

Multi response optimization in design of experiments considering capability index in bounded objectives method

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This paper presents a new method for optimization of multiple response problems in designing of experiment. An algorithm was developed for explicit determination of over bounded goals by combining mean and standard deviation of quality characteristics according to process capability (CP_K). The output of this algorithm will guide to use one of multiple objective decision making (MODM) methods. Proposed algorithm was successfully applied on a case study to minimize standard deviations while maximizing means of two quality characteristics and minimizing price of the product. Output of the algorithm directed to use bounded objective methods. Thus, non linear problem was solved with generalized reduced gradient (GRG) algorithm.

Keywords: Design of experiments, Multiple objective decisions making, Process capability

Introduction

Response surface methodology (RSM) typically involves experimental design, regression models, and optimization. Since response variables are different in some characteristics (scale, measurement unit, type of optimality and their preferences), there are different methods in model building and optimization of these problems¹. There are two types of optimization in RSM²: i) Dual response surface optimization (DRSO); and ii) multiple response optimizations (MRO). DRSO allows practitioners to optimize primary response subject to an appropriate constraint on the value of secondary response. In MRO, optimize mean response of quality characteristics simultaneously to find an optimal setting without considering standard deviation of responses.

Montgomery³ cited Graphic method applicable for less than or equal to three response variables. Using weight for each objective and combining them together is one of the methods used in multi-criteria decision making (MCDM). Various other methods include decision maker (DM) idea, linear programming

technique for multidimensional analysis of preference (LINMAP), and Eigenvector and analytical hierarchy process (AHP) for determining weights^{4,5}. Del Castillo⁶ discussed about gradient of each response and found weighted direction considering responses confidence regions for linear objectives. Ames *et al*⁷ proposed a quality loss function approach. Tong & Su⁸ used technique for order preference by similarity to ideal solution (TOPSIS). Del Castillo & Montgomery⁹ discussed that non-linear programming solution [generalized reduced gradient (GRG) algorithm] can lead to better solutions than those obtained with DRSO. Derringer & Suich¹⁰ described application of desirability functions for optimization of multi-response problems (MRS) in situation that sets quality objectives around bounded target. Kim & Lin¹¹ demonstrated non-linear desirability of DM using exponential desirability functions. Myers & Carter Jr¹² optimized primary response with respect to other responses by using a Lagrange multiplier approach. One popular method to MRO is dimensionality reduction. In such approaches, multiple-response problems are converted to one single aggregated function. Lin & Tu¹³ proposed mean squared error (MSE) in DRSO instead of using Lagrangian multipliers.

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Jeyapaul *et al*¹⁴ applied genetic algorithm as a heuristic search procedure in MRS and compared performance of proposed method with the performance of method that combines desirability functions with GRG optimization method. Wang & He¹⁵ improved TOPSIS method considering standard deviation of quality characteristics. Amiri & Salehi-Sadaghiani¹⁶ presented a method for optimizing statistical MRS. Köksoy¹⁷ presented a method to optimize multiple quality characteristics based on MSE criterion when data were collected from a combined array and GRG algorithm was used for nonlinear programming. Noorossana *et al*¹⁸ proposed artificial neural network to form implicit relationships between responses and control factors estimated by polynomial regression models. In optimization phase, a genetic algorithm (GA) was considered in conjunction with an unconstrained desirability function to determine optimal settings for control factors.

This study presents a new method for optimization of multiple response problems in designing of experiment considering capability index in bounded objectives method.

Proposed Algorithm for Optimization of MRS

Steps of using design of experiment (DOE) according to Deming cycle are suggested in Fig. 1. Proposed optimization method will be used in step 13 (Fig. 1) for analyzing results. Four steps are suggested for analyzing and optimizing of multi-response problems in DOE.

Step 1

It involves building regression model of mean and standard deviation for all quantitative responses with replication.

Step 2

It involves converting quality objectives with regression model of mean and regression model of standard deviation to one equation by using capability process index¹⁹ as

$$CP_{ki} = \text{Min} \left\{ \frac{USL_i - \mu_i}{3\sigma_i}, \frac{\mu_i - LSL_i}{3\sigma_i} \right\}$$

where USL_i , USL of quality characteristic (QC) i ; LSL_i , LSL of QC i ; μ_i , regression model of mean in QC i ; σ_i , regression model of standard deviation in QC i ; CP_{ki} , non linear model of capability process for QC i .

Step 3

It determines lower limit of CP_{ki} for QCs according to customer requirements.

