

Determination of sheet-like geological structures parameters using Marquardt inversion of the magnetic data

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A stable inversion method to estimate the depth to top, width and dip of a sheet-like geological structure from pole-reduced magnetic field is introduced. The inverse modelling is based on Marquardt optimization algorithm. The performance of the proposed method is considered by the theoretical magnetic data due to some dyke-shape models, with and without random noise. The inverted parameters convergence demonstrates the ability of the inversion approach as a powerful and useful tool, especially where the data are corrupted with noise. We employ this method for interpreting a real magnetic data set produced by a tabular structure from Iran. The inferred structure has approximately a depth to top of 17.12 m, a width of 12.74 m and a dip of 100.8 degree anticlockwise from horizontal, i.e. 10.8 degree from vertical towards east.

[Key words: Inversion, Magnetic, Marquardt optimization algorithm, Tabular structure]

Introduction

A sheet-like geological structure generated from intrusive igneous rock while cut through the strata is named dyke and if run in the same direction of the strata and be parallel with the bedding of the enclosing rocks is named sill. These structures have different slopes, thicknesses, lateral dimension's extent and are magnetically strong as are easily detectable in the residual magnetic field maps. Because of existence of important minerals in the igneous rock, such as Chromite, Magnetite and so on, these tabular structures are among the very considerable exploratory targets in geophysical investigations, especially in the potential fields methods. Various approaches have been proposed for estimating the geometric parameters and position of the dyke-form sources such as width, slope and depth from the magnetic data.

A least-squares minimization approach has been presented by Abdelrahman *et al.*¹ for the specification of depth, index parameter and amplitude coefficient from thin dyke's magnetic anomalies. Asfahani and Tlas² employed a nonlinearly mathematical optimization to compute the geophysical parameters of the vertical faults and thin dyke's models from magnetic anomalies. Cooper³ introduced a semi-automatic technique for interpreting the magnetic

anomalies caused by the dyke, based on ratios of the gradients of the anomaly. Ekinci⁴ used the higher order horizontal derivatives of magnetic anomaly due to thin dyke-like structure to estimate the depth to top of source. A literature review of the articles published in recent years suggests that there are different studies, which emphasize on determining the depth to the top of sheet-like magnetic sources⁵⁻¹¹. A fast-simulated annealing global optimization technique has been proposed by Biswas¹²⁻¹³ to the interpretation of gravity and magnetic anomaly over thin sheet-type structure. Biswas *et al.*¹³ also applied a nonlinear optimization method for the determination of dyke-type source parameters based on the calculation of first order horizontal and vertical derivatives of the gravity and magnetic anomalies.

The main problem in potential field inversion is non-uniqueness of solutions as measured magnetic/gravity anomalies in the plane of observation can be attributed to many of susceptibility/density distributions in different depths and various shapes¹⁵⁻¹⁶. Initial conjectures of the position and shape of the anomaly causative mass based on geological and geophysical information of the area under consideration is a way for minimizing non uniqueness in inverse modeling¹⁷. Accordingly, in this paper, Marquardt's algorithm¹⁸ is applied for

inverting the 2-D observed magnetic anomaly due to infinite dyke-shape model in order to evaluate the depth to top, width and slope of buried structure. We exemplify the capability of the method by a theoretical model with and without a random noise. Further, this inversion technique is employed for the interpretation of the real magnetic data from Iran.

Materials and Methods

The Marquardt inversion of magnetic anomalies is implicitly a repetitive optimizer mathematical process which estimate the three structural parameters of a sheet-like shape, i.e. depth (z), slope or dip (d) and width or thickness (w) in each iteration. This method, during inversion, try to fit the computed magnetic anomalies correspond to the evaluated parameters to the observed ones in the least-squares approach in each iteration. The difference between the observed magnetic $f_{obs}(x_i)$, and calculated magnetic anomaly of an initial assumed model $f_{cal}(x_i)$, can be estimated by a misfit function, J , as¹⁹

