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Detection, isolation and reconstruction of faulty sensors using principal component analysis[+]

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A strategy based on principal component analysis (PCA) is presented for detection, identification and reconstruction of faulty sensors. In this strategy, sensor fault detection is carried out by using multivariate statistics, faulty sensors are isolated using principal component score contributions and reconstruction of faulty sensors is accomplished through the analysis of fault direction vector. The performance of the strategy is evaluated by applying to a closed-loop controlled CSTR system. The simulation results demonstrate the ability of the strategy for detection, identification and reconstruction of single and multiple faulty sensors.

Keywords: Sensor, principal component analysis, controlled CSTR system

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Performance monitoring and fault diagnosis is an integral part of successful operation of a process. Modern chemical industries are well equipped with measurement sensors, which form a crucial link in providing information to controllers and operators. The sensor measurements are highly correlated under normal operating conditions, thus providing redundancy for sensor fault detection and identification. The correlations in sensor measurements are mostly due to physical and chemical principles governing the process operation. Sensor faults can cause process disturbances, loss of control performance, profit loss or even catastrophic accidents. Hard or catastrophic sensor failures can be detected easily, whereas non-catastrophic sensor degradations are difficult to detect and identify due to correlative interactions. Therefore, automatic detection and identification of sensor faults is an asset for efficient monitoring and control of the process.

In recent years, various techniques have been reported for monitoring and diagnosis of faulty sensors. Luo *et al*.[1] proposed a wavelet based sequential approach for real time sensor fault detection and applied for fault detection in CSTR temperature sensor. This approach mainly focuses on fault detection based on the analysis of a single sensor signal. Ying and Joseph[2] proposed an approach of sensor fault detection using analysis of high frequency noise present in the measurements. This method has been applied for fault detection in temperature sensor of a hot air blower and pressure sensor of a tank system, with single control loop in each system. Dunia *et al*.[3] proposed a method based on principal component analysis for detection and identification of faulty sensors and applied to a boiler process. In their method, squared prediction error (SPE) is used to test the breakdown of sensor correlations, and sensor validity index is used to determine the status of each sensor. Reconstruction of faulty sensors is carried out using PCA based successive substitution and optimisation. This method concentrates on the identification of a single sensor assuming that one sensor fails at a time. Doymaz *et al*.[4] proposed a method involving T^2 and *SPE* statistics for sensor fault detection, correlation coefficients for sensor fault isolation and calibration model for reconstruction of faulty sensors, and applied to a liquid fed ceramic melter. Most of these techniques have been evaluated by applying to open-loop systems or systems with single control loop. In closed loop process operation, the controller's effectiveness depends on getting reliable data from the sensors. If the sensor is faulty, the controller action may become ineffective and can lead to degraded process behaviour. Then it is necessary that its value

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be reconstructed so as to send the correct information to the controller. Moreover, it is also important to diagnose sensors when faults occur simultaneously in more than one sensor in processes with multi-loop control operations.

In this study, a strategy based on principal component analysis (PCA) is presented for detection, identification and reconstruction of faulty sensors. The PCA model is constructed by transforming the high dimensional original data matrix into a representation space of significantly reduced dimension. The PCA model so created based on the data of normal operating conditions is then used as a basis of reference for monitoring and diagnosis of process sensors.

The PCA based strategy of this study employs multivariate statistics for sensor fault detection, principal component score contributions for isolation of faulty sensors and analysis of fault direction vector for reconstruction of faulty sensors. The performance of the proposed strategy is evaluated through simulation by applying to a closed-loop operated non-isothermal CSTR system.

Principal component analysis

Principal component analysis (PCA) is a well-established multivariate statistical process control technique, which builds low dimensional models from the analysis and monitoring of routine process operations[5]. PCA transforms a number of correlated variables into a smaller number of uncorrelated variables called principal components (PCs), which are linear combinations of the measured variables in the data set. The PCA based fault diagnostic approach is useful because it focuses attention on a small number of PCs representing large number of process variables. The PCA model is established by the eigen value decomposition of the covariance matrix of the original data matrix. The data matrix can be denoted by X with rows as samples and columns as variables. The covariance matrix of the normalised data matrix X with m rows and n columns is defined by

