

CAN WE *SHRINK* OUR KIDS?

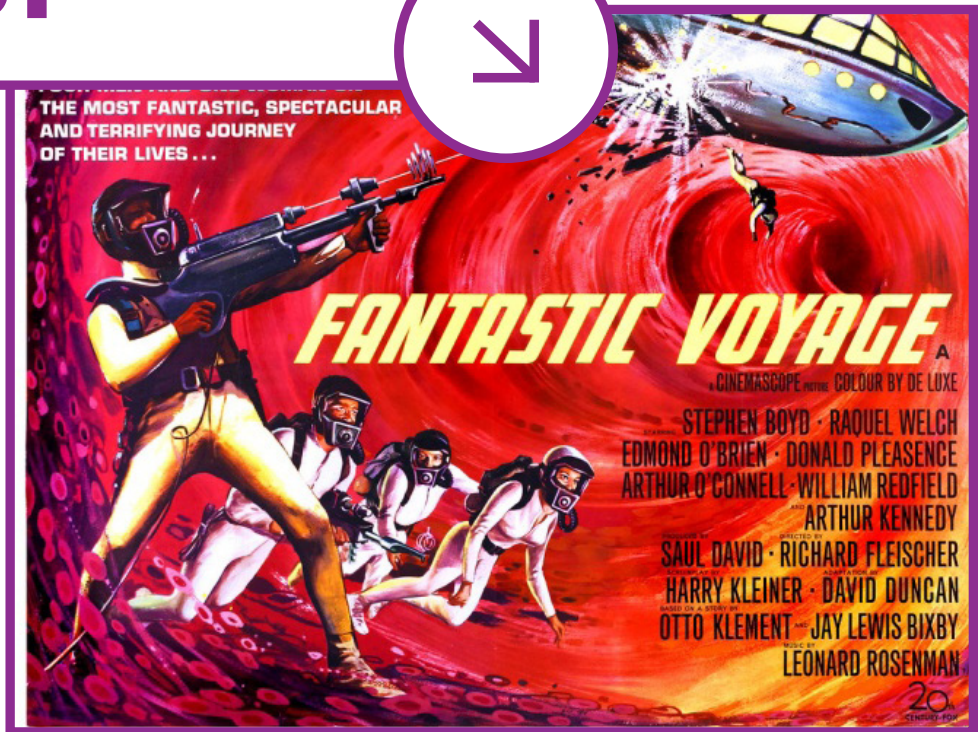
Amitabha Ghosh

MANY readers can recognize that the title of this article has been coined from the famous Walt Disney science fiction movie of 1989, “*Honey, I Shrunk the Kids*”, directed by Joe Johnston. The movie was based on a story where the children were shrunk to microscopic size.

Another popular 1966 science fiction movie “*Fantastic Voyage*” directed by Richard Fleischer was also based on a miniaturised submarine team sent to the brain through the veins of a patient with a cerebral clot. However, the idea of a world with miniaturized people was first published by Jonathan Swift in his classic 1892 novel “*Gulliver’s Travels*”. Subsequently, the term ‘Lilliput’ got coined in the English language to indicate small size.

Unfortunately, the basic ideas behind the stories are scientifically impossible (even if one accepts the fact that such miniaturisation can be achieved) as the scaling laws do not allow a human being to maintain the basic form when miniaturized to such levels.

The author of this article remembers a very funny experience when he was visiting one of his friends in a German university. He brought a toy train and a toy car for his two sons, 13 and 10 years old. The elder one was constantly asking his father why the toy car or the toy train were not getting damaged when they were either colliding with the wall or were getting derailed contrary to what is experienced in case of real life cars and trains! My friend was an electrical engineer and I watched with amusement how he struggled (of course unsuccessfully) to satisfy his son’s thirst for a clear answer. He was not aware that the clue to the answer was hidden in the scaling laws.