Step 4

It selects method of solving non-linear multi objective problem using Fig. 2.

Output of proposed algorithm (Fig. 2) provides a guideline for choosing optimization method in combination of DRSO and MRO. Bounded objectives method, goal programming and L-P are techniques of MCDM⁴.

Case Study

This study was performed in a company that produce copper-brass radiator of automobile. Lightness, strength and efficiency are desirable properties of radiators. Additionally, customers demand low cost products. Four factors in production process of copper-brass radiators were chosen in order to minimize production cost or final price of product ($f(x)$), maximize mean of durability against corrosion ($g_1(x)$), minimize standard deviation of durability against corrosion ($g_2(x)$), maximize mean of strength or viscosity between radiator's fins and tubes ($h_1(x)$) and minimize standard deviation of strength ($h_2(x)$). Factors are as follows: NH_4Cl % (x_1) used before furnace, thickness of tin on tubes (x_2), temperature of furnace (x_3) and tin alloy % on pipes (x_4). Also there were some technical limitations in factors.

A plan of experiments based on DOE PACK software in four central points and 1.44 star points was designed. Experimental objective was to optimize production cost, strength and durability of copper-brass radiators. Table 1 shows factors, responses, design and result of experiments. Each experiment condition was repeated two times. Two results for quality characteristics and one result for final price of product in design points were obtained.

Step 1: Regression Model of Objectives

Factors were changed to coded variables (Table 1) and regression models were built in software of Statgraph.

$$f(x) = \text{price} = 4438 + 4.67X_1 + 42.19X_2 + 4.17X_3 + 15.51X_4 - 4.5X_1^2 - 4.5X_2^2 - 4.5X_3^2 + 6.25X_1X_4 + 3.25X_2X_4 + 6.25X_3X_4$$