$$J = \sum_{i=1}^N [f_{obs}(x_i) - f_{cal}(x_i)]^2 \quad \dots (1)$$

N is the number of observed magnetic data. The Marquardt optimization algorithm¹⁸ described by Chakravarthi and Sundararajan¹⁹ is employed for minimizing the misfit function in order to determine the all modifications of the three unknown structural parameters namely dz , $d\beta$ and dw , by solving the nonlinear normal equations, as

$$\sum_{i=1}^N \sum_{k=1}^3 \frac{\partial f(x_i)}{\partial a_j} \frac{\partial f(x_i)}{\partial a_k} (1 + \delta\lambda) da_k = \sum_{i=1}^N [f_{obs}(x_i) - f_{cal}(x_i)] \frac{\partial f(x_i)}{\partial a_j}, \text{ for } j = 1, 2, 3 \quad \dots (2)$$

where da_k , $k = 1, 2$ and 3 are the improvements to the three model parameters of the tabular geometric structure, i.e. depth, angle Beta β and width which are calculated in each iteration. Also,

$$\delta = \begin{cases} 1 & \text{for } k = j, \\ 0 & \text{for } k \neq j, \end{cases}$$

and λ is the damping factor. If the values of the misfit function, J , in i -th iteration is less or more than its previous value in iteration number $i-1$, λ is decreased by a factor of 0.5 or is increased by a factor of 2, respectively.

The general expression of a total field magnetic anomaly due to infinite thin dyke-like structure sat any point on the surface $f(x_i)$, along a line perpendicular to its strike, is given by²⁰

$$f(x_i) = \frac{A(x_i \sin \beta - z \cos \beta)}{(x_i^2 + z^2)} \quad \dots (3)$$

where, x_i and z are the horizontal and vertical distances of the current measurement point to the structure, $\beta = 2I - d - 90^\circ$, d is angle of dip, $\tan I = \tan i / \cos \alpha$, i is the geomagnetic field inclination angle, $A = 2k.F.c.w.\sin(d)$, k is the susceptibility of the structure, w is the dyke width, F is the geomagnetic field strength, $c = 1 - \cos 2I \cdot \sin^2 \alpha$, and α is the angle between the positive x axis and North. If the strike of the magnetic sampling profile were along north-south direction, then the geomagnetic field declination angle α (azimuth) is zero. Moreover, the geomagnetic field is the pole-reduced one if the geomagnetic field inclination angle i were considered as 90° .

Partial derivatives required in the normal system of equation (2) are calculated numerically by the relations derived from equation (3) considering each parameter to be solved. The partial derivatives of the sheet-like source than the three shape parameters, namely depth, β and width can be computed, respectively, as

$$\frac{\partial f(x_i)}{\partial z} = \frac{2Bwz(z \cos \beta - x_i \sin \beta)}{(x_i^2 + z^2)^2} - \frac{Bw \cos \beta}{x_i^2 + z^2} \quad \dots (4)$$

$$\frac{\partial f(x_i)}{\partial \beta} = \frac{Bw(x_i \cos \beta + z \sin \beta)}{x_i^2 + z^2} \quad \dots (5)$$

$$\frac{\partial f(x_i)}{\partial w} = \frac{B(x_i \sin \beta - z \cos \beta)}{x_i^2 + z^2} \quad \dots (6)$$

where $B = 2k.F.c.\sin(d)$. The dip value is obtained from the β value estimated in each iteration. The advancements, da_k , $k = 1, 2$ and 3 estimated by equation (2) are then added to or subtracted from the available parameters estimated from previous iteration and the inversion process repeats until the misfit, J , in equation (1) descends below a predetermined allowable error or the damping factor grows from the initial value to a large value which is greater than predefined amount or the considered number for iterations is terminated.