Scaling & Scaling Laws

What are scaling laws and what is their importance in engineering and science? Scaling laws are quantitative relationships of various parameters of a system with its size. It has been emphasised by many experts that miniaturization is the route to the impending 3rd Industrial Revolution and that is why micro-system technology and nano-science are increasingly gaining importance.

However, without proper design of the miniaturized systems these may be totally non-functional or improperly functional. It must be remembered that designing miniaturized systems cannot be done successfully depending on our common engineering senses based on our day to day experiences with the usual surroundings. The performance of miniaturized devices and systems will be successful only when the ‘Scaling Laws’ are taken into account at the design stage.

However, except for cases involving major civil engineering structures or systems involving earth science, not much scope exists for any major scaling up. On the contrary very drastic scaling down (viz. scaling down by a factor of even million to billion times) is being attempted in case

of micro and nano-technology activities. As size (or scale) changes different physical phenomena also change in their relative predominance. As a result, the characteristics of a system's components also change differently with the size. These changes can be predicted with the help of scaling laws and system design can be accomplished when the size changes drastically.

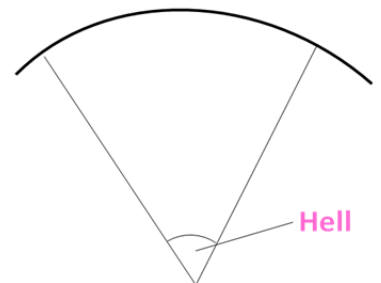
There are many examples of such phenomena taking place around us in our daily lives but we normally take these as obvious without any deeper understanding. For example, when a tumbler full of water is upturned the water gets spilled; but when the same water is kept in a thin tube (made of the same material as the tumbler) it takes a lot of effort to get the water out of the tube. Similarly, it is a common fact that when a piece of stone is dropped it readily falls to the ground (due to gravity). But when the same piece of stone is pulverized to a very fine powder the fine grains can float in air with the slightest breeze! There is no alteration in the physical laws since gravity is present in all such cases, but the relative magnitude of the gravitational pull becomes less significant compared to the force that acts against the gravity (surface tension in cases of the water in the tumbler/tube and the air frictional drag in the case of the stone/grain of the pulverized stone) when the size reduces.

Historical Background

Although scaling laws have gained importance in science and engineering in recent times, the subject had its first scientific and systematic study in the hands of none other than the father of modern science – Galileo Galilei (1564 – 1642). He was delivering a public lecture as a young aspirant for the position of the Professor of Mathematics at Pisa in front of the Grand Duke of Tuscany. He chose the topic of the model of Hell – a very popular topic in those days. However, the Seienise scholar Alessandro Vellutello proposed in 1544 a different and competing model as depicted in Figure (c).



(b) Concept of hell proposed by Antonio Manetti of Florence

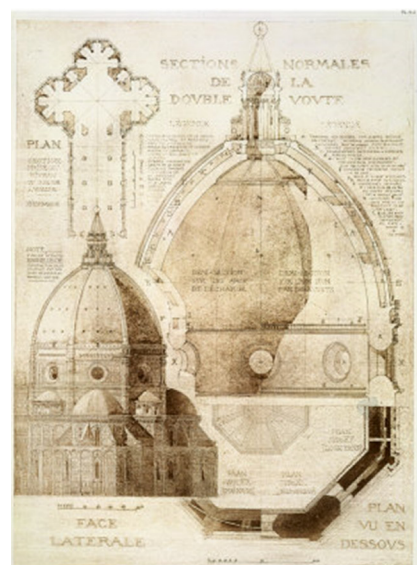


(c) Vellutello's model of hell
Models of hell that was popularly believed in Florence

There was severe rivalry regarding the correctness of the model and the people of Florence were strongly in favour of their model. In his characteristic style, Galileo first praised the opponent's logic that the shell of the earth covering the Hell would be too thin to support itself without collapsing. But then Galileo came to his point giving the example of Brunelleschi's dome which has no problem supporting itself and established the correctness of the Florentine philosopher's proposed model. The audience and the Grand Duke were all happy and Galileo got the job.