$$\text{Covariance}\,(X) = (X^T X) / (m\text{-}1) \qquad \ldots\ (1)$$

The PCA model divides the monitoring space into two orthogonal subspaces, called the principal component (PC) subspace and the residual subspace (RS). Out of these two subspaces, normal data variation will be captured by the PC subspace, and the abnormal variation and random noise may be captured by the RS. Thus, the PCA model decomposes X as the sum of the product of n pairs of vectors, each pair consisting of a $n\times1$ vector called the *loading*s, P_i , and a $m\times1$ vector called the *score*s, T_i plus a residual matrix E:

$$X = T_1P_1^T + T_2\,P_2^T + \ldots\ldots + T_k\,P_k^T + E \qquad \ldots\ (2)$$

where the residual matrix E represents the information not captured by the model. Each of the scores vectors T_i is simply the projection of X onto the new basis vector P_i

$$T_i = X\,P_i \qquad \ldots\ (3)$$

Thus, the PCA forms latent variables (PCs or scores) which are linear combinations of the input variables and the loadings. The first few PCs capture the maximum amount of variance in the measurements. Therefore, it is important to choose the adequate number of PCs to represent the system in an optimal way. Various methods have been reported for selecting the number of PCs[6]. In this study, an average eigen value method is used for selecting the optimum number of PCs.

Strategy for detection, identification and reconstruction of faulty sensors

Fault detection

In PCA, latent variables encapsulate the major variation in the process. Hence, monitoring these variables using various statistical measures can provide indispensable insight about the process characteristics. In this strategy, fault detection is carried out for both PC subspace and RS. The PC subspace is monitored through the use of Hotelling's τ^2 statistic and RS is monitored by using Q statistic[7]. These tests provide better understanding of unanticipated changes in each sensor characteristic.

Hotelling's τ^2 statistic

Having established the PCA model, process behaviour can be referenced against the in-control model, which is built using fault-free data. The new observations can be projected onto the plane defined by loading vectors to obtain their scores. Hotelling's τ^2 statistic determines the measure of variation within the PCA model based on the retained principal components. τ^2 is the sum of normalized squares defined by

$$\tau^2{}_i = t_i\,\lambda_k^{-1}\,t_i^T = x_i\,P_k\,\lambda_k^{-1}\,P_k^T\,x_i^T \qquad \ldots\ (4)$$

If k is the retained principal components, then P_k and T_k represent the corresponding loadings and scores. In the above equation, x_i is the observation vector at i th sampling instant, t_i is the i th row of the retained principal component scores and λ_k is the eigen values corresponding to the retained principal components. In general, the higher the τ^2 value, the more distant is the observation from the mean. The matrix λ^{-1} is a diagonal matrix containing the inverse of eigen values associated with the k th eigen vector. The Hotelling's control chart then plots the sample τ^2 test statistic on the vertical axis and the subgroup identifier on the horizontal axis. An upper control limit on this chart is obtained using F-distribution[8].

Q Statistic

Q statistic is a measure of the amount of variation not captured by the PCA model and it represents the sum of squared errors. The Q statistic is also referred as the squared prediction error (SPE). When a new type of event occurs, this type of event can be detected by computing the SPE of the new observations. This statistic is defined as

$$Q_i = e_i \, e_i^T \qquad \ldots (5)$$

where $e_i = (I - P_k P_k^T)$. Q_i is the sum of squares of each row of E. When the process is in-control, the SPE static represents unstructured residuals that cannot be accounted for by the PCA model. When an unusual event occurs, it results a change in the process mean or variance structure which will be detected by a high value of this statistic. The upper control limits for this statistic can be computed based on the reference data[9].

Fault identification

When sensor faults occur, there will be a change in correlation of the faulty sensor with other sensors. This phenomenon can be used to find the identifiability conditions for a fault. If m is the total number of principal components and k is the number of principal components retained, then the dimension of residual subspace is m-k. Hence, the maximum degree of freedom available for any observation vector is m-k. It has been shown that the necessary condition[10] for identifiability is $m\text{-}k \geq 2$.