Results and discussion

Figure 1(a) show the observed and calculated magnetic anomalies due to the initial and assumed models, which are shown in Figure 1(b). The considered values for the susceptibility is $k = 0.52$ SI (0.0414 c.g.s.), azimuth is $\alpha = 0$ (the profile strike is south (left) to north (right)) and magnetic field

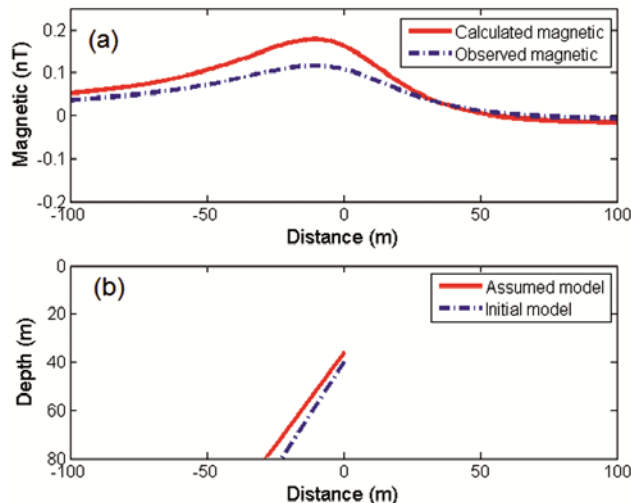


Fig. 1 — a) The observed and calculated magnetic anomalies along a profile with S-N direction due to b) the initial ($z=40$ m, $w = 5$ m and $d= 60$ degree towards the south) and assumed ($z = 36$ m, $w = 7$ m and $d = 57$ degree towards the south) models with a susceptibility of 0.52 SI (0.0414 c.g.s.)

Table 1 — Numerical results obtained from the Marquardt inversion of the synthetic magnetic data, with and without noise, due to the theoretical models.

Parameter	Without noise			With noise		
	Depth (m)	Dip (deg.)	Width (m)	Depth (m)	Dip (deg.)	Width (m)
Initial	40	60	5	40	60	5
Assumed	36	57	7	43	63	3.5
Estimated	39.98	59.95	5	39.62	60.124	4.993
Error %	0.05	0.083	0	0.95	0.21	0.14
Misfit (nT)	0.53×10^{-7}			0.00031		
Lambda λ	5.6×10^{-24}			16.6		
Iteration	79			171		

Table 2 — Inverted parameters from analysis of free-noise magnetic anomalies for different models

Parameter	Without noise					
	Model 1			Model 2		
	Depth (m)	Dip (deg.)	Width (m)	Depth (m)	Dip (deg.)	Width (m)
Initial	25	76	12	55	40	20
Assumed	20	67	19.5	46	53	14
Estimated	25	76.014	11.98	55.021	40.02	19.974
Error %	0	0.018	0.17	0.038	0.05	0.13
Misfit (nT)	0.0000000067			0.0000000075		
Lambda λ	3.1×10^{-15}			7.4×10^{-19}		
Iteration	48			63		

inclination is $i = 90$ deg. which are constant during inversion. The selected values for the parameters, which improve during inversion, i.e. depth, dip and width for the initial model are 40 m, 60 deg. toward the south (i.e. 120° clockwise from the horizontal) and 5 m, respectively and for the assumed model are 36 m, 57 deg. to the south and 7 m, respectively. The numerical parameters considered for the theoretical models are summarized in Tables 1 and 2.

The predefined values for misfit function, J , iteration and maximum damping factor (λ) are 10^{-7} nT, 100 and 13, respectively. The initial damping factor is given as 0.5.

The misfit, J , reduces intensely from its initial value of 0.195 nT at the first iteration to 0.0043 nT at the end of the 5th iteration and then incrementally reaches 0.53×10^{-7} nT at the 79th iteration (Fig. 2d). Because of the misfit, J obtained a value smaller than the allowable error value at the 79th repetition, as a result the iteration process ceases and therefore, the optimum estimates for the depth, dip and width are corresponding to the evaluated quantity at 79th iteration.

Figures 2(a), 2(b) and 2(c) showed the variations of the model parameters w , d , and z versus the iteration number. The values of the width and depth parameters remain fixed after 11th and 13th iteration. The conclusive obtained parameters values are $w = 5$ m, $d = 59.95$ degree to the south and $z = 39.98$ m. Figure 3(a) exhibited the inverted magnetic anomaly from the resulted model parameters which is shown in Figure 3(b). The percentage of error in the determination of the width, dip and depth parameters is zero, about 0.083 m and 0.05 m, respectively. The numerical results for the synthetic magnetic data noise are tabulated in Table 1.