(a) Dantes' inferno (1423-1497)



The inner structure of the Brunelleschi dome of Florence



Later Galileo shifted to Padua University near the Republic of Venice and his most productive years were spent there. There Galileo came close to the Venetian ship-building industry; Venice had to depend on a strong navy being a maritime republic. Later when studying the Venetian ship building industry Galileo realised his blunder. He observed that ship models were strong but the full-sized ships often broke when out of water under their self-weight. He then studied the strengths of beams and their different behaviour at different scales. He also observed that thin ebonite sheets could float in water when divided into small pieces. The matter of scaling laws dawned upon him and he discussed it in his last book “*Dialogues on Two New Sciences*”. But he kept the mistake he made at the age of 21 a secret to his heart all his life.

What can be inferred from the performance of an existing system about that of a system of similar configuration but scaled down to a much smaller size? The answer to this question is provided by the scaling laws and so these laws play a very important role in designing microsystems now. Besides, the performances of different subsystems of a system/device scale differently. For example, the actuator performance can scale in a completely different way from the structural strength. Thus the configuration/look of a system can be very different from that at a drastically different scale.

Geometric Scaling

In the case of isomorphic scaling the shape of a geometric figure remains unchanged. Since the shape of a figure remains unchanged its area scales as the square of the scale. The scale is indicated by any important length of a figure. On the other hand, the perimeter of a geometric figure scales as ‘ l ’ unlike the area that scales as ‘ l^2 ’. This scaling can be sometimes very useful.

In the case of non-isomorphic scaling different dimensions of a figure scale differently and, therefore, as the scale changes the shape of the figure also changes.

The scaling of a spherical reservoir mounted on a supporting pillar is important to understand. The scaling is *allometric* since the weight of the spherical reservoir scales as the volume *i.e.* cube of the size but the capacity to bear the load by the pillar scales as the square of its diameter (the cross sectional area). Thus, as the size increases the weight increases faster and the pillar radius needs to become disproportionately larger.

Non-isomorphic scaling is termed as *allometric* scaling. The study of the changes in various proportions of a system/organism due to the change of size is called *allometry*.

Scaling of Natural Forces

As all systems/organisms are subjected to physical forces or other natural phenomena like heat conduction, it is desirable to get some idea about the way the forces in nature scale with the size of the objects on which a force acts. The allometry is primarily dependent on the scaling laws followed by the forces.

The most common forces that are involved in the functioning of various living and non-living objects, scale differently with size.

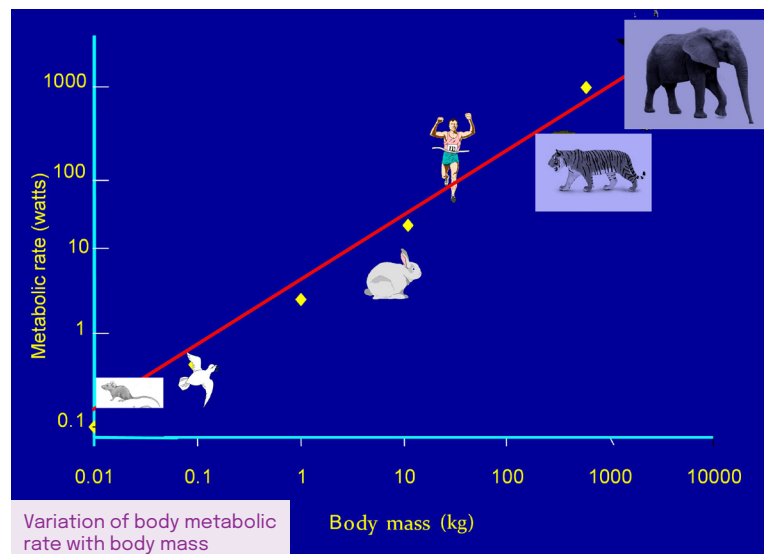
If a thin and flat object is placed over the water surface, a force due to the surface tension acts along its perimeter that acts in the upward direction resisting the downward force of gravity. For such cases, the gravitational force is proportional to the area (assuming very thin plates with no variation in thickness) that scales as square of length since the area scales as square of length. On the other hand the upward pull along the perimeter scales as proportional to length since the perimeter scales as linearly proportional to length. With the decrease in size the weight reduces at a faster rate than the upward force due to the surface tension and beyond certain level the upward force overtakes the downward force due to gravity. This is the reason why we see water bugs and mosquitoes can easily float on water but bigger things cannot. A full grown human being can also stand on water without sinking if the total perimeter of her/his two feet is about 8 km!