Once a fault is detected by multivariable statistical tests, the next task is to isolate the faulty sensors. When these tests violate the confidence control limits, the contribution of the individual variables can be plotted and those variables having large contributions are examined to identify the faulty sensors. Contribution plots can reveal the process variables which make the highest contribution to the PCA model or to the residues. When the SPE statistic violates its upper control limit, the contribution of the individual variables can be plotted and those variables having larger contributions are examined to indicate possible faults. Similarly, if the variation in the model space (τ^2) becomes large, then contribution of each variable (x_i) to a large value of the j th suspect PC score (t_j) is given by

$$\text{Contribution } (x_i) = P_{ij} \, x_i \qquad \ldots (6)$$

where P_{ij} is the weight of the i th variable (x_i) in the jth latent variable P_j, and x_i is the observation vector over the time period in consideration. These values are plotted against variable number. More details can be found in the literature[11].

Sensor fault reconstruction

Once the faulty sensors are identified, the immediate task is to rectify them prior to sending the signal to controllers or in estimating the present state of the process. In this work, fault free sensor information is used for reconstruction of faulty sensors.

The task of fault reconstruction is to best estimate x^*, using the PCA model and fault direction ξ. The sample vector for normal operation is denoted by x^*, which is unknown when a fault occurs. In the presence of a fault, the sample vector x can be represented using a fault direction vector ξ_i, which characterises the effect of fault on the actual measurements,

$$x = x^* + f\,\xi_i \qquad \ldots (7)$$

where ξ_I is the normalised vector, and scalar f represents the magnitude of the fault. The reconstructed vector x_i is obtained by correcting x in the direction ξ_i

$$x_i = x - f_i \, \xi_i \qquad \ldots (8)$$

where f_i is an estimate of the fault magnitude f which measures the displacement in the direction ξ_i. The distance between x_i and the PC subspace is given by the magnitude of residual vector, $\|\tilde{x}_i\|$, or the SPE of x_i:

$$\begin{aligned} \text{SPE}_i &= \|\tilde{x}_i\|^2 \\ &= \|\tilde{x} - f_i \xi_i\|^2 \\ &= \|\tilde{x} - \tilde{f}_i \xi_i^0 \|^2 \end{aligned} \qquad \ldots (9)$$

Here $\tilde{f}_i = f_i \left\| \tilde{\xi}_i \right\|$ and $\tilde{\xi}_i^0 = \frac{\xi_i}{\left\| \xi_i \right\|}$. The minimization of SPE$_i$ leads to,

$$\frac{d\,\text{SPE}_i}{d\tilde{f}_i} = 2\tilde{\xi}_i^{0^T}\left(\tilde{x} - \tilde{f}_i\tilde{\xi}_i^0\right) = 0$$

or $$\tilde{f}_i = \tilde{\xi}_i^{0^T}\tilde{x} = \tilde{\xi}_i^{0^T} x \qquad \dots (10)$$

and $$f_i = \frac{\tilde{f}_i}{\left\| \tilde{\xi}_i \right\|} = \frac{\tilde{\xi}_i^{0^T} x}{\left\| \tilde{\xi}_i \right\|}$$

The reconstructed normal vector is

$$x_i = x - f_i\xi_i = \left(I - \frac{\xi_i\tilde{\xi}_i^{0^T}}{\left\| \tilde{\xi}_i \right\|} \right) x \qquad \dots (11)$$

and $$\tilde{x}_i = \left(I - \tilde{\xi}_i^0 \tilde{\xi}_i^{0^T} \right)\tilde{x}$$

The feasibility of calculating f_i assures the existence of x_i, which is the best estimate for x^* by reconstruction in the direction ξ_i. Further details on faults reconstruction can be obtained from literature[10].

Application system

The sensor diagnosis strategy is evaluated by applying to a closed-loop controlled CSTR system, the schematic of which is shown in Fig.1. The reactor has cross sectional area A with cooling surface area A_c. A single feed stream containing a reactant A with flow rate of Q_F and concentration C_{AF} enters the tank. The reactant A undergoes a first-order exothermic reaction. A coolant flowing through the jacket around the tank cools the reactor contents. The dynamic model equations for the reactor system are:

$$\frac{dC_A}{dt} = -k_0 e^{-E/RT} C_A + \frac{Q_F C_{AF} - C_V\sqrt{h}\; C_A}{Ah}$$

