

General Expression for Spectrum of Magnetic Anomaly due to Long Tabular Body & Its Characteristics

D C MISHRA

National Geophysical Research Institute, Hyderabad 500 007

and

K S R MURTHY & T C S RAO

National Institute of Oceanography, Regional Centre, Waltair 530 003

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A general expression for spectrum of magnetic anomalies—vertical, horizontal and total intensity—due to a long tabular body is derived which is used to estimate the body parameters. The analysis is extended to a marine magnetic anomaly recorded over the outershelf off Visakhapatnam. The results obtained show good correlation with those from methods of curve matching and characteristic curves.

Importance of long tabular models in the form of intrusions or fracture zones has attracted attention. Any interpretation scheme in exploration geophysics using different types of models usually takes into consideration first a tabular model which is modified through 'trial and error' as the analysis proceeds. However, interpretation using trial and error procedure is usually tedious and time consuming. Therefore, efforts have been made to discover methods which provide an estimate of unknown parameters in the first instance. Frequency domain analysis has acquired a wide application in this regard. In this paper an attempt is made to derive a general expression for the spectrum of magnetic anomalies—vertical, horizontal and total intensity—due to a tabular body and discuss its various characteristics, in estimating the unknown parameters of the causative source. The analysis is tested on a field example from the Visakhapatnam outershelf and the results are found to be in good correlation with those obtained by curve matching and method of characteristic curves.

Theory

General expression for magnetic anomaly due to a tabular body (Fig. 1) as given by Gay¹ as

$$\Delta F = C_F [(\psi_1 - \psi_2)/R \cos \theta_F + 1/R l_n (\cos \psi_2 / \cos \psi_1) \sin \theta_F] \quad \dots (1)$$

where C_F is $2KT_0^1 t/Z$ for ΔZ anomaly, $2KT_0^1 t/Z \sin \alpha$ for ΔH anomaly, $2KT_0^1 t/Z \frac{\sin I_0}{\sin I_0^1}$ for ΔT anomaly, θ_F is $I_0^1 - d$ for ΔZ anomaly, $I_0^1 - d - 90$ for ΔH anomaly and $2I_0^1 - d - 90$ for ΔT anomaly and t is thickness, $2W$

the width, and Z the depth to the dike, respectively. T_0^1 is effective total intensity, I_0^1 the effective magnetic inclination, K susceptibility contrast and $R = 2W/Z$, T_0^1 and I_0^1 are given as

$$\tan I_0^1 = \tan I_0 / \sin \alpha \quad \text{and} \quad T_0^1 = T_0 \sin I_0 / \sin I_0^1$$

where T_0 and I_0 are the magnitude and inclination of the earth's magnetic field and α is the strike angle of the body from magnetic north. ψ_1 , ψ_2 and d are as shown in Fig. 1.

Rewriting Eq. 1 after simplification, we get

$$\Delta F = C_F [(\psi_1 - \psi_2) \cos \theta_F + l_n (\cos \psi_2 / \cos \psi_1) \sin \theta_F] \quad (2)$$

where C_F is now $2KT_0^1 \sin d$ for ΔZ , $2KT_0^1 \sin \alpha \sin d$ for ΔH and $2KT_0^1 \sin d \sin I_0 / \sin I_0^1$ for ΔT .

Therefore, C_F is a function of T_0 , I_0 , α and k and θ_F a function of α , d and I_0 . $(\psi_1 - \psi_2)$ and $l_n (\cos \psi_2 / \cos \psi_1)$ are functions of depth, width and distance on surface