The performance of the described inversion method has been studied by adding 10 % random noise to the magnetic response of the initial model, shown in Figure 1(a) using the following expression:

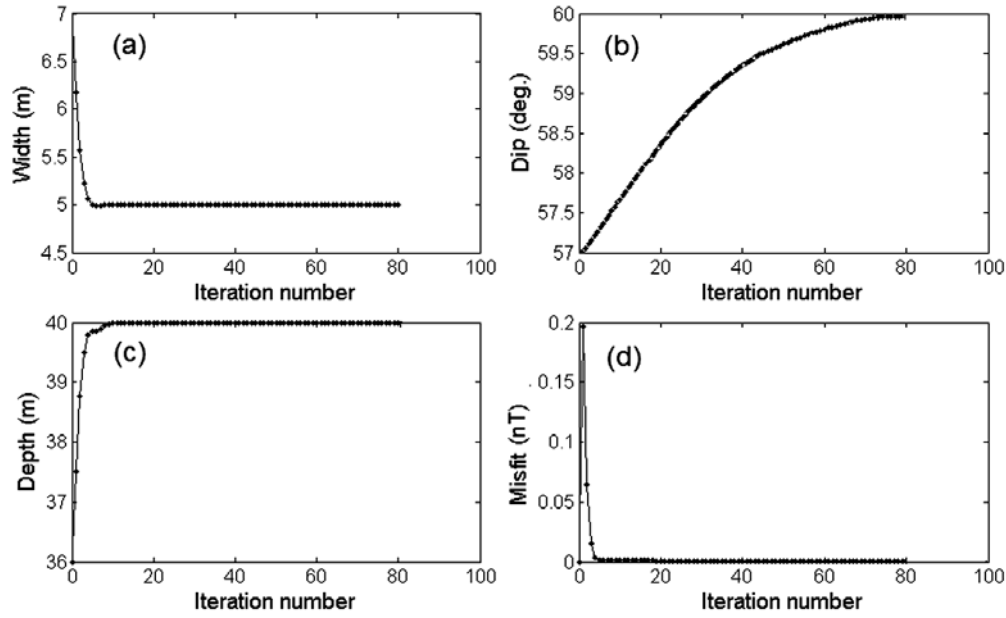


Fig. 2 — The variations of a) width b) dip c) depth and d) misfit function versus iteration number for free-noise synthetic data in figure 1.

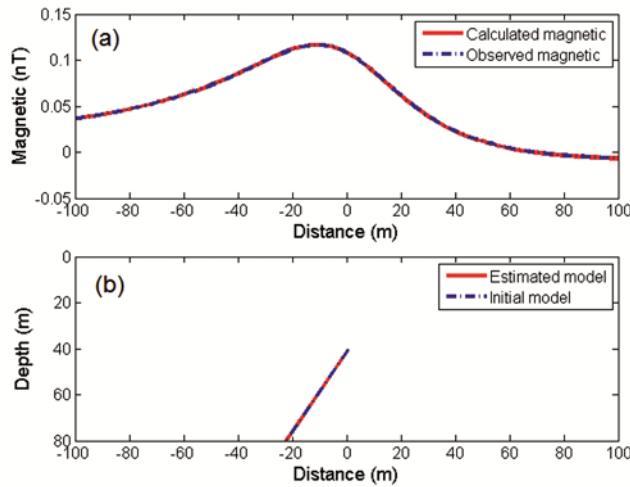


Fig. 3 — a) The observed and calculated magnetic anomalies along a profile with S-N direction due to b) the initial ($z = 40$ m, $w = 5$ m and $d = 60$ degree towards the south) and estimated ($z = 39.98$ m, $w = 5$ m and $d = 59.95$ degree towards the south) models with a susceptibility of 0.52 SI (0.0414 c.g.s.)

$$f_{nois}(x_i) = f_{obs}(x_i) + \left[\left(\frac{f_{max} + f_{min}}{2} \right) (rand(i) - 0.5) \times 0.1 \right] \quad \dots (11)$$

Where, $f_{nois}(x_i)$ is the noise corrupted synthetic data at x_i , while f_{max} and f_{min} are the maximum and minimum values of the observed magnetic, respectively, and *rand* function generate a set of random number whose range is between 0 to 1. The observed magnetic data after added 10 % random noise is show in Figure 4(a).