$$\frac{dT}{dt} = \frac{k_0 e^{-E/RT} C_A\left(-\Delta H\right)}{\rho C_p} + \frac{Q_F(T_F - T)}{Ah} + \frac{UA_C(T_C - T)}{\rho C_p Ah} \qquad \dots (12)$$

$$\frac{dT_C}{dt} = \frac{Q_C(T_{CF} - T_C)}{V_C} + \frac{UA_C(T - T_C)}{\rho_C C_{pC} V_C}$$

$$\frac{dh}{dt} = \frac{Q_F - C_v\sqrt{h}}{A}$$

where k_0 is Arrhenius constant; E is the activation energy for the reaction, R is the universal gas constant; C_v is the valve constant; ΔH is the heat of reaction; U is the overall heat transfer coefficient; and ρ, ρ_c, and C_p, C_{PC} are densities and specific heats of reaction and cooling liquid, respectively. The data used in the system is reported elsewhere[12].

The system is controlled by proportional integral (PI) controllers. The concentration C_A is controlled by adjusting the coolant flow rate Q_C, the reactor temperature T is controlled by manipulating the feed flow rate Q_F, and the height of the liquid in the reactor h is controlled by changing the flow rate of the output stream Q. The proportional and integral constants used for the controllers are 0.05 and 100.0 for concentration controller, 0.05 and 100.0 for reactor temperature controller, and 2.0 and 25.0 for the liquid level controller, respectively. The system is simulated with the help of fourth order Runge-Kutta method. In order to represent the actual data, random Gaussian noise of about 5% to the magnitude of the variables is added to the simulated data.

Results and Discussion

The ability of the proposed strategy is evaluated through simulation by applying to a non-isothermal CSTR. The process operation for a time period of 25

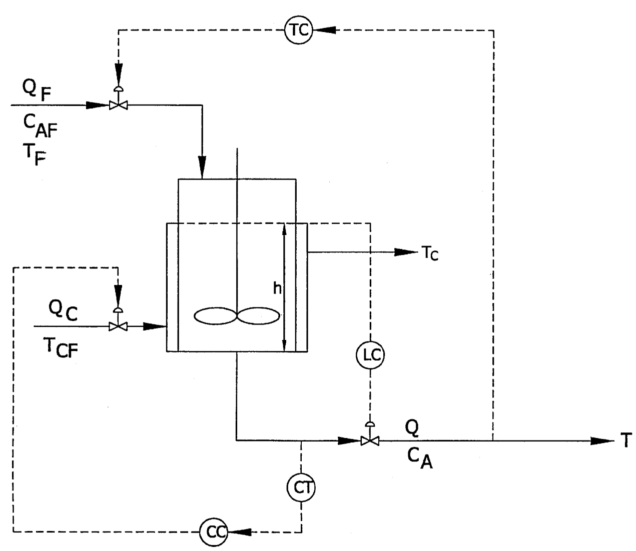

Fig. 1 — Jacketed CSTR with control scheme

min is considered for the analysis and diagnosis of sensors. This operation period corresponds to the treatment of 500 data sets, each data set involving 4 sensor measurements, namely, reactant concentration, reactor temperature, coolant temperature and liquid level in the reactor, respectively. The first 100 data sets of the operation period are considered as fault free data, and used to build the PCA model. The remaining data sets represent faulty operation. Single and multiple sensor faults are introduced in measurement sensors in the form of high percentage noise. Confidence limits for normal operation are obtained assuming that the data is under normal distribution. The measurements of normal operation are corrupted using random Gaussian noise of 5% to the magnitude of the original variables. The sensor measurements below 5% noise limit is considered to be under normal status. Sensors with noisy disturbance of above 5% random Gaussian noise are considered to be faulty. The covariance matrix of the normal data is used to calculate eigen values and eigen vectors. The average eigen value technique is used to evaluate the optimum number of PCs to be retained.