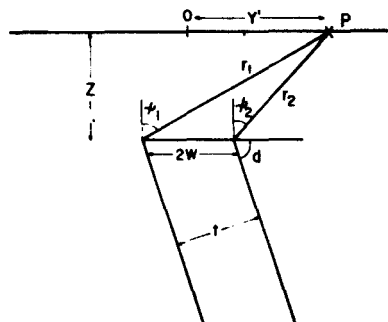


Fig. 1—Cross-section of a tabular body

from the top of the model. The Fourier transform of the magnetic anomaly ΔF (Eq. 2) is

$$\Delta F(w) = \int_{-\infty}^{\infty} \Delta F \exp(-iwy') dy' \quad \dots (3)$$

$$= C_F \left[\int_{-\infty}^{\infty} (\psi_1 - \psi_2) \exp(-iwy') dy' \cdot \cos \theta_F + \int_{-\infty}^{\infty} I_n (\cos \psi_2 / \cos \psi_1) \exp(-iwy') dy' \cdot \sin \theta_F \right] \dots (4)$$

$$= C_F [I_1 \cos \theta_F + I_2 \sin \theta_F] \quad \dots (5)$$

$$\text{where } I_1 = \int_{-\infty}^{\infty} (\psi_1 - \psi_2) \exp(-iwy') dy' \quad \dots (6)$$

$$I_2 = \int_{-\infty}^{\infty} I_n (\cos \psi_2 / \cos \psi_1) \exp(-iwy') dy' \quad \dots (7)$$

Solution of Eq. 6 and Eq. 7 with the help of integral transforms of Erdelyi²

$$I_1 = (2\pi/w) \sin(wW) \exp(-wZ) \quad \dots (8)$$

$$I_2 = i(2\pi/w) \sin(wW) \exp(-wZ) \quad \dots (9)$$

Substituting Eq. 8 and Eq. 9 in Eq. 5, we get

$$\Delta F(w) = 2\pi C_F (\sin wW/w) \exp(-wZ) [\cos \theta_F + i \sin \theta_F] \quad \dots (10)$$

from which amplitude spectrum $A(w)$ can be obtained as

$$A(w) = 2\pi C_F (\sin wW/w) \exp(-wZ) \quad \dots (11)$$

and phase spectrum

$$\phi(w) = \arctan(\sin \theta_F / \cos \theta_F) \quad \dots (12)$$

Dividing both sides of Eq. 11 by $(\sin wW)/w$ and denoting the LHS by $AF(w)$, we get

$$AF(w) = A(w) \cdot w / (\sin wW) = 2\pi C_F \exp(-wZ) \quad \dots (13)$$

The log. of $AF(w)$ versus w gives a straight line whose slope is equal to the depth to the body with intercept on y-axis (at $w=0$) equal to $2\pi C_F$. Dip of the body d can be calculated from Eq. 12, since α and I_0^1 can be assumed depending on the strike of the body and the inclination of the earth's magnetic field. Knowing dip of the body, susceptibility k can be estimated from the intercept value $2\pi C_F$, since all other parameters are known. Half-width W can be calculated from Eq. 11 for any particular frequency (except at $w=0$) and by substituting the values of k , I_0^1 , T_0^1 and d .

Theoretical Example

Tabular model with the following parameters was chosen to calculate the amplitude spectrum of magnetic anomaly.

$$Z = 2.5; W = 2.5; I_0 = 30^\circ, I_0^1 = 50^\circ, \alpha = 30^\circ,$$

$$d = 45^\circ; T_0^1/T_0 = 0.6; KT_0 = 1$$

Amplitude spectra $AF(w)$ for the magnetic anomalies (ΔZ , ΔH and ΔT) for such a model are calculated using Eq. 13 and are shown in Fig. 2. The spectra for the 3 anomalies (ΔZ , ΔH and ΔT) differ only in their intercepts and the slope of the spectrum remains the same in all cases. The depth obtained from the slope is 2.475 in all the 3 cases against an actual value of 2.5. The intercepts on y-axis for ΔZ , ΔH and ΔT are 5.8, 2.9 and 3.8 as against the actual values of 5.78, 2.89 and 3.77, respectively. The half-width W calculated at frequency $w = 3.0$ by assuming α and d yielded a value of 2.5.