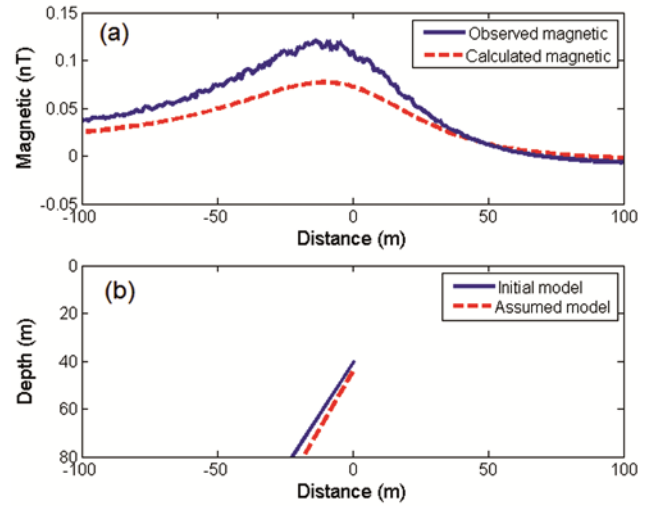


Fig. 4 — The observed magnetic anomaly with a added random noise of 10 % and calculated magnetic anomaly along a profile with S-N direction due to b) the initial ($z = 40$ m, $w = 5$ m and $d = 60$ degree towards the south) and assumed ($z = 43$ m, $w = 3.5$ m and $d = 63$ degree towards the south) models with a susceptibility of 0.52 SI (0.0414 c.g.s.)

The initial values for the parameters of the assumed model are given as $z = 43$ m, $d = 63$ degree towards the south and $w = 3.5$ m (Fig. 4b). The predefined values for error (misfit, J), iteration and damping factor (λ) are 0.00001 nT, 200 and 13, respectively. The initial damping factor is considered as 0.2. The misfit, J , lessens quickly from its initial value of 0.092 nT at the first iteration to 0.0026 nT at the end of the 4th iteration and then gradually attains 0.00031 nT after the 58th iteration and stay unchanged

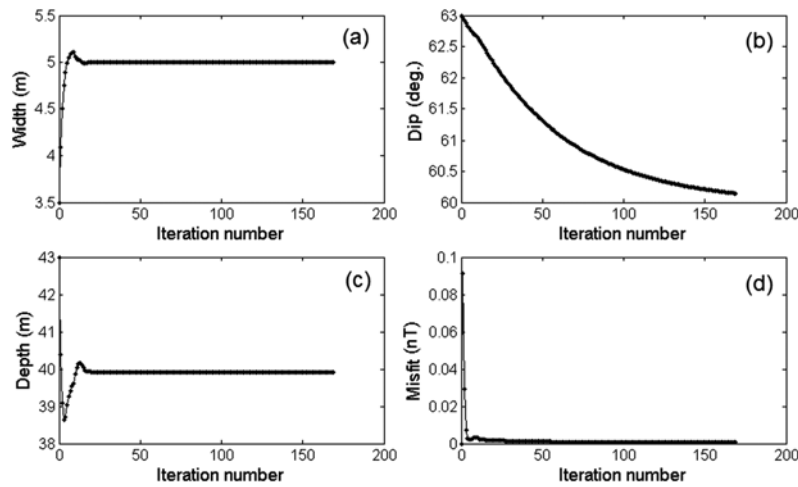


Fig. 5 — The variations of a) width b) dip c) depth and d) misfit function versus iteration number for noise corrupted synthetic data in Figure 4.

until the repetition stopped (Fig. 5d). The iteration finished at the 171st iteration where the damping factor whose value reached 16.6, then exceeded from the predefined value. The concluded values for the width and depth parameters beyond 22 and 25 iterations, respectively, remain unchanged. The final evaluated values for the width, dip and depth are given as 39.62 m, 61.124 degree and 4.993, respectively (Figs. 5a, 5b and 5c) and the percentage of their computational error are about 0.14 m, 0.21 degree and 0.95 m, respectively (see Table 1). Figure 6a shows the generated magnetic anomaly of the final inferred structure corresponding to the estimated parameters as shown in Figure 6b.