Galileo observed that small ship models remain intact when held by hand in any fashion but the full-sized real ships break under self-weight when taken out of water. This is also the clue to the query of my friend’s son -“Why the toy car does not break when it collides with other objects like the wall?” The stress developed scales as proportional to length and beyond a certain size the developed stress exceeds the limiting value and the beam breaks. There can be many more examples from engineering but the reader can refer to more detailed articles on scaling laws.

Importance of Surface to Volume Ratio

The scaling effects play a very important role when one examines the issues related to the size of objects. Surface to volume ratio decides many important matters in cooling phenomena and the influence of the ratio becomes very apparent. Since the ratio is large for small bodies they cool down very fast. It is seen that when the micro-sized chips come off grinding wheels they are brilliantly luminous and white-hot; but before they reach the ground taking only some fractions of a second, they cool down to almost room temperature. Similarly, the small-sized hails during a hailstorm melt soon, while the big chunks of ice coming from an ice factory take a long time to melt away.

In the living world also this ratio plays a very significant role in the affairs of various animals. The amount of heat generated in the body depends on the conversion of food into heat through metabolism. Furthermore, it is necessary to



maintain a definite body temperature for continuing the life process in case of most living objects. Thus, to maintain a specified body temperature a small animal (like rabbit) needs to eat more frequently in comparison with that for a large animal like an elephant.

In 1839, two scientists of Strassburg, Sarrus, a mathematician and Rameaux, a medical doctor, presented a thesis to the Royal Academy of France, in which they claimed that the body metabolic rate varies as the $2/3^{\text{rd}}$ power of the body mass as expected from the variation of the surface area to the volume ratio. But strangely it was discovered that the body metabolic rate varies as per the $3/4^{\text{th}}$ power of body mass (known as Kleiber's law) over a very wide range as shown. Recently it has been shown that this is because of the fractal nature of the body fluid circulation in all living entities. All other major parameters like heart beat rate, longevity, etc. also follow very simple scaling laws!

Use of Scaling Laws by Nature

There can be many advantages of appropriate application and use of the scaling laws of natural forces. Nature does it on all occasions and one example can be presented here. Actuators (which cause the movement of body parts on getting the required signal) are essential for the functioning of any machine and so is the case with all living objects also. Nature makes use of the scaling laws to develop actuators that can develop large actuation forces by miniaturising an actuator and using a very large number of such actuator elements in parallel *i.e.* exerting the force by all elements simultaneously. This is called massive parallelism which is so common in nature.