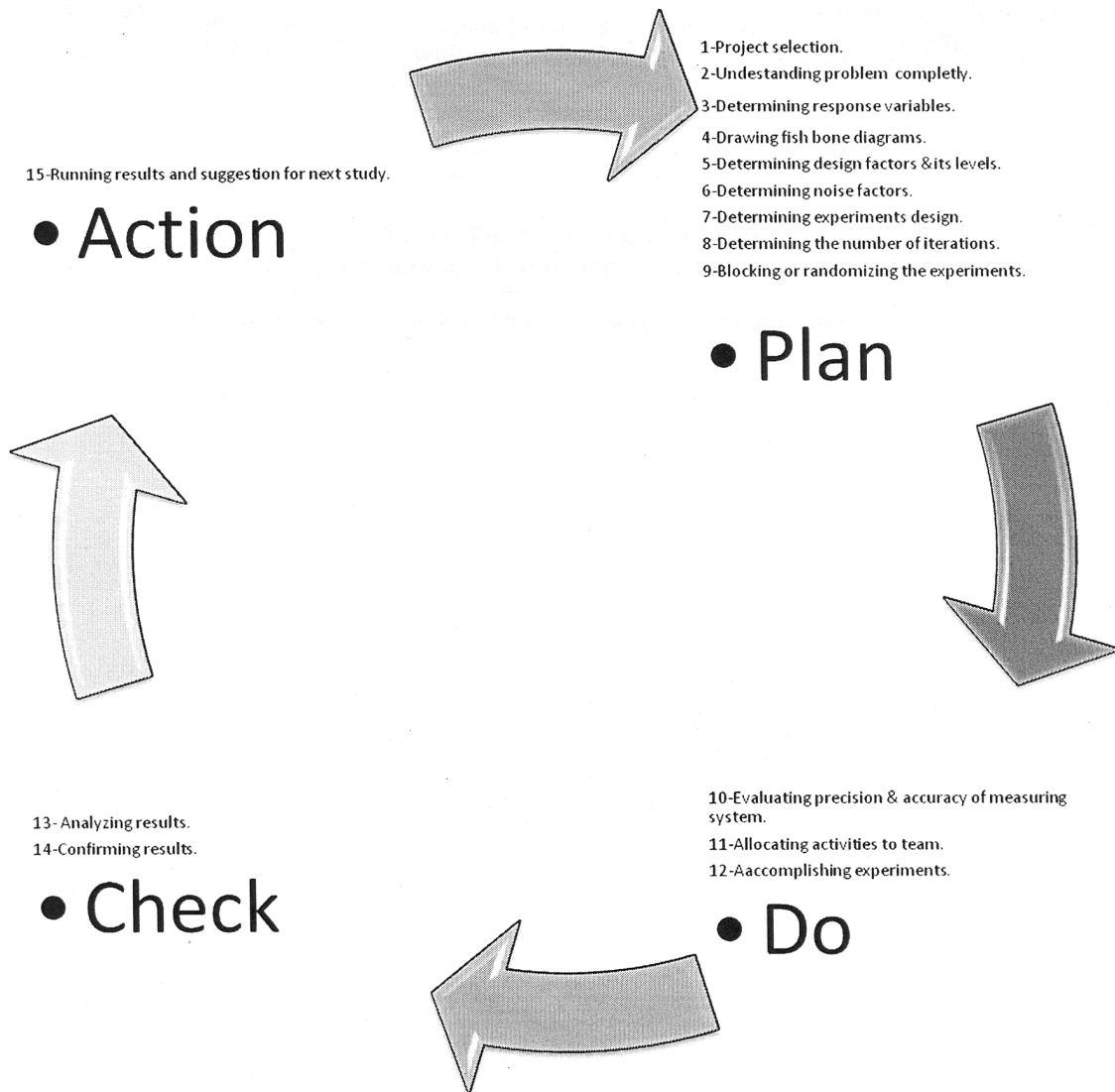


Fig. 1— Steps of DOE according to Deming cycle

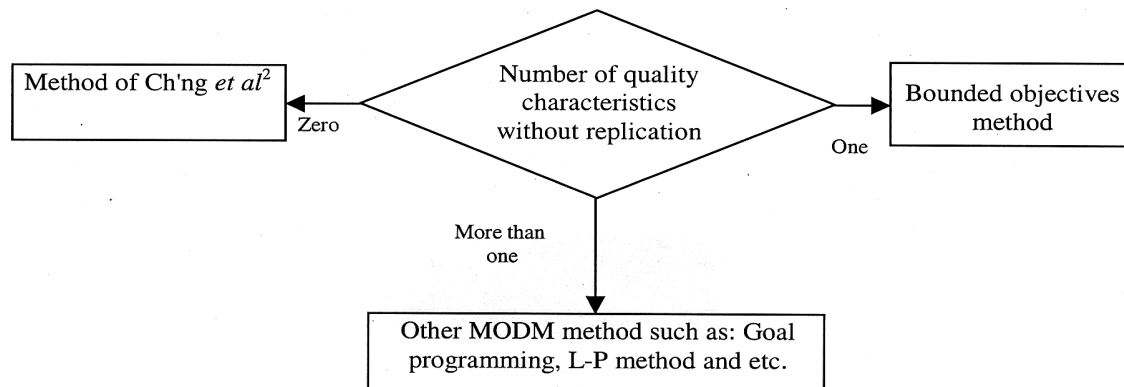


Fig. 2— Proposed flowchart for choosing optimization method

Table 1—Design and results of experiments

Row	NH ₄ Cl%	Thickness of Sn on tube	Temperature of furnace	SN% in alloy	Hours of durability in salt spray		Viscosity		Price
	X ₁	X ₂	X ₃	X ₄					
1	2	18	330	25	330	120	52	50	4367
2	7	18	360	25	360	70	50	45	4368
3	7	23	330	25	330	100	120	117	4448
4	2	23	360	25	360	256	170	159	4447
5	7	18	330	30	330	60	120	110	4382
6	2	18	360	30	360	91	94	90	4381
7	2	23	330	30	330	290	186	178	4449
8	7	23	360	30	360	164	180	176	4500
9	4.5	20.5	345	27.5	345	230	166	160	4433.5
10	4.5	20.5	345	27.5	345	225	165	163	4433.5
11	4.5	20.5	345	27.5	345	232	167	165	4433.5
12	4.5	20.5	345	27.5	345	231	161	166	4433.5
13	4.5	20.5	345	31.04	345	230	172	169	4470.3
14	4.5	20.5	345	23.96	345	215	160	157	4396.7
15	4.5	24.04	345	27.5	345	240	173	174	4490.15
16	4.5	16.96	345	27.5	345	210	155	150	4376.85
17	4.5	20.5	366.2	27.5	366.2	221	171	167	4433.5
18	4.5	20.5	323.8	27.5	323.8	231	157	159	4433.5
19	8.035	20.5	345	27.5	345	207	169	161	4434.2
20	0.965	20.5	345	27.5	345	235	162	159	4432.8