For evaluating the convergence of the responses of the Marquardt inversion, two different initial and assumed models were supposed. The interpreted parameters from the theoretical magnetic anomalies, without and with a random noise of 10 % have been summarized in Tables 2 and 3, respectively. The estimated structural parameters approximately conform to the initial ones.

Figure 7 shows the pole-reduced residual magnetic field of the area under investigation, after elimination the geomagnetic field, situated in Kerman Province, Iran. Since the magnetization of the existing geological formations and rocky layers in this region is almost high (Fig. 8), for removing the regional magnetic field effect and obtaining the residual magnetic anomaly, a second degree polynomial surface was fitted to the pole-reduced magnetic field. In Figure 7, the extended strong magnetic field with a direction of south to north indicated a sheet-like structure (dyke). The intrusive igneous mass in the region under evaluation include Magnetite with a

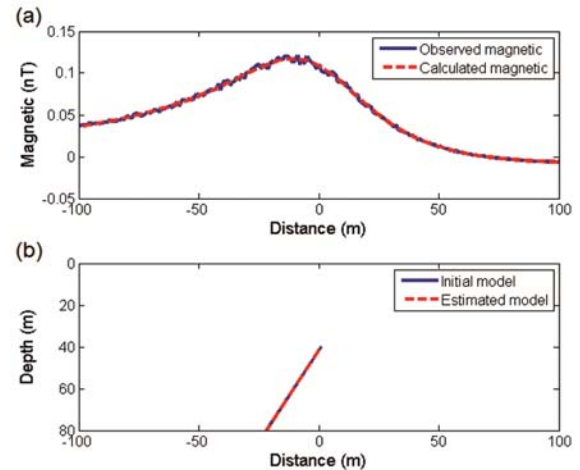


Fig. 6 — a) The observed magnetic anomaly with a added random noise of 10 % and calculated magnetic anomaly along a profile with S-N direction due to b) the initial ($z = 40$ m, $w = 5$ m and $d = 60$ degree towards the south) and estimated ($z = 39.62$ m, $w = 4.993$ m and $d = 60.124$ degree towards the south) models with a susceptibility of 0.52 SI (0.0414 c.g.s.)

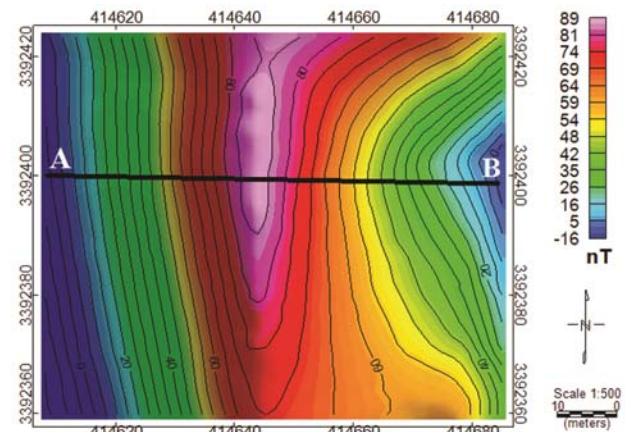


Fig. 7 — The pole-reduced residual magnetic field of the area under study where the west to east profile AB run across the positive magnetic anomaly due to a dyke structure

Table 3 — Inverted parameters from analysis of 10 % noise-corrupted magnetic anomalies for different models

Parameter	With 10 % random noise					
	Model 1			Model 2		
	Depth (m)	Dip (deg.)	Width (m)	Depth (m)	Dip (deg.)	Width (m)
Initial	25	76	12	55	40	20
Assumed	29	69	7	61	55	24.5
Estimated	25.13	76.37	11.83	55.21	39.69	19.84
Error %	0.52	0.49	1.42	0.382	0.775	0.8
Misfit (nT)		0.0000034			0.000048	
Lambda λ		5.2×10^{-21}			20.6	
Iteration		96			127	

Table 4 — The initial parameters values and final parameters values from interpretation of the real magnetic data

Parameter	Depth (m)	Dip (deg.)	Width (m)
Assumed	15	110	10
Estimated	17.12	100.7	12.73
Misfit (nT)		0.98	
Lambda λ		18.36	
Iteration		98	

range from about 5 % to 20 %. Therefore, according to the standard charts compiled by Clark and Emerson (1991) and Hunt *et al.* (1995), the susceptibility of the tabular body can be considered as 0.6 SI (0.048 c.g.s.). The magnetic anomaly variation along a profile AB, which runs across the dyke in W–E direction (Fig. 7), is interpreted using the Marquardt inversion.

