (1) and relation between rectangular coordinate and spherical coordinate. For the sake of simplicity, parts of results are presented here without showing in detail.

The relation of scale transformation of differential functor is first derived as

$$\nabla = T \nabla' \quad \dots (4)$$

The relations of scale transformation of the other parameters are derived as

$$\rho'(x', y', z') = abc \rho(x, y, z).$$

The surface-charge density is defined as

$$\begin{bmatrix} \sigma'_x \\ \sigma'_y \\ \sigma'_z \end{bmatrix} = M^{-1} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix} \quad \dots (5)$$

where

$$M^{-1} = \begin{bmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{bmatrix}$$

The transformation relation about electric current density (ECD) is

$$J' = M^{-1} \cdot J \quad \dots (6)$$

The scale transformation of electromagnetic field is made, keeping the form of Maxwell equations unchanged in different coordinate systems, as follows

$$D' = M^{-1} D$$

$$B' = M^{-1} B$$

$$E' = T^{-1} E$$

$$H' = T^{-1} H$$

$$\varepsilon'_0 = \varepsilon_0 M^{-1} T$$

$$\mu'_0 = \mu_0 M^{-1} T$$

$$k' = T^{-1} k \quad \dots (7)$$

The boundary conditions of electromagnetic field are of great importance in theoretical study and engineering. Problems such as solving the equation of wave propagation, scattering characteristics of a target relative to electromagnetic field are difficult to be investigated if there exists no boundary conditions. Among many boundary conditions, the three kinds of conditions are often used, namely, conditions of electromagnetic field, conditions of electric current density and electric charge density and conditions of propagating vector. These conditions are not changed due to the fact that the Maxwell equations have the same formation and we use the same derivation in the two systems.

3 Applications

3.1 Process of usage for scale transformation

Taking an ellipsoidal target, its shape parameters are shown in Fig. 1. The following are the processes of usage for scale-transformation.

(i) A plane wave propagating along z -direction and polarizing along x -direction and the target in the original of coordinate system Σ are transformed into the system Σ' , so we find a new incident with the same propagating direction and the same polarization and a sphere in the original.

(ii) The scattering fields of the transformed target are obtained in the system Σ' with the scattering fields about a sphere^{6,7}.

(iii) The scattering fields of transformed target is again transformed into the original system Σ and now the scattering field of an ellipsoid is obtained.

3.2 Effect of application of transformation

The scale transformation of electromagnetic theory is first presented. Its effect of application, therefore, must be tested. This is demonstrated by both theoretical accuracy and the using accuracy. Speaking theoretically all the results in system Σ' are equal to those in system Σ , when scale factors a , b and c are all of 1, which has demonstrated the validity of the theory we present in this paper. The result in Ref. (7) and the simulation result of scale transformation are compared in Fig. 2, which indicates the change of the squared scattering field with the distance. This difference in Fig. 2 is about 10^{-18} , which should produce no effects on practical applications. The Rayleigh scattering on the other hand is a kind of good approximation at very low frequencies.

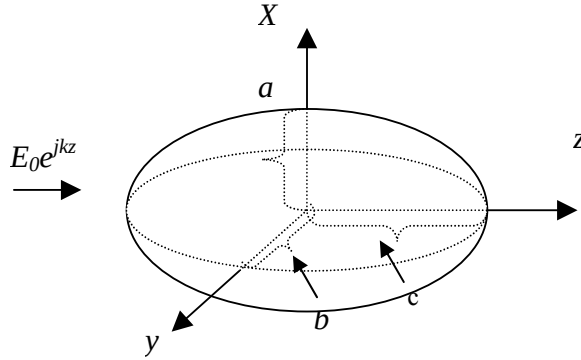


Fig. 1—An EM wave irradiates an ellipsoid

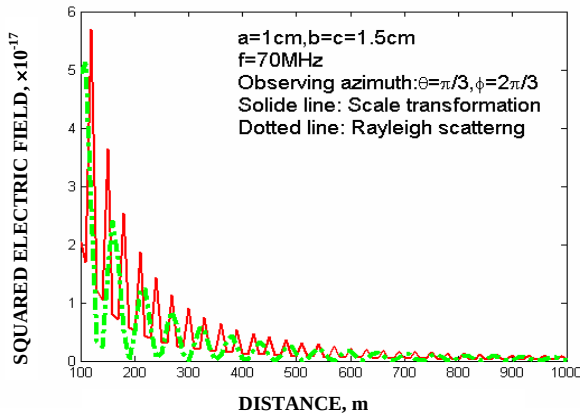


Fig. 2—Squared of scattering field versus distance

3.3 Applications

The theory of scale transformation for electromagnetic field has an extensive area in usage. Obviously it establishes a theoretical foundation for the electromagnetic simplification of target model. Now the electromagnetic simplification in the literatures only changes the targets and not changes the incident wave and its medium around, which must cause error in application. The theory of scale transformation can be used to investigate the scattering characteristics of the coated ellipsoidal target and to model the finite ellipsoidal columniation as the parameters satisfy $a \approx b \ll c$. The above targets are typical in estimating attenuation of EM wave induced by rain, sand dust storm, cloud and fog. They are of quintessence in radar target identification due to the fact that many fake targets can be considered as ellipsoids. The resonant problems and transmission subjects relative to the objects, such as elliptical sphere and elliptical cylinder, can be solved further with the scale transformation theory. A medium sphere is changed into an anisotropic ellipsoid after the scale transformation, which implies that we can investigate the scattering characteristics for an anisotropic ellipsoid utilizing the famous Mie theory and scale transformation theory presented.

4 Electromagnetic field in elliptical cylinder system

In reality, many scattering objects such as trees and flyers that can be described as elliptical cylinders lie across the propagation of electromagnetic wave. Thus, investigation of the electromagnetic scattering characteristics for an elliptical cylinder is of great importance in engineering. If a scattering problem about an elliptical cylinder is converted into that of a cylinder, we simplify computations, but obtain an analytical solution by utilizing the references available.

Assuming the symmetrical axis of an elliptical cylinder is parallel to the z -axis, the scale transformation made only in the x - y plane. For an elliptical cylinder, it can be supposed that

$$\text{when } \begin{matrix} c=1, \\ a>0, b>0. \end{matrix}$$

All the obtained results and conclusions are valid and Maxwell equations have the same forms as those in general situation. The transforming matrixes T and M can be written as

$$T^{-1}(a, b, c) = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M^{-1} = \begin{bmatrix} b & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & ab \end{bmatrix} \quad \dots (8)$$

The relations for the correlative quantities are the same. In reality, the expressions derived before are universal.

5 Conclusions

The scale transformation for coordinates is first introduced. The relations between spherical coordinates and the differential functors are presented. The transformation relations of charge density and current density vector are developed based on the form of conservation law of electric charge, being unchanged in different coordinate systems. The scale-transformation of electromagnetic field is made with the form of Maxwell equations unchanged in different coordinate systems. The boundary conditions are investigated. A theoretical basis is established for utilizing the results available for sphere and cylinder to investigate the scattering problems. Finally its potential usage and using fields are introduced.

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