$$g_1(X) = \mu_{durability} = 250.34 - 34.26 X_1 + 43.91 X_2 - 1.55 X_3 + 6.75 X_4 - 24.09 X_1^2 - 22.97 X_2^2 - 21.84 X_3^2 - 23.22 X_4^2 + 4.56 X_1 X_4 + 16.94 X_2 X_4 + 25.31 X_3 X_4$$

$$g_2(x) = \sigma_{durability} = 1.945 - 1.002 X_1 + 1.321 X_2 - 1.827 X_3 + 0.093 X_4 + 0.884 X_1^2 + 0.707 X_2^2 + 0.53 X_3^2 + 1.061 X_4^2 - 0.265 X_1 X_2 - 2.21 X_2 X_3 - 0.265 X_2 X_4$$

$$h_1(x) = \mu_{adherence} = 176.75 - 2.01 X_1 + 30.6 X_2 + 2.59 X_3 + 16.87 X_4 - 13.31 X_1^2 - 13.19 X_2^2 - 12.94 X_3^2 - 12.44 X_4^2 + 8.56 X_1 X_4 - 3.94 X_2 X_4 - 8.69 X_3 X_4$$

$$h_2(x) = \sigma_{adherence} = 2.172 + 0.24 X_1 - 0.039 X_2 + 0.226 X_3 - 0.295 X_4 + 1.099 X_1^2 + 0.215 X_2^2 + 0.215 X_3^2 + 0.215 X_4^2 - 1.856 X_1 X_2 - 0.795 X_2 X_4 + 0.619 X_2 X_3$$

General Goal

$$\text{Min } f(x), \quad \text{Max } g_1(x), \quad \text{Min } g_2(x), \quad \text{Max } h_1(x), \quad \text{Min } h_2(x)$$

Step 2: Convert Quality Characteristics to Capability Process Index

According to control plan of company, minimum durability of radiator in salt spray should be 200 h, and

minimum amount of adherence between fins and tubes of radiator must be 130 units. Thus

$$CP_{k1} = CP_k(durability) = \text{Min} \left\{ \frac{\infty - \mu_{durability}}{3\sigma_{durability}}, \frac{\mu_{durability} - 200}{3\sigma_{durability}} \right\}$$

$$= \frac{\mu_{durability} - 200}{3\sigma_{durability}} = \frac{g_1(X) - 200}{3g_2(X)}$$

$$CP_{k2} = CP_k(adherence) = \text{Min} \left\{ \frac{\infty - \mu_{adherence}}{3\sigma_{adherence}}, \frac{\mu_{adherence} - 130}{3\sigma_{adherence}} \right\}$$

$$= \frac{\mu_{adherence} - 130}{3\sigma_{adherence}} = \frac{h_1(X) - 130}{3h_2(X)}$$

Step 3: Determining Lower Limit of Capability Process Coefficients

According to customer requirements and quality objectives of company, CP_{k1} and CP_{k2} were determined as 2 and 1.4 respectively.

Step 4: Selecting Method of Optimizing Objectives

Since objectives of $g_1(x)$ and $g_2(x)$ are converted into CP_{k1} and objectives of $h_1(x)$ and $h_2(x)$ are changed into CP_{k2} , there is only one objective without lower limit (final price of product). So according to

Table 2—Optimum factor level of process

Coded variables				Actual variable										
x_1	x_2	x_3	x_4	NH4CL	Thickness	Temp.	SN	CP_{k1}	CP_{k2}	$g_1(x)$	$g_2(x)$	$h_1(x)$	$h_2(x)$	$f(x)$
-0.61	-0.81	0.1	-0.21	2.97	18.47	346.5	26.97	2	1.4	212.8	2.13	136.2	1.46	4394.46

proposed algorithm, BMO method should be selected. In BMO, according to DM point of view, most important objective is selected as main objective function and other goals considering their desirability targets are converted into limitations.

Optimize Y

Subject to $l_j \leq Y_j \leq u_j$ and $x \in R$

where Y , main objective; Y_j , secondary objective; l_j and u_j , desirable area of secondary objective; R , design area.

Thus, in this study, mean regression model of cost was chosen as main objective function and QCs of durability and adherence appeared in form of capability process index with lower limit. Technical limitation of factors in form of code also added to constraint.