Field Example

Total intensity marine magnetic anomaly record along a profile over the outers shelf off Visakhapatnam

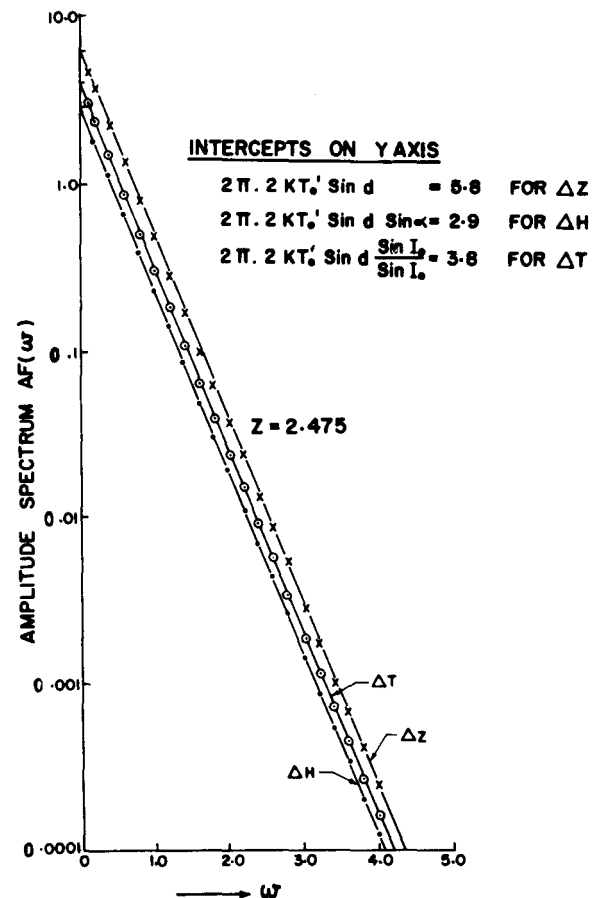


Fig. 2—Amplitude spectrum of ΔZ , ΔH and ΔT anomalies

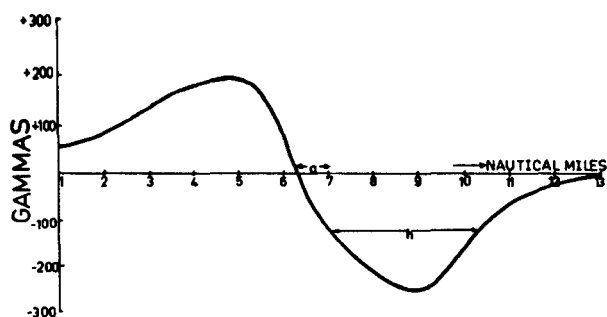


Fig. 3—Total intensity of marine magnetic anomaly over the outershell off Visakhapatnam (h and a are parameters used in the method of characteristic curves, Zurflueh³)

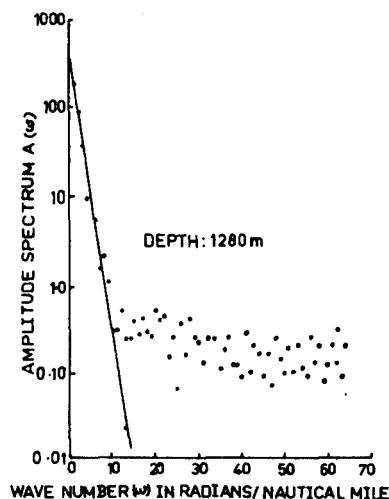


Fig. 4— Amplitude spectrum of magnetic anomaly of Fig. 3

(Fig. 3) is taken to study the spectral characteristics. The anomaly is digitized at an interval of 0.1 nautical mile resulting in a total of 130 points. The amplitude spectrum $A(w)$ is calculated using the discrete Fourier analysis (Fig. 4). The spectrum repeats itself after the

Table 1—Magnetic Anomaly off Visakhapatnam Calculated by Different Methods

Method	Depth (m) Z	Width (m) $2w$	Dip d°
Amplitude spectrum	1280	5100	60°
Curve matching method ¹	1260	5040	50°
Characteristic curves ³	1250	5000	—

first half of the data length and is therefore shown only for 65 points. The slope and intercept of the amplitude spectrum are used to get the parameters of the causative source.

The anomaly in Fig. 3 is also interpreted by curve matching method¹ and by characteristic curves³ assuming a wide tabular body. Table 1 shows the parameters of the causative source obtained from these 3 methods. It can be seen that they are in good agreement.

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