The parameters values for the assumed model are given as $z = 15$ m, $d = 110$ degree (i.e. 70 deg. to the east or 20 deg. from the vertical towards the east), $w = 10$ m (Fig. 9 b). Further, more it is assumed that the target is free of the remnant magnetization.

Moreover, the assigned values for misfit (J), iteration and damping factor (λ) are 0.5 nT, 100 and 15, respectively. The changes in each parameter and misfit against the iteration number during inversion process are shown in Figure 10 a - d. The algorithm performed 98 iterations, before it was ceased, as in the end of this iteration number, the damping factor obtained a value greater than the predefined value (Table 4). The misfit function variations versus the iteration number (Fig. 10 d) showed a fast decrease from its first value of 4.8 nT to its value at the 9th iteration and then this downswing continued with a low slope until 98th iteration whose value is 0.98 nT. The dip parameter descended steadily from its initial value to final value at the 98th iteration whose value is 100.7 degree towards the east (Fig. 10b). The width (Fig. 10 a) and depth (Fig. 10 c) parameters have varied significantly until 9th iteration and then gradually achieved 12.73 m and 17.12 m, respectively, at 98th iteration. The resulted tabular

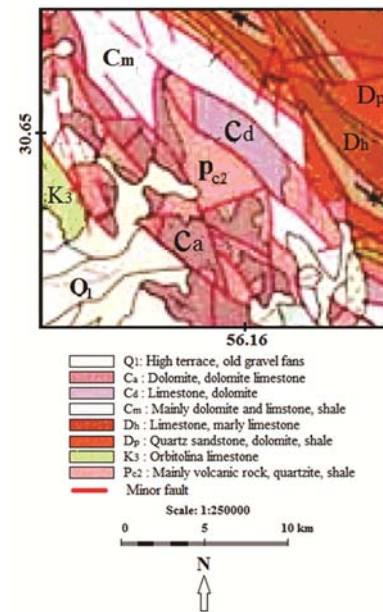


Fig. 8 — The geological map of the region under investigation

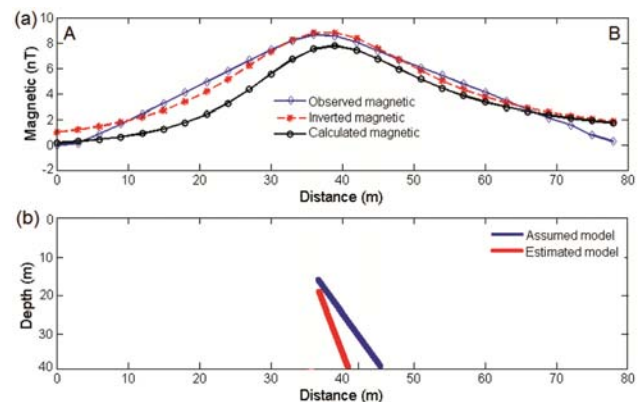


Fig. 9 — The observed, calculated and inverted magnetic anomalies along a profile AB due to b) the variations of the magnetic field along profile AB, assumed ($z = 15$ m, $w = 10$ m and $d = 110$ degree towards the east) model and inferred ($z = 17.12$ m, $w = 12.73$ m and $d = 100.8$ degree towards the east) model with a susceptibility of 0.6 SI (0.048 c.g.s.), respectively.

model is delineated in Figure 9 b. The inverted structural parameters are given in Table 4.