Min $f(x)$

s.t

$CP_k(\text{durability}) \geq 2$

$CP_k(\text{adherence}) \geq 1.4$

$-1.8 \leq x_1 \leq 3$

$-2.2 \leq x_2 \leq 1.8$

$-1.67 \leq x_3 \leq 1.67$

$-3 \leq x_4 \leq 3$

Table 2 shows result of solving nonlinear problem using GRG algorithm on Microsoft Excel software. In fact, this result is optimum factor level production process in order to achieve minimum production cost and optimum condition for two quality characteristics.

In order to evaluate anticipated improvements under optimum conditions, confirmation experiments were conducted under an optimal factor level combination to verify whether quality performance is enhanced. Results helped to improve quality of products and decrease cost of production in comparison with previous setting of process. Better result may need more factors or more experiments, but proposed method showed that it can be very useful in special case of minimizing mean regression model of cost and simultaneously optimizing regression model of mean and standard deviation of quality characteristics. Applying bounded objective method instead of goal programming or other MCDM methods helped quality and production manager to better build and understand mathematical model in order to make a compromise among objectives.

Conclusions

This paper proposed an algorithm for optimization of quantitative multiple-response problems with and without replication of responses. Using suggested algorithm in a case study, following advantages are gained: 1) Combination of average and standard deviation of each quality characteristics in the form of capability process causes objectives to be combined suitably together and reduced their numbers; 2) According to customer's requirement, lower limits of capability process coefficients are deterministic; and 3) Since suggested method is very simple, it may be readily applied by quality and process engineers to find optimum combination of dual response surface methods and multiple response problems.

References

- 1 Kazemzadeh R B, Bashiri M, Atkinson A C & Noorossana R, A general framework for multiresponse optimization problems based on goal programming, *Eur J Oper Res*, **189** (2008) 421-429.
- 2 Ch'ng C K, Quah S H & Low H C, Index c^*pm in multiple response optimization, *Qual Engg*, **17** (2005) 165-171.

- 3 Montgomery D C, *Design and Analysis of Experiments* (John Willey and Sons, New York) 1997, 388-486.
- 4 Asgharpour M J, *Multiple Criteria Decision making* (Tehran university publications, Tehran) 1999, 144-308.
- 5 Eshlaghy A T, Radfar R & Iau T, A new approach for classification of weighting methods, in *IEEE Int Conf on Management of Innovation and Technology* (IEEE, Singapore) 2006.
- 6 Del Castillo E, Multiresponse process optimization via constrained confidence regions, *J Qual Technol*, **28** (1996) 61-70.
- 7 Ames A E, Mattucci N, Macdonald S, Szonyi G & Hawkins D M, Quality loss functions for optimization across multiple response surfaces, *J Qual Technol*, **29** (1997) 339-346.
- 8 Tong L I & Su C T, Optimizing multi-response problems in the taguchi method by fuzzy multiple attribute decision making, *Qual Reliab Eng Int*, **13** (1997) 25-34.
- 9 Del Castillo E & Montgomery D C, A nonlinear programming solution to the dual response problem, *J Qual Technol*, **25** (1993) 199-204.
- 10 Derringer G & Suich R, Simultaneous optimization of several response variables, *J Qual Technol*, **12** (1980) 214-219.
- 11 Kim K J & Lin D K J, Dual response surface optimization: A fuzzy modeling approach, *J Qual Technol*, **30** (1998), 1-10.
- 12 Myers R H & Carter Jr, W H, Response surface techniques for dual response systems, *Technometrics*, (1973) 301-317.
- 13 Lin D K J & Tu W, Dual response surface optimization, *J Qual Technol*, **27** (1995) 34-39.
- 14 Jeyapaul R, Shahabudeen P & Krishnaiah K, Simultaneous optimization of multi-response problems in the taguchi method using genetic algorithm, *Int J Adv Manuf Technol*, **30** (2006) 870-878.
- 15 Wang Y & He Z, A new tosis based methodology for multiple responses problems, in *Int Conf on Wireless Communications, Networking and Mobile Computing* (IEEE< Dalian) WiCOM 2008, 2008.
- 16 Amiri M & Salehi-Sadaghiani J, A methodology for optimizing statistical multi-response problems using fuzzy goal programming, *Sci Iran*, **15** (2008) 389-397.
- 17 K ksoy O, A nonlinear programming solution to robust multi-response quality problem, *Appl Math Comput*, **196** (2008) 603-612.
- 18 Noorossana R, Tajbakhsh S D & Saghaei A, An artificial neural network approach to multiple-response optimization, *Int J Adv Manuf Technol*, **40** (2009) 1227-1238.
- 19 Kane V E, Process capability indices, *J Qual Technol*, **18** (1986) 41-52.