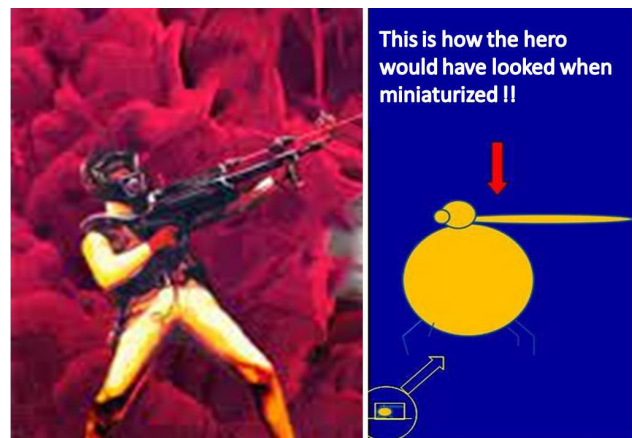
Natural systems like our bodies employ electrostatic force (that scales as the square of the scale) for actuation, unlike our regular machines which are run by electric motors using electromagnetic forces that scale as the cube of the scale. The advantage of using electrostatic force is because the number of actuator elements packed in a given volume increases when the size is reduced as the cube of the scaling factor; on the other hand, the actuation force by each unit reduces as the square of the scale. As a result the total force developed (that is given by the product of the number of actuator elements and the force developed by each unit) is magnified according to the scale.

A very simple scaling law (relating the magnitude and the frequency of occurrence) followed by a complex geophysical

phenomenon like an earthquake is extremely intriguing. Quite possibly, this law hides a very important secret of this phenomenon not yet understood (like the discovery of the fractal nature of body fluid circuits in all living objects was shown to result in Kleiber's law). Flying of objects, cruising of vehicles, running of animals also obey nice but simple scaling laws. The lift generated in a flying object must be equal to the weight and lift depends on both speed and the wing area.

Extreme Miniaturization

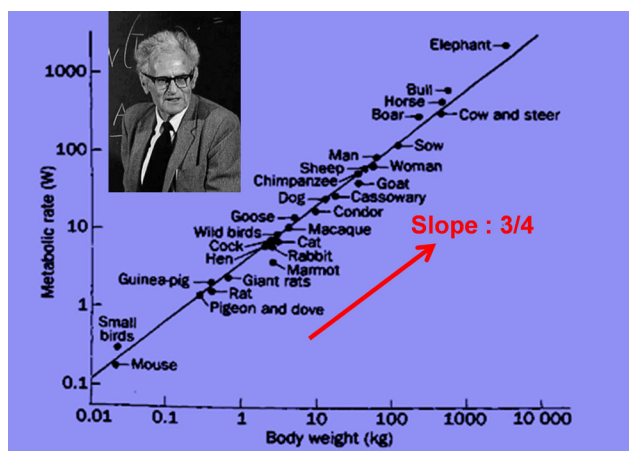
It will be interesting to explore the possible shape of a human when miniaturized to an extreme level. It must be remembered that it has no scientific value but just to evaluate the scientific correctness of the science fiction movies mentioned earlier. One must remember that even in science fiction the very basic scientific principles need to be followed.



Approximate shape of the hero in the film "Fantastic Voyage", should look something like this!

When a man (or a woman) is miniaturised the surface to volume ratio of the main body increases drastically and to maintain the body temperature the shape of the body should be such that the surface area is minimum (to minimize the heat loss). Hence the main torso should be spherical in shape. To support the reduced weight, the legs can be very thin and for maintaining stability four legs will be necessary. However, the hands that need to bear bending load will have to be disproportionately thick. The distance of the retina from the cornea will not decrease to the extent like the other parts as the light wavelength does not change with scaling. So the eyes will be projected behind the head.

As noted by most experts, the impending next industrial revolution will depend heavily on the science of miniaturization and the understanding of the scaling laws will become very critical. Designing and fabrication of micro and nano-systems successfully will depend very heavily on the understanding and application of the scaling laws by scientists and engineers. Besides, as mentioned earlier, understanding of the scaling laws can reveal many deeper secrets of nature as experienced while explaining Kleiber's law.



Kleiber's law for variation of metabolic rate as the $3/4^{\text{th}}$ power of body mass

Dr Amitabha Ghosh is Honorary Scientist, Indian National Science Academy, New Delhi