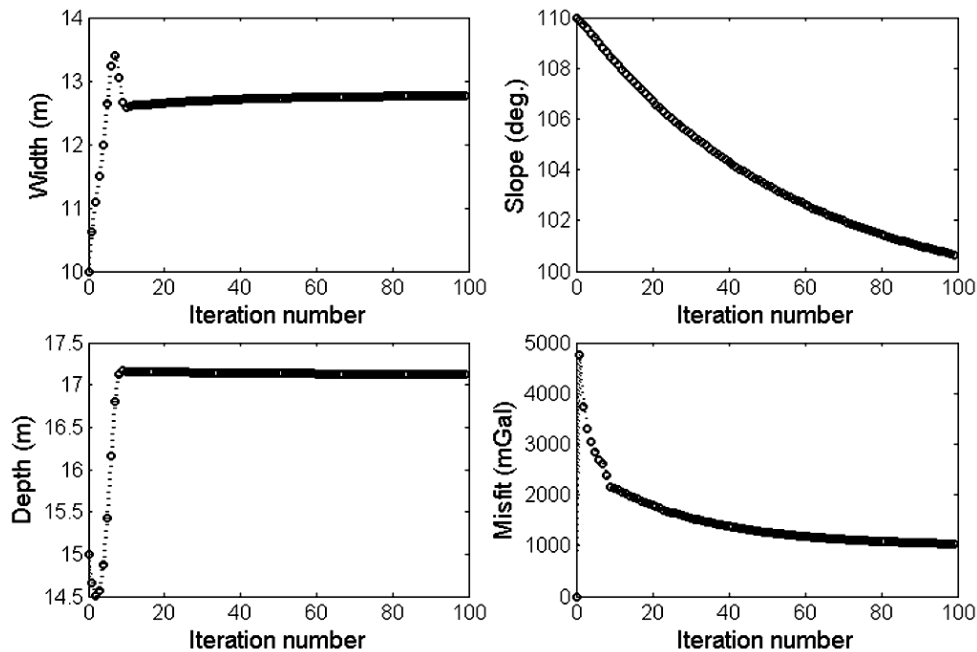


Fig. 10 — The variations of a) width b) dip c) depth and d) misfit function versus iteration number for real magnetic data during inverse modelling

Table 5 — The different initial parameters and interpreted parameters from inversion of the real magnetic data

Parameter	Model 1			Model 2		
	Depth (m)	Dip (deg.)	Width (m)	Depth (m)	Dip (deg.)	Width (m)
Assumed	23	91	17.5	11	115.5	16
Estimated	17.11	101.2	12.78	17.12	100.6	12.71
Misfit (nT)		0.87			0.93	
Lambda λ		6.7×10^{-9}			21.64	
Iteration		76			114	

The calculated magnetic due to the assumed model and inverted magnetic due to the estimated model are shown in Figure 9 a. The convergence and stability of the evaluated parameters from real magnetic data using the Marquardt inversion is tested by two different assumed models. The considered initial values for model 1 are $J = 1$ nT and $\lambda = 15$ and for model 2 are $J = 0.6$ nT and $\lambda = 15$. Moreover, the number of the algorithm iteration is given as 150. The numerical results have been summarized in Table 5. The inferred structural parameters from the different initial models are almost similar.

Conclusion

In this paper, a new inversion technique based on the Marquardt optimization algorithm has been employed to estimate the depth to top and structural features of a sheet-like body from magnetic field data. The method is examined with synthetic magnetic data due to a dyke model. The capability of the method is

evaluated with magnetic data as corrupted with noise. The convergence in the interpreted parameters proved the efficiency and insensitivity to noise of the proposed method. The predefined values for the maximum damping factor and misfit function can be effective on the final results as their optimum values can be determined with trial and error as well as, different initial guesses for the model parameters, even if they are far from reality, yield the acceptable results.

We have applied this approach to analyse the pole-reduced real magnetic anomaly caused by a sheet-like structure from Iran. The Marquardt inversion response of the real magnetic data to the different assumed parameters is approximately the similar structures. Considering the inferred parameters, the dyke-form structure, on an average, have a depth to top of 17.12 m, a width of 12.74 m and a dip of 100.8 degree towards the east. Considering the geological structures and outcrops in the region under investigation, these results are expected.

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