The multivariate control charts, namely, τ^2 chart, Q/SPE chart are used for fault detection. The results of τ^2 statistic and Q statistic for normal status of the process are shown in Fig. 2. The squared prediction error to the Q statistic is calculated for every new observation and plotted against the sampling time. Confidence limit for this Q test is calculated to be 0.002. Fig. 2(a) shows that the normal process operation of the entire run (25min) is below the limit of 0.002. Fig. 2(b) shows the results of Hotelling τ^2 statistic for normal operation. The calculated confidence limit for this test is 4. The results show that the plot is completely below the limit of 4 indicating the normal behaviour of the process. The results of plots shown in Fig. 3 indicate the violation of the confidence limits from normal operation due to sensor faults. Initially the process is under normal operation, and after 5 min of elapsed time, faults are introduced into different sensors. A random Gaussian noise of 10% to the magnitude of original variables is considered to be the case of sensor faults. The results of Q plot and τ^2 plot shown in Fig. 3 clearly indicate the faulty operation. Thus, the τ^2 and Q statistics are able to successfully detect the sensor faults.

Once the fault is detected, the individual contributions of each sensor variable to the suspected principal components are computed to identify the faulty sensors. Isolation of faulty sensors is carried out using the procedure described in the previous section. Fig. 4(a) shows the results of fault isolation when a fault is considered as 20% noise in reactor temperature measurement sensor. Fault is also introduced in more than one sensor simultaneously in a single plant run. The

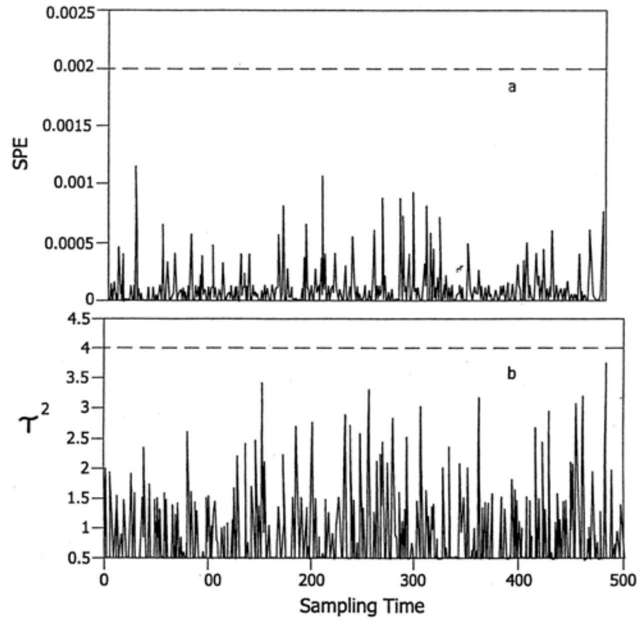

Fig. 2 — Tests for normal operation: (a) Q statistic, (b) τ^2 statistic

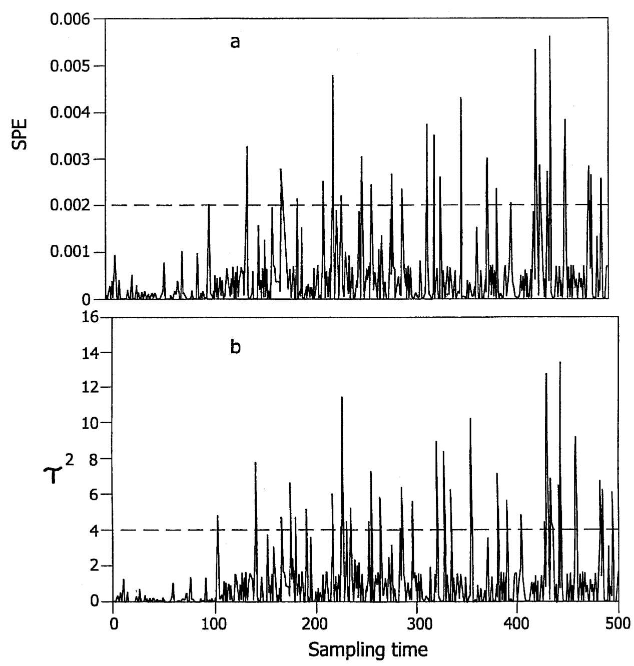

Fig. 3 — Tests for faulty operation: (a) Q statistic, (b) τ^2 statistic

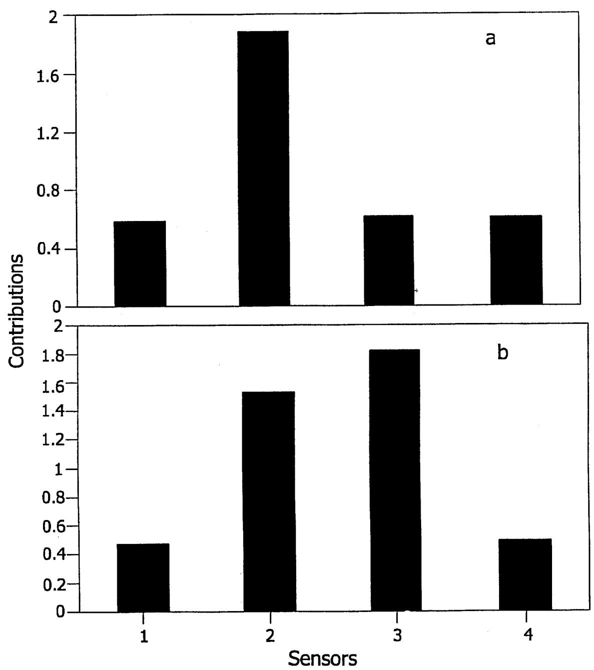

Fig. 4 — Identification of sensor faults: (a) Fault in reactor temperature sensor, (b) faults in reactor and coolant temperature sensors

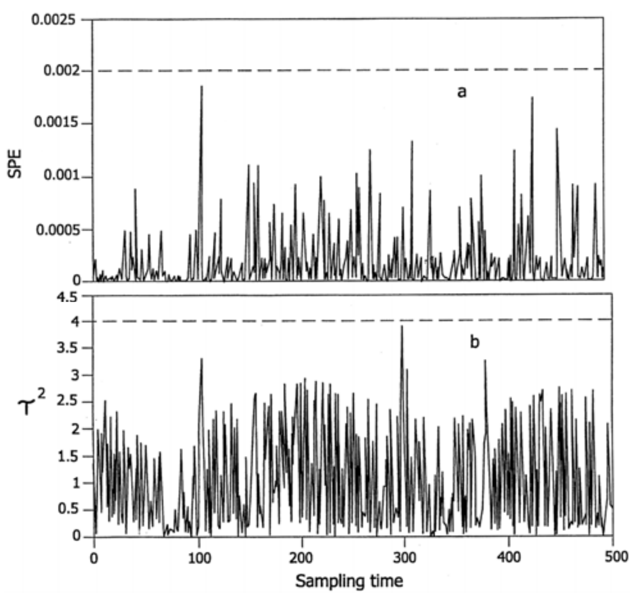

Fig. 5 — Fault detection tests after reconstruction: (a) Q statistic, (b) τ^2 statistic

results of the strategy when faults occur as 20% noise in reactor and coolant temperature measurement sensors are shown in Fig. 4(b). These results show the effectiveness of the diagnostic strategy for identification of single and multiple sensor faults.

Sensor fault rectification is carried out using the reconstruction procedure based on the analysis of fault direction vector as illustrated in the previous section. When reactor temperature sensor is faulty, its reconstruction is carried out using the fault direction vector ξ as [0 1 0 0]. When both the reactor temperature and coolant temperature sensors are faulty, the fault direction vector used is [0 1 1 0]. The sensors after reconstruction are further tested through fault detection tests. The results of these tests shown in Fig. 5 indicate the rectification of sensor faults due to reconstruction. The information on reconstruction is fed to the controllers to maintain the specified process operation. Previously reported methods for sensor fault diagnosis[10,13] employ a procedure in which sensor faults are isolated after reconstruction of all the sensors in the process. The sensor fault diagnostic approach presented in this study employs a procedure in which faulty sensors are diagnosed prior to reconstruction. Thus, by this strategy, only faulty sensors can be reconstructed instead of reconstructing all the sensors. Diagnosis of sensor faults prior to reconstruction has the advantage that the reconstruction can be carried only if fault exists in sensors.

Conclusion

In this study, a principal component analysis based strategy is presented for detection, isolation and reconstruction of faulty sensors. The performance of the strategy is evaluated by applying to a non-isothermal CSTR system. The results demonstrate the effectiveness of the strategy for identification and reconstruction of single as well as multiple sensor faults.

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